Mark Scheme (Results)

Summer 2016

Pearson Edexcel IAL in Further Pure Mathematics 2 (WFM02/01)

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General Marking Guidance

- All candidates must receive the same treatment. Examiners must mark the first candidate in exactly the same way as they mark the last.
- Mark schemes should be applied positively. Candidates must be rewarded for what they have shown they can do rather than penalised for omissions.
- Examiners should mark according to the mark scheme not according to their perception of where the grade boundaries may lie.
- There is no ceiling on achievement. All marks on the mark scheme should be used appropriately.
- All the marks on the mark scheme are designed to be awarded. Examiners should always award full marks if deserved, i.e. if the answer matches the mark scheme. Examiners should also be prepared to award zero marks if the candidate's response is not worthy of credit according to the mark scheme.
- Where some judgement is required, mark schemes will provide the principles by which marks will be awarded and exemplification may be limited.
- When examiners are in doubt regarding the application of the mark scheme to a candidate's response, the team leader must be consulted.
- Crossed out work should be marked UNLESS the candidate has replaced it with an alternative response.

PEARSON EDEXCEL IAL MATHEMATICS

General Instructions for Marking

1. The total number of marks for the paper is 75

2. The Edexcel Mathematics mark schemes use the following types of marks:

- **M** marks: Method marks are awarded for 'knowing a method and attempting to apply it', unless otherwise indicated.
- **A** marks: Accuracy marks can only be awarded if the relevant method (M) marks have been earned.
- **B** marks are unconditional accuracy marks (independent of M marks)
- Marks should not be subdivided.

3. Abbreviations

   These are some of the traditional marking abbreviations that will appear in the mark schemes.

   - bod – benefit of doubt
   - ft – follow through
   - the symbol $\surd$ will be used for correct ft
   - cao – correct answer only
   - cso - correct solution only. There must be no errors in this part of the question to obtain this mark
   - isw – ignore subsequent working
   - awrt – answers which round to
   - SC: special case
   - oe – or equivalent (and appropriate)
   - d… or dep – dependent
   - indep – independent
   - dp decimal places
   - sf significant figures
   - $\ast$ The answer is printed on the paper or ag- answer given
   - $\lfloor$ or d… The second mark is dependent on gaining the first mark

4. All A marks are ‘correct answer only’ (cao.), unless shown, for example, as A1 ft to indicate that previous wrong working is to be followed through. After a misread however, the subsequent A marks affected are treated as A ft.

5. For misreading which does not alter the character of a question or materially simplify it, deduct two from any A or B marks gained, in that part of the question affected.

6. If a candidate makes more than one attempt at any question:
   - If all but one attempt is crossed out, mark the attempt which is NOT crossed out.
   - If either all attempts are crossed out or none are crossed out, mark all the attempts and score the highest single attempt.

7. Ignore wrong working or incorrect statements following a correct answer.

General Principles for Further Pure Mathematics Marking

(But note that specific mark schemes may sometimes override these general principles).

Method mark for solving 3 term quadratic:

1. Factorisation

$(x^2+bx+c)=(x+p)(x+q)$, where $|pq|=|c|$, leading to x = ...

$(ax^2+bx+c)=(mx+p)(nx+q)$, where $|pq|=|c|$ and $|mn|=|a|$, leading to x = ...

2. Formula

Attempt to use the correct formula (with values for a, b and c).

3. Completing the square

Solving $x^2+bx+c=0$: $\left(x\pm\frac{b}{2}\right)^2\pm q\pm c=0,\quad q\neq 0$, leading to x = ...

Method marks for differentiation and integration:

1. Differentiation

Power of at least one term decreased by 1. ($x^n\to x^{n-1}$)

2. Integration

Power of at least one term increased by 1. ($x^n\to x^{n+1}$)

Use of a formula

Where a method involves using a formula that has been learnt, the advice given in recent examiners' reports is that the formula should be quoted first.

Normal marking procedure is as follows:

Method mark for quoting a correct formula and attempting to use it, even if there are small errors in the substitution of values.

Where the formula is not quoted, the method mark can be gained by implication from correct working with values, but may be lost if there is any mistake in the working.

Exact answers

Examiners' reports have emphasised that where, for example, an exact answer is asked for, or working with surds is clearly required, marks will normally be lost if the candidate resorts to using rounded decimals.

Question Number	Scheme	Notes	Marks
1(a)	$\frac{1}{4r^2-1}$		
	$\frac{1}{2(2r-1)}-\frac{1}{2(2r+1)}$ or $\frac{\frac{1}{2}}{(2r-1)}-\frac{\frac{1}{2}}{(2r+1)}$ or equivalent or $\frac{1}{4r^2-1}\equiv\frac{A}{2r-1}+\frac{B}{2r+1}\Rightarrow A=\frac{1}{2}, B=-\frac{1}{2}$	Correct partial fractions or correct values of '*A*' and '*B*'. Isw if possible so if correct values of '*A*' and '*B*' are found, award when seen even if followed by incorrect partial fractions.	B1
			(1)
(b)	$\sum_{r=1}^{n}\frac{1}{4r^2-1}=\frac{1}{2}\left(1-\frac{1}{3}+......+\frac{1}{2n-1}-\frac{1}{2n+1}\right)$ Attempt at least first and last terms using their partial fractions. May be implied by e.g. $\frac{1}{2}\left(1-\frac{1}{2n+1}\right)$		M1
	$\frac{1}{2}\left(1-\frac{1}{2n+1}\right)$ or $\frac{1}{2}-\frac{1}{2(2n+1)}$ or $\frac{1}{2}-\frac{1}{4n+2}$	Correct expression	A1
	$\frac{n}{2n+1}$ *	Correct completion with no errors	A1*
	Allow a different variable to be used in (a) and (b) but final answer in (b) must be as printed i.e. in terms of *n*.		
			(3)
(c)	$\sum_{r=9}^{25}\frac{5}{4r^2-1}=(5)(\mathrm{f}(25)-\mathrm{f}(8))$	$\mathrm{f}(25)-\mathrm{f}(8)$ where $\mathrm{f}(n)=\frac{n}{2n+1}$	M1
	$=5\left(\frac{25}{51}-\frac{8}{17}\right)=\frac{5}{51}$	cao	A1
	Correct answer with no working in (c) scores both marks.		
			(2)
			Total 6

Question Number	Scheme	Notes	Marks
2	$\lvert x^2-9\rvert < \lvert 1-2x \rvert$ **(ignore use of "<" instead of "=" when finding cv's)**		
	$x^2-9=1-2x \Rightarrow x^2+2x-10=0 \Rightarrow x=...$ or $x^2-9=-1+2x \Rightarrow x^2-2x-8=0 \Rightarrow x=...$	Attempts to solve $x^2-9=1-2x$ **OR** $x^2-9=-1+2x$ to obtain two non-zero values of x	M1
	$x=\dfrac{-2\pm\sqrt{44}}{2}$ **OR** $x=-2,\ 4$	**One** correct pair of values. Allow the irrational roots to be at least as given here or $-1\pm\sqrt{11}$ or awrt 2.32, -4.32 or truncated 2.3, -4.3	A1
	$x^2-9=-1+2x \Rightarrow x^2-2x-8=0 \Rightarrow x=...$	Attempts to solve $x^2-9=1-2x$ **AND** $x^2-9=-1+2x$ to obtain four non-zero values of x	M1
	$x=\dfrac{-2\pm\sqrt{44}}{2}$ **AND** $x=-2,\ 4$	**Both** pairs of values correct. Allow the irrational roots to be at least as given here or $-1\pm\sqrt{11}$ or awrt 2.32, -4.32 or truncated 2.3, -4.3	A1
	$-1+\sqrt{11}<x<4$ **or** $-1-\sqrt{11}<x<-2$	One correct inequality.	B1
	For $-1+\sqrt{11}$ allow $\dfrac{-2+\sqrt{44}}{2}$, for $-1-\sqrt{11}$ allow $\dfrac{-2-\sqrt{44}}{2}$ but must be exact here. Allow alternative notation e.g. $\left(-1+\sqrt{11},\ 4\right)$, $\left(-1-\sqrt{11},\ -2\right)$ $4>x>-1+\sqrt{11}$, $x>-1+\sqrt{11}$ **and** $x<4$, $-2>x>-1-\sqrt{11}$, $x>-1-\sqrt{11}$ **and** $x<-2$		
	$-1+\sqrt{11}<x<4$ **and** $-1-\sqrt{11}<x<-2$	Both inequalities correct.	B1
	For $-1+\sqrt{11}$ allow $\dfrac{-2+\sqrt{44}}{2}$, for $-1-\sqrt{11}$ allow $\dfrac{-2-\sqrt{44}}{2}$ but must be exact here. Allow alternative notation e.g. $\left(-1+\sqrt{11},\ 4\right)$, $\left(-1-\sqrt{11},\ -2\right)$ $4>x>-1+\sqrt{11}$, $x>-1+\sqrt{11}$ **and** $x<4$, $-2>x>-1-\sqrt{11}$, $x>-1-\sqrt{11}$ **and** $x<-2$		
			(6)
			Total 6

	Q2 Alternative by squaring (ignore use of "<" instead of "=" when finding cv's)		
	$\left(x^2-9\right)^2=\left(1-2x\right)^2 \Rightarrow x^4-18x^2+81=1-4x+4x^2$		
	$x^4-22x^2+4x+80=0 \Rightarrow x=...$	Squares and attempts to solve a quartic equation to obtain at least two values of x that are non-zero.	M1
	$x=\dfrac{-2\pm\sqrt{44}}{2}$ **or** $x=-2,\ 4$	One pair of values correct as defined above	A1
	$x=\dfrac{-2\pm\sqrt{44}}{2}$ **and** $x=-2,\ 4$	M1: Obtains four non-zero values of x. A1: Both pairs of values correct as defined above	M1A1
	$-1+\sqrt{11}<x<4$ **or** $-1-\sqrt{11}<x<-2$	See notes above	B1
	$-1+\sqrt{11}<x<4$ **and** $-1-\sqrt{11}<x<-2$	See notes above	B1
	In an otherwise fully correct solution, if any extra incorrect regions are given, deduct the final B mark.		

	Scheme	Notes	Marks
3	$(1+x)\dfrac{\mathrm{d}y}{\mathrm{d}x}+ky=x^{\frac{1}{2}}(1+x)^{2-k}$		
	$\dfrac{\mathrm{d}y}{\mathrm{d}x}+\dfrac{ky}{(1+x)}=\dfrac{x^{\frac{1}{2}}(1+x)^{2-k}}{(1+x)}$	Divides by (1 + x) including the ky term	M1
	$\mathrm{I}=\mathrm{e}^{\int\frac{k}{1+x}\mathrm{d}x}=(1+x)^k$	dM1: Attempt integrating factor. $\mathrm{I}=\mathrm{e}^{\int\frac{k}{1+x}\mathrm{d}x}$ is sufficient for this mark but must include the k. Condone omission of "dx". A1: $(1+x)^k$	dM1A1
	$y(1+x)^k=\int x^{\frac{1}{2}}(1+x)\mathrm{d}x$	Reaches $y\times(\text{their I})=\int x^{\frac{1}{2}}(1+x)^{1-k}\times(\text{their I})\mathrm{d}x$	M1
	$\int x^{\frac{1}{2}}(1+x)\mathrm{d}x=\frac{2}{3}x^{\frac{3}{2}}+\frac{2}{5}x^{\frac{5}{2}}(+c)$ or by parts $\int x^{\frac{1}{2}}(1+x)\mathrm{d}x=\frac{2}{3}x^{\frac{3}{2}}(1+x)-\frac{4}{15}x^{\frac{5}{2}}(+c)$	Correct integration	A1
	$y=\dfrac{\frac{2}{3}x^{\frac{3}{2}}+\frac{2}{5}x^{\frac{5}{2}}+c}{(1+x)^k}$ or e.g. $y=\dfrac{\frac{2}{3}x^{\frac{3}{2}}(1+x)-\frac{4}{15}x^{\frac{5}{2}}+c}{(1+x)^k}$ or e.g. $y=\frac{2}{3}x^{\frac{3}{2}}(1+x)^{1-k}-\frac{4}{15}x^{\frac{5}{2}}(1+x)^{-k}+c(1+x)^{-k}$ or e.g. $y=\dfrac{10x^{\frac{3}{2}}(1+x)-4x^{\frac{5}{2}}+c}{15(1+x)^k}$ or e.g. $y=\frac{2}{3}x^{\frac{3}{2}}(1+x)^{-k}+\frac{2}{5}x^{\frac{5}{2}}(1+x)^{-k}+c(1+x)^{-k}$ **Correct answer with the constant correctly placed.** **Allow any equivalent correct answer.**		A1
			(6)
			Total 6

Question Number	Scheme	Notes	Marks
4(a)	$\mathrm{f}(x)=\sin\left(\tfrac{3}{2}x\right)$ $\mathrm{f}'(x)=\tfrac{3}{2}\cos\left(\tfrac{3}{2}x\right)$ $\mathrm{f}''(x)=-\tfrac{9}{4}\sin\left(\tfrac{3}{2}x\right)$ $\mathrm{f}'''(x)=-\tfrac{27}{8}\cos\left(\tfrac{3}{2}x\right)$ $\mathrm{f}''''(x)=\tfrac{81}{16}\sin\left(\tfrac{3}{2}x\right)$	M1: Attempt first 4 derivatives. Should be $\sin\to\cos\to\sin\to\cos\to\sin$. I.e. ignore signs and coefficients. A1: $\mathrm{f}'=\tfrac{3}{2}\cos\left(\tfrac{3}{2}x\right)$ and $\mathrm{f}''=-\tfrac{9}{4}\sin\left(\tfrac{3}{2}x\right)$ A1: $\mathrm{f}'''=-\tfrac{27}{8}\cos\left(\tfrac{3}{2}x\right)$ and $\mathrm{f}''''=\tfrac{81}{16}\sin\left(\tfrac{3}{2}x\right)$ Allow un-simplified e.g. $\mathrm{f}''=-\tfrac{3}{2}\cdot\tfrac{3}{2}\sin\left(\tfrac{3}{2}x\right)$	M1A2
	$y\left(\tfrac{\pi}{3}\right)=1,\ y'\left(\tfrac{\pi}{3}\right)=0,\ y''\left(\tfrac{\pi}{3}\right)=-\tfrac{9}{4},\ y'''\left(\tfrac{\pi}{3}\right)=0,\ y''''\left(\tfrac{\pi}{3}\right)=\tfrac{81}{16}$ Attempts at least 1 derivative at $x=\dfrac{\pi}{3}$		M1
	$\mathrm{f}(x)=1-\dfrac{9}{8}\left(x-\dfrac{\pi}{3}\right)^2+\dfrac{27}{128}\left(x-\dfrac{\pi}{3}\right)^4$	dM1: Correct use of Taylor series. I.e. $\mathrm{f}(x)=\mathrm{f}\left(\tfrac{\pi}{3}\right)+\left(x-\tfrac{\pi}{3}\right)\mathrm{f}'\left(\tfrac{\pi}{3}\right)+\left(x-\tfrac{\pi}{3}\right)^2\dfrac{\mathrm{f}''\left(\tfrac{\pi}{3}\right)}{2!}+\ldots$ Evidence of at least one term of the correct structure i.e. $\left(x-\tfrac{\pi}{3}\right)^n\dfrac{\mathrm{f}^n\left(\tfrac{\pi}{3}\right)}{n!}$ and not a Maclaurin series. **Dependent on the previous method mark.** A1: Correct expansion. Allow equivalent **single** fractions for $\dfrac{9}{8}$ and/or $\dfrac{27}{128}$ and allow decimal equivalents i.e. 1.125 and 0.2109375. Ignore any extra terms.	dM1A1
			(6)
(b)	$\mathrm{f}\left(\dfrac{1}{3}\right)=0.4815$	M1: Attempts $\mathrm{f}\left(\dfrac{1}{3}\right)$ or states $x=\dfrac{1}{3}$ A1: 0.4815 cao	M1A1
			(2)
			Total 8

Question Number	Scheme	Notes	Marks
5	$z=\dfrac{3w+1}{2-w}$	M1: Attempt to make *z* the subject as far as *z* = … A1: Correct equation	M1A1
	$\lvert z\rvert=1\Rightarrow\left\lvert\dfrac{3w+1}{2-w}\right\rvert=1\Rightarrow\left\lvert\dfrac{3(u+\mathrm{i}v)+1}{2-(u+\mathrm{i}v)}\right\rvert=1$	Uses $\lvert z\rvert=1$ and introduces $w=u+\mathrm{i}v$	M1
	$(3u+1)^2+(3v)^2=(u-2)^2+v^2$	M1: Correct use of Pythagoras. Condone missing brackets provided the intention is clear and allow e.g. $(3v)^2=3v^2$ but there should be no i's.	M1
	$u^2+v^2+\dfrac{10}{8}u-\dfrac{3}{8}=0$		
	$\left(u+\dfrac{5}{8}\right)^2-\dfrac{25}{64}+v^2=\dfrac{3}{8}$	Attempt to complete the square on the equation of a circle. I.e. an equation where the coefficients of u^2 and v^2 are the same and the other terms are in *u*, *v* or are constant. (Allow slips in completing the square). **Dependent on all previous M marks.**	ddddM1
	$\left(u+\dfrac{5}{8}\right)^2+v^2=\dfrac{49}{64}\Rightarrow\left(-\dfrac{5}{8},0\right),\dfrac{7}{8}$	A1: Centre $\left(-\dfrac{5}{8},0\right)$ A1: Radius $\dfrac{7}{8}$	A1A1
			(7)
			Total 7
	Alternative for the first 3 marks		
	$z=\dfrac{3w+1}{2-w}$	M1: Attempt to make *z* the subject A1: Correct equation	M1A1
	$x+\mathrm{i}y=\dfrac{3(u+\mathrm{i}v)+1}{2-(u+\mathrm{i}v)}=\dfrac{(3u+1+3\mathrm{i}v)}{2-u-\mathrm{i}v}\times\dfrac{2-u+\mathrm{i}v}{2-u+\mathrm{i}v}=\dfrac{5u+2-3(u^2+v^2)}{(2-u)^2+v^2}+\dfrac{7v}{(2-u)^2+v^2}\mathrm{i}$ $x^2+y^2=1\Rightarrow\left(\dfrac{5u+2-3(u^2+v^2)}{(2-u)^2+v^2}\right)^2+\left(\dfrac{7v}{(2-u)^2+v^2}\right)^2=1$		M1
	Introduces $w=u+\mathrm{i}v$, multiplies numerator and denominator by the complex conjugate of the denominator and uses $x^2+y^2=1$ correctly to obtain an equation in *u* and *v*.		

Question Number	Scheme	Notes	Marks
	$\frac{d^2y}{dx^2}+3\frac{dy}{dx}+2y=3x^2+2x+1$		
6(a)	$m^2+3m+2=0 \Rightarrow m=-1,-2$	Correct roots (may be implied by their CF)	B1
	$y=Ae^{-2x}+Be^{-x}$	M1: CF of the correct form A1: Correct CF	M1A1
	$y=ax^2+bx+c$	Correct form for PI	B1
	$\frac{dy}{dx}=2ax+b,\ \frac{d^2y}{dx^2}=2a \Rightarrow 2a+3(2ax+b)+2(ax^2+bx+c)=3x^2+2x+1$		M1
	M1: Differentiates twice and substitutes into the lhs of the given differential equation and puts equal to $3x^2+2x+1$ or substitutes into the lhs of the given differential equation and compares coefficients with $3x^2+2x+1$. For the substitution, at least one of $y,\ y'$ or y'' must be correctly placed.		
	$a=\frac{3}{2}$		A1
	$6a+2b=2 \Rightarrow b=-\frac{7}{2} \Rightarrow c=\frac{17}{4}$	M1: Solves to obtain one of *b* or *c* A1: Correct *b* and *c*	M1A1
	$y=Ae^{-2x}+Be^{-x}+\frac{3}{2}x^2-\frac{7}{2}x+\frac{17}{4}$	Correct ft (their CF + their PI) but must be $y=\ldots$	B1ft
			(9)
(b)	$0=A+B+\frac{17}{4}$	Substitutes $x = 0$ and $y = 0$ into their GS	M1
	$\frac{dy}{dx}=-2Ae^{-2x}-Be^{-x}+3x-\frac{7}{2} \Rightarrow 0=-2A-B-\frac{7}{2}$ Attempts to differentiate and substitutes $x = 0$ and $y' = 0$		M1
	$0=A+B+\frac{17}{4},\ 0=-2A-B-\frac{7}{2} \Rightarrow A=..,B=..$	Solves simultaneously to obtain values for *A* and *B*	M1
	$A=\frac{3}{4},\quad B=-5$	Correct values	A1
	$y=\frac{3}{4}e^{-2x}-5e^{-x}+\frac{3}{2}x^2-\frac{7}{2}x+\frac{17}{4}$	Correct ft (their CF + their PI) but must be $y=\ldots$	B1ft
			(5)
			Total 14

Question Number	Scheme	Notes	Marks
7.	$C_1: \ r = \frac{3}{2}\cos\theta, \quad C_2: \ r = 3\sqrt{3} - \frac{9}{2}\cos\theta$		
(a)	$\frac{3}{2}\cos\theta = 3\sqrt{3} - \frac{9}{2}\cos\theta \Rightarrow \theta = ...$ or $\cos\theta = \frac{2r}{3} \Rightarrow r = 3\sqrt{3} - 3r \Rightarrow r = ...$	Puts $C_1 = C_2$ and attempt to solve for θ or Eliminates $\cos\theta$ and solves for r	M1
	$\theta = \frac{\pi}{6}$ or $r = \frac{3\sqrt{3}}{4}$	Correct θ or correct r. Allow θ = awrt 0.524, r = awrt 1.3	A1
	$r = \frac{3\sqrt{3}}{4}$ and $\theta = \frac{\pi}{6}$	Correct r and θ (isw e.g. $\left(\frac{\pi}{6}, \frac{3\sqrt{3}}{4}\right)$) Allow θ = awrt 0.524, r = awrt 1.3	A1
			(3)

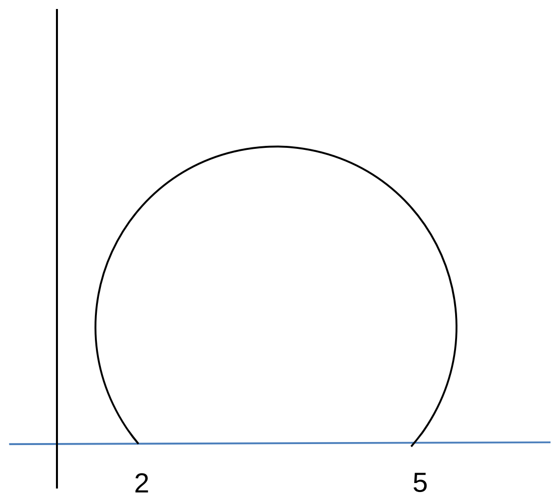
7(b)	$\frac{1}{2}\int\left(3\sqrt{3}-\frac{9}{2}\cos\theta\right)^2 \mathrm{d}\theta$ or $\frac{1}{2}\int\left(\frac{3}{2}\cos\theta\right)^2 \mathrm{d}\theta$	M1
	Attempts to use correct formula on either curve. The ½ may be implied by later work.	
	$\left(3\sqrt{3}-\frac{9}{2}\cos\theta\right)^2 = 27-27\sqrt{3}\cos\theta+\frac{81}{4}\cos^2\theta = 27-27\sqrt{3}\cos\theta+\frac{81}{4}\frac{(\cos 2\theta+1)}{2}$	M1
	Expands to obtain an expression of the form $a+b\cos\theta+c\cos^2\theta$ and attempts to use $\cos^2\theta = \pm\frac{1}{2}\pm\frac{\cos 2\theta}{2}$	
	$\left(\frac{1}{2}\right)\int\left(3\sqrt{3}-\frac{9}{2}\cos\theta\right)^2 \mathrm{d}\theta = \left(\frac{1}{2}\right)\left[\frac{297}{8}\theta-27\sqrt{3}\sin\theta+\frac{81}{16}\sin 2\theta\right]$	M1A1
	M1: Attempts to integrate to obtain at least two terms from $\alpha\theta,\ \beta\sin\theta,\ \gamma\sin 2\theta$ A1: Correct integration with or without the ½ (NB $\frac{297}{8}=27+\frac{81}{8}$)	
	$\left(\frac{1}{2}\right)\left[\frac{297}{8}\theta-27\sqrt{3}\sin\theta+\frac{81}{16}\sin 2\theta\right]_0^{\frac{\pi}{6}} = \left(\frac{1}{2}\right)\left\{\left(\frac{297}{8}.\frac{\pi}{6}-27\sqrt{3}.\sin\frac{\pi}{6}+\frac{81}{16}\sin 2.\frac{\pi}{6}\right)(-0)\right\}$	M1
	M1: Uses the limits 0 and their $\frac{\pi}{6}$ If the substitution for $\theta = 0$ evaluates to 0 then the substitution for $\theta = 0$ does not need to be seen but if it does not evaluate to 0, the substitution for $\theta = 0$ needs to be seen.	
	$\frac{1}{2}\int\left(\frac{3}{2}\cos\theta\right)^2 \mathrm{d}\theta = \frac{9}{16}\int(\cos 2\theta+1)\mathrm{d}\theta = \frac{9}{16}\left[\frac{1}{2}\sin 2\theta+\theta\right]_{\frac{\pi}{6}}^{\frac{\pi}{2}} = \frac{9}{16}\left(\frac{\pi}{3}-\frac{\sqrt{3}}{4}\right)$	M1
	M1: Uses $\cos^2\theta = \pm\frac{1}{2}\pm\frac{\cos 2\theta}{2}$, integrates to obtain at least $k\sin 2\theta$ and uses the limits of their $\frac{\pi}{6}$ and $\frac{\pi}{2}$ to find the other area NB can be done as a segment : $\frac{1}{2}\left(\frac{3}{4}\right)^2\left(\frac{2\pi}{3}\right)-\frac{1}{2}\left(\frac{3}{4}\right)^2\sin\left(\frac{\pi}{3}\right)$ Allow $\frac{1}{2}\left(\frac{3}{4}\right)^2\left(\pi-2\times\text{their}\frac{\pi}{6}\right)-\frac{1}{2}\left(\frac{3}{4}\right)^2\sin\left(\pi-2\times\text{their}\frac{\pi}{6}\right)$	
	$\frac{297}{96}\pi-\frac{351\sqrt{3}}{64}+\frac{9}{16}\left(\frac{\pi}{3}-\frac{\sqrt{3}}{4}\right) = \frac{105}{32}\pi-\frac{45}{8}\sqrt{3}$	M1A1
	M1: Adds their two areas both of which are of the form $a\pi+b\sqrt{3}$ A1: Correct answer (allow equivalent fractions for $\frac{105}{32}$ and/or $\frac{45}{8}$)	
		(8)
		Total 11

Special Case – Uses $\pm(C_1 - C_2)$

(b)	$\frac{1}{2}\int\left(3\sqrt{3}-\frac{9}{2}\cos\theta-\frac{3}{2}\cos\theta\right)^2 \mathrm{d}\theta$	M1
	Attempts to use correct formula on $\pm(C_1 - C_2)$. The ½ may be implied by later work.	
	$\left(3\sqrt{3}-6\cos\theta\right)^2 = 27-36\sqrt{3}\cos\theta+36\cos^2\theta = 27-36\sqrt{3}\cos\theta+36\frac{(\cos 2\theta+1)}{2}$	M1
	Expands to obtain an expression of the form $a+b\cos\theta+c\cos^2\theta$ and attempts to use $\cos^2\theta = \pm\frac{1}{2}\pm\frac{\cos 2\theta}{2}$	
	$\left(\frac{1}{2}\right)\int\left(3\sqrt{3}-6\cos\theta\right)^2\mathrm{d}\theta = \left(\frac{1}{2}\right)\left[45\theta-36\sqrt{3}\sin\theta+9\sin 2\theta\right]$	M1
	Attempts to integrate to obtain at least two terms from $\alpha\theta,\ \beta\sin\theta,\ \gamma\sin 2\theta$	
	No more marks available	

Question Number	Scheme	Notes	Marks
8(a) WAY 1	$\left(z+\frac{1}{z}\right)^5 = z^5+5z^3+10z+\frac{10}{z}+\frac{5}{z^3}+\frac{1}{z^5}$	M1: Attempt to expand $\left(z\pm\frac{1}{z}\right)^5$ A1: Correct expansion with correct powers of z.	M1A1
	$z=\cos\theta+\mathrm{i}\sin\theta \Rightarrow z+\frac{1}{z}=2\cos\theta$	May be implied	B1
	$=z^5+\frac{1}{z^5}+5\left(z^3+\frac{1}{z^3}\right)+10\left(z+\frac{1}{z}\right)=2\cos 5\theta+10\cos 3\theta+20\cos\theta$ Uses at least one of $z^5+\frac{1}{z^5}=2\cos 5\theta$ or $z^3+\frac{1}{z^3}=2\cos 3\theta$		M1
	$\left(z+\frac{1}{z}\right)^5=32\cos^5\theta$		B1
	$\cos^5\theta=\frac{1}{16}\cos 5\theta+\frac{5}{16}\cos 3\theta+\frac{5}{8}\cos\theta$	Correct expression	A1
			(6)
	WAY 2 (Using $\mathrm{e}^{\mathrm{i}\theta}$)		
	$\left(\mathrm{e}^{\mathrm{i}\theta}+\mathrm{e}^{-\mathrm{i}\theta}\right)^5=\mathrm{e}^{5\mathrm{i}\theta}+5\mathrm{e}^{3\mathrm{i}\theta}+10\mathrm{e}^{\mathrm{i}\theta}+10\mathrm{e}^{-\mathrm{i}\theta}+5\mathrm{e}^{-3\mathrm{i}\theta}+\mathrm{e}^{-5\mathrm{i}\theta}$	M1: Attempt to expand $\left(\mathrm{e}^{\mathrm{i}\theta}\pm\mathrm{e}^{-\mathrm{i}\theta}\right)^5$ A1: Correct expansion	M1A1
	$2\cos\theta=\mathrm{e}^{\mathrm{i}\theta}+\mathrm{e}^{-\mathrm{i}\theta}$	May be implied	B1
	$=\mathrm{e}^{5\mathrm{i}\theta}+\mathrm{e}^{-5\mathrm{i}\theta}+5\left(\mathrm{e}^{3\mathrm{i}\theta}+\mathrm{e}^{-3\mathrm{i}\theta}\right)+10\left(\mathrm{e}^{\mathrm{i}\theta}+\mathrm{e}^{-\mathrm{i}\theta}\right)=2\cos 5\theta+10\cos 3\theta+20\cos\theta$ Uses one of $\mathrm{e}^{5\mathrm{i}\theta}+\mathrm{e}^{-5\mathrm{i}\theta}=2\cos 5\theta$ or $\mathrm{e}^{3\mathrm{i}\theta}+\mathrm{e}^{-3\mathrm{i}\theta}=2\cos 3\theta$		M1
	$\left(\mathrm{e}^{\mathrm{i}\theta}+\mathrm{e}^{-\mathrm{i}\theta}\right)^5=32\cos^5\theta$		B1
	$\cos^5\theta=\frac{1}{16}\cos 5\theta+\frac{5}{16}\cos 3\theta+\frac{5}{8}\cos\theta$	Correct expression	A1
	WAY 3 (Using De Moivre on cos 5θ and identity for cos 3θ)		
	$(\cos\theta+\mathrm{i}\sin\theta)^5=c^5+5\mathrm{i}c^4s+10c^3\mathrm{i}^2s^2+10c^2\mathrm{i}^3s^3+5c\mathrm{i}^4s^4+\mathrm{i}^5s^5$ M1: Attempts to expand. NB may only consider real parts here. A1: Correct real terms (may include i's) (Ignore imaginary parts for this mark)		M1A1
	$\cos 5\theta=\cos^5\theta-10\cos^3\theta\sin^2\theta+5\cos\theta\sin^4\theta$	Correct real terms with no i's	B1
	$=\cos^5\theta-10\cos^3\theta\left(1-\cos^2\theta\right)+5\cos\theta\left(1-\cos^2\theta\right)^2$	Uses $\sin^2\theta=1-\cos^2\theta$ to eliminate $\sin\theta$	M1
	$16\cos^5\theta=\cos 5\theta+20\cos^3\theta-5\cos\theta$		
	$\cos 3\theta=4\cos^3\theta-3\cos\theta$	Correct identity for $\cos 3\theta$	B1
	$16\cos^5\theta=\cos 5\theta+5\cos 3\theta+10\cos\theta$		
	$\cos^5\theta=\frac{1}{16}\cos 5\theta+\frac{5}{16}\cos 3\theta+\frac{5}{8}\cos\theta$	Correct expression	A1
			(6)

(b)	$\int\left(\frac{1}{16}\cos 5\theta+\frac{5}{16}\cos 3\theta+\frac{5}{8}\cos\theta\right)\mathrm{d}\theta=\frac{1}{80}\sin 5\theta+\frac{5}{48}\sin 3\theta+\frac{5}{8}\sin\theta$ M1: Attempt to integrate – Evidence of $\cos n\theta\to\pm\frac{1}{n}\sin n\theta$ where $n=5$ or 3 A1ft: Correct integration (ft their *p*, *q*, *r*)		M1A1ft
	$\left[\frac{1}{80}\sin 5\theta+\frac{5}{48}\sin 3\theta+\frac{5}{8}\sin\theta\right]_{\frac{\pi}{6}}^{\frac{\pi}{3}}=\left(\frac{1}{80}\sin\frac{5\pi}{3}+\frac{5}{48}\sin\pi+\frac{5}{8}\sin\frac{\pi}{3}\right)-\left(\frac{1}{80}\sin\frac{5\pi}{6}+\frac{5}{48}\sin\frac{\pi}{2}+\frac{5}{8}\sin\frac{\pi}{6}\right)$ Substitutes the given limits into a changed function and subtracts the right way round. There should be evidence of the substitution of $\frac{\pi}{3}$ and $\frac{\pi}{6}$ into their changed function for at least 2 of their terms and subtraction. If there is no evidence of substitution and the answer is incorrect, score M0 here.		M1
	$=\frac{49\sqrt{3}}{160}-\frac{203}{480}$	Allow exact equivalents e.g. $=\frac{1}{16}\left(4.9\sqrt{3}-\frac{203}{30}\right)$	A1
	If they use the letters *p*, *q* and *r* or their values of *p*, *q* and *r*, even from no working, the M marks are available in (b) but <u>not</u> the A marks.		
			(4)
			Total 10

Question Number	Scheme	Notes	Marks
	$\arg\left(\frac{z-5}{z-2}\right)=\frac{\pi}{4}$		
9(a)		M1 A circle or an arc of a circle anywhere. Allow dotted or dashed. A1 A circle or an arc of a circle (allow dotted or dashed) passing through or touching at 2 and 5 on the positive real axis. (Imaginary axis may be missing) A1 Fully correct diagram with 2 and 5 marked correctly with no part of the circle below the real axis. It must be a major arc and not a semi-circle. The imaginary axis must be present and the arc must not cross or touch the imaginary axis.	
			(3)
(b)	Centre $C(x_c, y_c)$ is at $(3.5,\ 1.5)$	May be implied and may appear on the diagram. Can score anywhere e.g. from finding the equation of the circle in part (a) or as part of the calculation for *OC*.	B1
	$r=\sqrt{1.5^2+1.5^2}\left(=\frac{3}{\sqrt{2}}=\frac{3\sqrt{2}}{2}\right)$	$r=\sqrt{2\times y_c^{\ 2}}$ or equivalent work e.g. $r=\frac{1.5}{\cos 45}$, $r=\frac{1.5}{\sin 45}$ or $\frac{3\sqrt{2}}{2}$ seen	M1
	Max $\lvert z\rvert = OC+r=\sqrt{3.5^2+1.5^2}+r$		M1
	$=\frac{\sqrt{58}}{2}+\frac{3}{\sqrt{2}}$	Oe e.g $\sqrt{14.5}+\sqrt{4.5}$, $\frac{\sqrt{58}+3\sqrt{2}}{2}$	A1
			(4)
	Special Case – correct work with arc below the real axis:		
	Centre $C(x_c, y_c)$ is at $(3.5,\ -1.5)$		B0
	$r=\sqrt{1.5^2+1.5^2}\left(=\frac{3}{\sqrt{2}}=\frac{3\sqrt{2}}{2}\right)$	$r=\sqrt{2\times y_c^{\ 2}}$ or equivalent work e.g. $r=\frac{1.5}{\cos 45}$, $r=\frac{1.5}{\sin 45}$ or $\frac{3\sqrt{2}}{2}$ seen	M1
	Max $\lvert z\rvert = OC+r=\sqrt{3.5^2+1.5^2}+r$		M1
	$=\frac{\sqrt{58}}{2}+\frac{3}{\sqrt{2}}$	Oe e.g $\sqrt{14.5}+\sqrt{4.5}$, $\frac{\sqrt{58}+3\sqrt{2}}{2}$	A1
			Total 7