Mark Scheme (Results)

Summer 2021

Pearson Edexcel International Advanced Level
In Further Pure Mathematics F2
(WFM02/01)

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General Marking Guidance

- All candidates must receive the same treatment. Examiners must mark the first candidate in exactly the same way as they mark the last.
- Mark schemes should be applied positively. Candidates must be rewarded for what they have shown they can do rather than penalised for omissions.
- Examiners should mark according to the mark scheme not according to their perception of where the grade boundaries may lie.
- There is no ceiling on achievement. All marks on the mark scheme should be used appropriately.
- All the marks on the mark scheme are designed to be awarded. Examiners should always award full marks if deserved, i.e. if the answer matches the mark scheme. Examiners should also be prepared to award zero marks if the candidate's response is not worthy of credit according to the mark scheme.
- Where some judgement is required, mark schemes will provide the principles by which marks will be awarded and exemplification may be limited.
- When examiners are in doubt regarding the application of the mark scheme to a candidate's response, the team leader must be consulted.
- Crossed out work should be marked UNLESS the candidate has replaced it with an alternative response.

PEARSON EDEXCEL IAL MATHEMATICS

General Instructions for Marking

1. The total number of marks for the paper is 75

2. The Edexcel Mathematics mark schemes use the following types of marks:

- **M** marks: Method marks are awarded for 'knowing a method and attempting to apply it', unless otherwise indicated.
- **A** marks: Accuracy marks can only be awarded if the relevant method (M) marks have been earned.
- **B** marks are unconditional accuracy marks (independent of M marks)
- Marks should not be subdivided.

3. Abbreviations

These are some of the traditional marking abbreviations that will appear in the mark schemes.

- bod – benefit of doubt
- ft – follow through
- the symbol $\sqrt{}$ will be used for correct ft
- cao – correct answer only
- cso - correct solution only. There must be no errors in this part of the question to obtain this mark
- isw – ignore subsequent working
- awrt – answers which round to
- SC: special case
- oe – or equivalent (and appropriate)
- d… or dep – dependent
- indep – independent
- dp decimal places
- sf significant figures
- $\ast$ The answer is printed on the paper or ag- answer given
- $\sqsubset$ or d… The second mark is dependent on gaining the first mark

4. All A marks are 'correct answer only' (cao.), unless shown, for example, as A1 ft to indicate that previous wrong working is to be followed through. After a misread however, the subsequent A marks affected are treated as A ft, but manifestly absurd answers should never be awarded A marks.

5. For misreading which does not alter the character of a question or materially simplify it, deduct two from any A or B marks gained, in that part of the question affected.

6. If a candidate makes more than one attempt at any question:
   - If all but one attempt is crossed out, mark the attempt which is NOT crossed out.
   - If either all attempts are crossed out or none are crossed out, mark all the attempts and score the highest single attempt.

7. Ignore wrong working or incorrect statements following a correct answer.

General Principles for Further Pure Mathematics Marking

(But note that specific mark schemes may sometimes override these general principles).

Method mark for solving 3 term quadratic:

1. Factorisation

$(x^2+bx+c)=(x+p)(x+q),\ \text{where}\ |pq|=|c|$, leading to x = ...

$(ax^2+bx+c)=(mx+p)(nx+q),\ \text{where}\ |pq|=|c|\ \text{and}\ |mn|=|a|$, leading to x = ...

2. Formula

Attempt to use the correct formula (with values for a, b and c).

3. Completing the square

Solving $x^2+bx+c=0$: $\left(x\pm\frac{b}{2}\right)^2 \pm q \pm c = 0,\ \ q\neq 0$, leading to x = ...

Method marks for differentiation and integration:

1. Differentiation

Power of at least one term decreased by 1. ($x^n \rightarrow x^{n-1}$)

2. Integration

Power of at least one term increased by 1. ($x^n \rightarrow x^{n+1}$)

Use of a formula

Where a method involves using a formula that has been learnt, the advice given in recent examiners' reports is that the formula should be quoted first.

Normal marking procedure is as follows:

Method mark for quoting a correct formula and attempting to use it, even if there are small errors in the substitution of values.

Where the formula is not quoted, the method mark can be gained by implication from correct working with values, but may be lost if there is any mistake in the working.

Exact answers

Examiners' reports have emphasised that where, for example, an exact answer is asked for, or working with surds is clearly required, marks will normally be lost if the candidate resorts to using rounded decimals.

Summary of changes from Provisional Mark Scheme

A few minor changes were made to the Mark Scheme before marking on the marking service began.

Question Number	**Summary of changes**
Q01b	Totals for 1(b) and question corrected.
Q06	An alternative method was added. This was seen after standardization and was considered to satisfy the demands of the question and so was worthy of marks
Q08	Final mark should not have been “follow through”

Question Number	Scheme	Marks
1		
1(a)	$\dfrac{2}{r(r+1)(r-1)} = \dfrac{1}{r-1} - \dfrac{2}{r} + \dfrac{1}{r+1}$	M1A1A1 (3)
1(b)	$r=2 \quad 1-\dfrac{2}{2}+\dfrac{1}{3}$	
	$r=3 \quad \dfrac{1}{2}-\dfrac{2}{3}+\dfrac{1}{4}$	
	$r=4 \quad \dfrac{1}{3}-\dfrac{2}{4}+\dfrac{1}{5}$	M1
	$r=n-1 \quad \dfrac{1}{n-2}-\dfrac{2}{n-1}+\dfrac{1}{n}$	
	$r=n \quad \dfrac{1}{n-1}-\dfrac{2}{n}+\dfrac{1}{n+1}$	M1
	$\sum_{r=2}^{n}\left(\dfrac{1}{r-1}-\dfrac{2}{r}+\dfrac{1}{r+1}\right)=\left(1-\dfrac{2}{2}+\dfrac{1}{2}+\dfrac{1}{n}-\dfrac{2}{n}+\dfrac{1}{n+1}\right)$	A1
	$\dfrac{1}{2}\sum_{r=1}^{n}\dfrac{2}{r(r+1)(r-1)}=\dfrac{1}{2}\times\left(\dfrac{1}{2}-\dfrac{1}{n}+\dfrac{1}{n+1}\right)=\dfrac{n^2+n-2}{4n(n+1)}$	dM1A1 (5)
		[8]
(a)		
M1	Attempt PFs by any valid method (by implication if 3 correct fractions seen)	
A1A1	A1 any 2 fractions correct; A1 third fraction correct	
(b)		
	Method of differences with at least 3 terms at start and 2 at end OR 2 at start and 3 at end.	
M1	Must start at 2 and end at *n* One M mark for the initial terms and a second for the final.	
M1	Last lines may be missing $k/(n-1)$ and $c/(n-2)$ These 2 M marks may be implied by a correct extraction of terms. If starting from 1, M0M1 can be awarded.	
A1	Extract the remaining terms. $1-2/2$ may be missing and $1/n - 2/n$ may be combined	
dM1	Include the 1/2 and attempt a common denominator of the required form. Depends on both previous M marks	
A1	$\dfrac{n^2+n-2}{4n(n+1)}$	

Question Number	Scheme	Marks
1(a)	**Special Case:** $\frac{2}{r\left(r^2-1\right)}=\frac{2r}{r^2-1}-\frac{2}{r}$ seen, award M1A1A0 Award M1A0A0 provided of the form $\frac{2}{r\left(r^2-1\right)}=\frac{Ar}{r^2-1}-\frac{B}{r}$	
1(b)	Terms listed as described above – award M1M1. Further progress unlikely as too many terms needed to establish the cancellation.	

Question Number	Scheme	Marks
2	$w = \dfrac{z+2}{z-\mathrm{i}} \quad z \neq \mathrm{i}$	
	$z = \dfrac{2+\mathrm{i}w}{w-1}$	M1
	$\lvert z\rvert = 2 \Rightarrow \left\lvert \dfrac{2+\mathrm{i}w}{w-1} \right\rvert = 2 \Rightarrow \lvert 2+\mathrm{i}w\rvert = 2\lvert w-1\rvert$	
	$\lvert 2+\mathrm{i}u-v\rvert = 2\lvert u+\mathrm{i}v-1\rvert$	M1 A1
	$(2-v)^2+u^2 = 4\left((u-1)^2+v^2\right)$	M1 A1
	$3u^2+3v^2-8u+4v=0$ oe	
	$\left(u-\dfrac{4}{3}\right)^2+\left(v+\dfrac{2}{3}\right)^2=\dfrac{20}{9}$ or $u^2+v^2-\dfrac{8}{3}u+\dfrac{4}{3}v=0$	dM1
	(i) centre is $\left(\dfrac{4}{3},-\dfrac{2}{3}\right)$	A1
	(ii) radius is $\dfrac{2\sqrt{5}}{3}$ oe	A1 [8]
M1	Rearrange equation to $z=...$	
M1	Change w to u + iv and use $\lvert z\rvert=2$ Allow if a different pair of letters used.	
A1	Correct equation	
M1	Correct use of Pythagoras on either side. Allow with 2 or 4 (RHS)	
A1	Correct unsimplified equation	
dM1	Attempt the circle form. Coefficients for u^2 and v^2 must be 1. Depends on all 3 previous M marks	
(i)A1	Correct centre given (no decimals) (Use of rounded decimals changes the values)	
(ii)A1	Correct radius given, any equivalent form (but no decimals)	
	NB: These 2 A marks can only be awarded if the results have been deduced from a correct circle equation.	
ALT 1		
M1	Change w to u + iv Allow a different pair of letters.	
M1	Rearrange equation to $z=...$ and use $\lvert z\rvert=2$	
A1	Correct equation Then as above.	
ALT 2	Very rare but may be seen:	
	i maps to $\infty \Rightarrow \pm 2\mathrm{i}$ map to a diameter of C	M1A1
	So $\dfrac{2\mathrm{i}+2}{\mathrm{i}}$ and $\dfrac{-2\mathrm{i}+2}{-3\mathrm{i}}$ are ends of a diameter	M2A1
	Calculate centre and radius	M1A1A1

Question Number	Scheme	Marks
3(a)	$y = r\sin\theta = \sin\theta + \sin\theta\cos\theta$ OR $r\sin\theta = \sin\theta + \frac{1}{2}\sin 2\theta$	B1
	$\frac{dy}{d\theta} = \cos\theta - \sin^2\theta + \cos^2\theta$ OR $\frac{dy}{d\theta} = \cos\theta + \cos 2\theta$ $0 = \cos\theta + 2\cos^2\theta - 1 \quad = (2\cos\theta - 1)(\cos\theta + 1)$	M1
	$\cos\theta = \frac{1}{2}$ $(\cos\theta = -1$ outside range for $\theta)$ $\theta = \frac{\pi}{3}$	M1
	A is $\left(1\frac{1}{2}, \frac{\pi}{3}\right)$	A1 (4)
3(b)	Area $= \frac{1}{2}\int_0^{\frac{\pi}{3}} (1+\cos\theta)^2 d\theta$	B1
	$= \frac{1}{2}\int \left(1 + 2\cos\theta + \frac{1}{2}(\cos 2\theta + 1)\right) d\theta$	M1A1
	$= \frac{1}{2}\left[\frac{3}{2}\theta + 2\sin\theta + \frac{1}{4}\sin 2\theta\right]_0^{\frac{\pi}{3}}$	dM1A1
	$= \frac{\pi}{4} + \frac{\sqrt{3}}{2} + \frac{\sqrt{3}}{16} = \frac{\pi}{4} + \frac{9\sqrt{3}}{16}$	A1 (6) [10]
(a) **B1**	Use of $r\sin\theta$ Award if not seen explicitly but a correct result following use of double angle formula is seen.	
M1	Differentiate $r\sin\theta$ or $r\cos\theta$	
M1	Set $\frac{d(r\sin\theta)}{d\theta} = 0$ and solve the resulting equation. Only the solution used need be shown.	
A1	Correct coordinates of A	
(b)B1	Use of Area $= \frac{1}{2}\int r^2 d\theta$ with $r = 1 + \cos\theta$, limits not needed.	
M1	Attempt $(1+\cos\theta)^2$ (minimum accepted is $(1 + k\cos\theta + \cos^2\theta)$) and change $\cos^2\theta$ to an expression in $\cos 2\theta$ using $\cos^2\theta = \frac{1}{2}(\pm\cos 2\theta \pm 1)$	
A1	Correct integrand; limits not needed. $\frac{1}{2}$ may be missing.	
dM1	Attempt to integrate all terms. $\cos 2\theta \to \pm\frac{1}{k}\sin 2\theta$ $k = \pm 1$ or ± 2 Limits not needed. Depends on the previous M mark	
A1	Correct integration and correct limits seen	
A1	Substitute correct limits and obtain the correct answer in the required form.	

Question Number	Scheme	Marks
	Alternative for (b) *using integration by parts* (Very rare but may be seen)	
	Area $=\frac{1}{2}\int_0^{\frac{\pi}{3}}(1+\cos\theta)^2\mathrm{d}\theta$	B1
	$=\frac{1}{2}\left[\int(1+2\cos\theta)\mathrm{d}\theta+\int\cos^2\theta\,\mathrm{d}\theta\right]$	
	$=\frac{1}{2}\left[\int(1+2\cos\theta)\mathrm{d}\theta+\cos\theta\sin\theta+\int\sin^2\theta\,\mathrm{d}\theta\right]$	M1A1
	$=\frac{1}{2}\left[\theta+2\sin\theta+\sin\theta\cos\theta+\int(1-\cos^2\theta)\mathrm{d}\theta\right]_0^{\frac{\pi}{3}}$	
	$=\frac{1}{2}\left[\theta+2\sin\theta+\frac{1}{2}(\sin\theta\cos\theta+\theta)\right]_0^{\frac{\pi}{3}}$	dM1A1
	$=\frac{\pi}{4}+\frac{\sqrt{3}}{2}+\frac{\sqrt{3}}{16}=\frac{\pi}{4}+\frac{9\sqrt{3}}{16}$	A1
B1	Use of Area $=\frac{1}{2}\int r^2\mathrm{d}\theta$ with $r=1+\cos\theta$, limits not needed.	
M1	Attempt $(1+\cos\theta)^2$ (minimum accepted is $(1+k\cos\theta+\cos^2\theta)$) and attempt first stage of $\int\cos^2\theta\,\mathrm{d}\theta$ by parts. Reach $\int\cos^2\theta\,\mathrm{d}\theta=\cos\theta\sin\theta\pm\int\sin^2\theta\,\mathrm{d}\theta$ Limits not needed	
A1	Correct so far. Limits not needed.	
dM1	Attempt to integrate all terms. $\int(1+2\cos\theta)\mathrm{d}\theta$ **and** attempt to complete $\int\cos^2\theta\,\mathrm{d}\theta$ using Pythagoras identity. Limits not needed. Depends on the previous M mark	
A1A1	As main scheme	

Question Number	Scheme	Marks
4 (a)	$\frac{\mathrm{d}^2y}{\mathrm{d}x^2}=\frac{4}{y}\left(\frac{\mathrm{d}y}{\mathrm{d}x}\right)^2-3$	M1
	$\frac{\mathrm{d}^3y}{\mathrm{d}x^3}=-\frac{4}{y^2}\left(\frac{\mathrm{d}y}{\mathrm{d}x}\right)^3+\frac{8}{y}\times\frac{\mathrm{d}^2y}{\mathrm{d}x^2}\times\frac{\mathrm{d}y}{\mathrm{d}x}$	M1A1A1
	$\frac{\mathrm{d}^3y}{\mathrm{d}x^3}=-\frac{4}{y^2}\left(\frac{\mathrm{d}y}{\mathrm{d}x}\right)^3+\frac{8}{y}\left(\frac{4}{y}\left(\frac{\mathrm{d}y}{\mathrm{d}x}\right)^2-3\right)\left(\frac{\mathrm{d}y}{\mathrm{d}x}\right)$	
	$\frac{\mathrm{d}^3y}{\mathrm{d}x^3}=\frac{28}{y^2}\left(\frac{\mathrm{d}y}{\mathrm{d}x}\right)^3-\frac{24}{y}\left(\frac{\mathrm{d}y}{\mathrm{d}x}\right)$ *	A1* (5)
ALT	$\frac{\mathrm{d}y}{\mathrm{d}x}\frac{\mathrm{d}^2y}{\mathrm{d}x^2}+y\frac{\mathrm{d}^3y}{\mathrm{d}x^3}-8\frac{\mathrm{d}y}{\mathrm{d}x}\times\frac{\mathrm{d}^2y}{\mathrm{d}x^2}+3\frac{\mathrm{d}y}{\mathrm{d}x}=0$	M1A1A1
	$\frac{\mathrm{d}^3y}{\mathrm{d}x^3}=\frac{1}{y}\left(7\frac{\mathrm{d}y}{\mathrm{d}x}\right)\left(\frac{4}{y}\left(\frac{\mathrm{d}y}{\mathrm{d}x}\right)^2-3\right)-\frac{3}{y}\frac{\mathrm{d}y}{\mathrm{d}x}$	M1
	$\frac{\mathrm{d}^3y}{\mathrm{d}x^3}=\frac{28}{y^2}\left(\frac{\mathrm{d}y}{\mathrm{d}x}\right)^3-\frac{24}{y}\left(\frac{\mathrm{d}y}{\mathrm{d}x}\right)$ *	A1* (5)
4(b)	At $x=0$ $\frac{\mathrm{d}^2y}{\mathrm{d}x^2}=\frac{4}{8}(1)^2-3=-\frac{5}{2}$ oe	B1
	$\frac{\mathrm{d}^3y}{\mathrm{d}x^3}=\frac{28}{64}\times1^3-\frac{24}{8}\times1=-\frac{41}{16}$	M1
	$y=8+x-\frac{5}{2}\times\frac{x^2}{2!}-\frac{41}{16}\times\frac{x^3}{3!}+\ldots$	M1
	$y=8+x-\frac{5}{4}x^2-\frac{41}{96}x^3+\ldots$	A1 (4) [9]

Question Number	Scheme	Marks
5(a)		
M1	Divide through by y No need to re-arrange the equation until later	
M1	Attempt the differentiation using product rule and chain rule and obtain $\frac{d^3y}{dx^3} = \ldots$	
A1A1	A1 Either RHS term correct A1 Second RHS term correct and no extras	
A1*	Eliminate $\frac{d^2y}{dx^2}$ and obtain the **given** result	
ALT		
M1	Re-arrange the equation (Will probably be seen later in work)	
M1	Attempt the differentiation using product rule and chain rule	
A1A1	A1 Two terms correct A1 All correct and no extras	
A1*	Eliminate $\frac{d^2y}{dx^2}$ and obtain the correct result	
5(b)B1	Correct value for $\frac{d^2y}{dx^2}$	
M1	Use the *given* expression from (a) to obtain a value for $\frac{d^3y}{dx^3}$ Award if correct value seen.	
M1	Taylor's series formed using their values for the derivatives (2! or 2, 3! or 6)	
A1	Correct series, must start (or end) $y = \ldots$ Correct terms must be seen, order may be different. Can have f(x) = ... provided f(x) = y is defined somewhere.	

Question Number	Scheme	Marks
5		
NB	Question states "Use algebra..." so purely graphical solutions (using calculator?) score 0/7. A sketch and some algebra to find intersection points can score.	
	$2x^2+x-3\geq 0$ $2x^2+x-3=3(1-x)\Rightarrow 2x^2+4x-6=0$	M1
	$2x^2+4x-6\Rightarrow x^2+2x-3=(x+3)(x-1)=0$ $x=-3,\ 1$	A1
	$2x^2+x-3\leq 0$	
	$-2x^2-x+3=3(1-x)\Rightarrow 2x^2-2x=0$	M1
	$2x(x-1)=0,\ \ x=0,\ 1$	A1
	$x<-3\quad 0<x<1\quad x>1$	dM1A1A1 [7]
	The first 4 marks can be awarded with any inequality sign or =	
M1	Assume $2x^2+x-3\geq 0$ and obtain a 3TQ	
A1	Correct CVs obtained from a correct equation.	
M1	Assume $2x^2+x-3\leq 0$ and obtain a 2 or 3TQ	
A1	Correct CVs obtained from a correct equation.	
dM1	Form 3 distinct inequalities with their 3 CVs. Can have $<$ or $\leq$, $>$ or $\geq$. Must have scored both previous M marks. Accept $x<-3\quad 0<x\quad x\neq 1$	
A1	All 3 correct CVs used correctly	
A1	Inequalities fully correct. "and" between the inequalities is acceptable. If $\cap$ is used, award A0 here. Fully correct set language accepted.	
ALT	*Squaring both sides* $\left(2x^2+x-3\right)^2>9(1-x)^2$	
	$4x^4+4x^3-20x^2+12x>0$	M1A1
	$x(x+3)(x-1)(x-1)>0$	M1
	$\text{CVs}:\ x=0,-3,1$	A1
	Then as main scheme	
	These 4 marks can be awarded with any inequality sign or =	
M1	Square both sides and collect terms to obtain a quartic with 4 or 5 terms	
A1	Correct quartic	
M1	Factorise their quartic	
A1	3 correct CVs	

1

Question Number	Scheme	Marks
6(a)	$m^2 - 6m + 8 = 0$	
	$(m-2)(m-4) = 0,\ m = 2, 4$	M1
	$(\text{CF} =)\quad Ae^{2x} + Be^{4x}$	A1
	$\text{PI}:\ y = \lambda x^2 + \mu x + \nu$	B1
	$y' = 2\lambda x + \mu \quad y'' = 2\lambda$	
	$2\lambda - 6(2\lambda x + \mu) + 8(\lambda x^2 + \mu x + \nu) = 2x^2 + x$	M1
	$\lambda = \frac{1}{4},\ -12\lambda + 8\mu = 1,\ \ 2\lambda - 6\mu + 8\nu = 0$	M1
	$\lambda = \frac{1}{4},\ \mu = \frac{1}{2},\ \nu = \frac{5}{16}$	A1A1
	$y = Ae^{2x} + Be^{4x} + \frac{1}{4}x^2 + \frac{1}{2}x + \frac{5}{16}$	A1ft (8)
(a)M1	Form aux equation and attempt to solve (any valid method). Equation need not be shown if CF is correct or complete solution $(m = 2, 4)$ is shown	
A1	Correct CF $y =$.. not needed.	
B1	Correct form for PI	
M1	Their PI (minimum 2 terms) differentiated twice and substituted in the equation	
M1	Coefficients equated	
A1	Any 2 values correct	
A1	All 3 values correct	
A1ft	A complete solution, follow through their CF and PI. All 3 M marks must have been earned. Must start $y =$...	
6(b)	$y = Ae^{2x} + Be^{4x} + \frac{1}{4}x^2 + \frac{1}{2}x + \frac{5}{16}$	
	$1 = A + B + \frac{5}{16}$	M1
	$\frac{dy}{dx} = 2Ae^{2x} + 4Be^{4x} + \frac{1}{2}x + \frac{1}{2} \qquad 0 = 2A + 4B + \frac{1}{2}$	M1
	$A = \frac{13}{8} \quad B = -\frac{15}{16}$ oe	dM1A1
	$y = \frac{13}{8}e^{2x} - \frac{15}{16}e^{4x} + \frac{1}{4}x^2 + \frac{1}{2}x + \frac{5}{16}$ oe	A1ft (5)
		[13]
(b)		
M1	Substitute $y = 1$ and $x = 0$ in their complete solution from (a)	
M1	Differentiate and substitute $\frac{dy}{dx} = 0$, $x = 0$	
dM1	Solve the 2 equations to $A =$... or $B =$ .... Depends on the two previous M marks	
A1	Both values correct	
A1ft	Particular solution, follow through their general solution and A and B. Must start $y =$ ...	

Question Number	Scheme	Marks
7(a)	$\left(\cos\theta+\mathrm{i}\sin\theta\right)^4=\cos 4\theta+\mathrm{i}\sin 4\theta$	
	$\cos^4\theta+4\cos^3\theta\left(\mathrm{i}\sin\theta\right)+\frac{4\times3}{2!}\cos^2\theta\left(\mathrm{i}\sin\theta\right)^2+\frac{4\times3\times2}{3!}\cos\theta\left(\mathrm{i}\sin\theta\right)^3+\left(\mathrm{i}\sin\theta\right)^4$	M1
	$=\cos^4\theta+4\mathrm{i}\cos^3\theta\sin\theta+\mathrm{i}^2 6\cos^2\theta\sin^2\theta+4\mathrm{i}^3\cos\theta\sin^3\theta+\mathrm{i}^4\sin^4\theta$	A1
	$\cos 4\theta=\cos^4\theta-6\cos^2\theta\sin^2\theta+\sin^4\theta$	M1
	$\sin 4\theta=4\cos^3\theta\sin\theta-4\cos\theta\sin^3\theta$	A1
	$\tan 4\theta=\frac{\sin 4\theta}{\cos 4\theta}=\frac{4\cos^3\theta\sin\theta-4\cos\theta\sin^3\theta}{\cos^4\theta-6\cos^2\theta\sin^2\theta+\sin^4\theta}$	
	$\tan 4\theta=\frac{\sin 4\theta}{\cos 4\theta}=\frac{4\tan\theta-4\tan^3\theta}{1-6\tan^2\theta+\tan^4\theta}$ *	M1A1* (6)
7(b)	$x=\tan\theta \qquad \frac{2\tan\theta-2\tan^3\theta}{1-6\tan^2\theta+\tan^4\theta}=\frac{1}{2}\tan 4\theta=1$	
	$\tan 4\theta=2$	M1
	$x=\tan\theta=0.284,\ 1.79$	A1A1 (3) [9]
(a)		
M1	Correct use of de Moivre and attempt the complete expansion	
A1	Correct expansion. Coefficients to be single numbers but powers of i may still be present.	
M1	Equate the real and imaginary parts	
A1	Correct expressions for $\cos 4\theta$ and $\sin 4\theta$	
M1	Use $\tan 4\theta=\frac{\sin 4\theta}{\cos 4\theta}$ and divide numerator and denominator by $\cos^4\theta$ Only tangents now.	
A1*	Correct **given** answer, no errors seen.	
(b)		
M1	Substitute $x=\tan\theta$ and re-arrange to $\tan 4\theta=\pm 2$ or $\pm\frac{1}{2}$	
A1A1	A1 for either solution; A2 for both. Deduct one mark only for failing to round either or both to 3 sf (One correct answer but not rounded scores A0A0; two correct answers neither rounded scores A1A0; two correct answers, only one rounded, scores A1A0)	

Question Number	Scheme	Marks
	Alternative for first 4 marks of 7(a):	
	$\sin 4\theta = \frac{1}{2i}\left(z^4 - z^{-4}\right) = \frac{1}{2i}\left(\left(\cos\theta - i\sin\theta\right)^4 - \left(\cos\theta + i\sin\theta\right)^{-4}\right)$	M1
	$= \frac{1}{2i}\left(\cos^4\theta + 4i\cos^3\theta\sin\theta - 6\cos^2\theta\sin^2\theta - 4i\cos\theta\sin^3\theta + \sin^4\theta\right)$	M1
	$-\frac{1}{2i}\left(-\cos^4\theta + 4i\cos^3\theta\sin\theta + 6\cos^2\theta\sin^2\theta - 4i\cos\theta\sin^3\theta - \sin^4\theta\right)$	
	$= 4\cos^3\theta\sin\theta - 4\cos\theta\sin^3\theta$	A1
	Similar work leads to $\cos 4\theta = \cos^4\theta - 6\cos^2\theta\sin^2\theta + \sin^4\theta$	A1
	Remaining 2 marks as main scheme	
M1	For the expression derived from de Moivre for **either** $\sin 4\theta$ or $\cos 4\theta$	
A1	Both shown and correct	
M1	Attempt the binomial expansion for either, reaching a simplified expression	
A1	Both simplified expressions correct	

1

Question Number	Scheme	Marks
8(a)	$v = y^{-2} \quad \frac{dv}{dy} = -2y^{-3}$	B1
	$\frac{dy}{dx} = \frac{dy}{dv} \times \frac{dv}{dx} = -\frac{y^3}{2}\frac{dv}{dx}$	M1A1
	$-\frac{y^3}{2}\frac{dv}{dx} + 6xy = 3xe^{x^2}y^3$	
	$\frac{1}{2}\frac{dv}{dx} - \frac{6xy}{y^3} = -3xe^{x^2}$	
	$\frac{dv}{dx} - 12vx = -6xe^{x^2}$ *	dM1A1* (5)
ALT 1	$y = v^{-\frac{1}{2}} \quad \frac{dy}{dv} = -\frac{1}{2}v^{-\frac{3}{2}}$	B1
	$\frac{dy}{dx} = \frac{dy}{dv} \times \frac{dv}{dx} = -\frac{1}{2}v^{-\frac{3}{2}}\frac{dv}{dx}$	M1A1
	$-\frac{1}{2}v^{-\frac{3}{2}}\frac{dv}{dx} + 6xv^{-\frac{1}{2}} = 3xe^{x^2}v^{-\frac{3}{2}}$	dM1
	$-\frac{1}{2}\frac{dv}{dx} + 6xv = 3xe^{x^2}$	
	$\frac{dv}{dx} - 12vx = -6xe^{x^2}$ *	A1* (5)
ALT 2	$v = y^{-2} \quad \frac{dv}{dy} = -2y^{-3}$	B1
	$\frac{dv}{dx} = \frac{dv}{dy} \times \frac{dy}{dx} = -2y^{-3}\frac{dy}{dx}$	M1A1
	$-2y^{-3}\frac{dy}{dx} - 12y^{-2}x = -6xe^{x^2}$	dM1
	$\frac{dy}{dx} + 6xy = 3xe^{x^2}y^3 \qquad x > 0$	A1* (5)
8(a)	**All Methods:**	
B1	Correct derivative	
M1	Attempt $\frac{dy}{dx}$ or $\frac{dv}{dx}$ using the chain rule	
A1	Correct derivative	
dM1	Substitute in equation (I) to obtain an equation in v and x only OR in equation (II) to obtain an equation in x and y only (ALT 2)	
A1*	Correct completion with no errors seen	

Question Number	Scheme	Marks
8(b)	IF: $e^{\int -12x\,dx} = e^{-6x^2}$	M1A1
	$ve^{-6x^2} = \int -6xe^{x^2} \times \left(e^{-6x^2}\right)dx = \int -6xe^{-5x^2}\,dx$	dM1
	$ve^{-6x^2} = \frac{6}{10}e^{-5x^2}\ (+c)$	A1
	$v\left(= y^{-2}\right) = \frac{6}{10}e^{x^2} + ce^{6x^2}$	ddM1
	$y^2 = \dfrac{1}{\frac{6}{10}e^{x^2} + ce^{6x^2}}$ oe eg $y^2 = \dfrac{10}{6e^{x^2} + ke^{6x^2}}$	A1 (6) **[11]**
(b)		
M1	IF of form $e^{\int \pm 12x\,dx}$ and attempt the integration.	
A1	Correct IF	
dM1	Multiply through by their IF and integrate the LHS. Depends on first M mark of (b)	
A1	Correct integration of the complete equation with or without constant	
ddM1	Include the constant and multiply through by e^{6x^2} Depends on both previous M marks of (b)	
A1	Any equivalent to that shown. (No need to change letter used for constant when rearranging)	