Pearson Edexcel

Mark Scheme (Results)

October 2020

Pearson Edexcel International Advanced Level
In Further Pure Mathematics F1
(WFM01/01)

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General Marking Guidance

- All candidates must receive the same treatment. Examiners must mark the first candidate in exactly the same way as they mark the last.
- Mark schemes should be applied positively. Candidates must be rewarded for what they have shown they can do rather than penalised for omissions.
- Examiners should mark according to the mark scheme not according to their perception of where the grade boundaries may lie.
- There is no ceiling on achievement. All marks on the mark scheme should be used appropriately.
- All the marks on the mark scheme are designed to be awarded. Examiners should always award full marks if deserved, i.e. if the answer matches the mark scheme. Examiners should also be prepared to award zero marks if the candidate's response is not worthy of credit according to the mark scheme.
- Where some judgement is required, mark schemes will provide the principles by which marks will be awarded and exemplification may be limited.
- When examiners are in doubt regarding the application of the mark scheme to a candidate's response, the team leader must be consulted.
- Crossed out work should be marked UNLESS the candidate has replaced it with an alternative response.

PEARSON EDEXCEL IAL MATHEMATICS

General Instructions for Marking

1. The total number of marks for the paper is 75

2. The Edexcel Mathematics mark schemes use the following types of marks:

- **M** marks: Method marks are awarded for 'knowing a method and attempting to apply it', unless otherwise indicated.
- **A** marks: Accuracy marks can only be awarded if the relevant method (M) marks have been earned.
- **B** marks are unconditional accuracy marks (independent of M marks)
- Marks should not be subdivided.

3. Abbreviations

   These are some of the traditional marking abbreviations that will appear in the mark schemes.

   - bod – benefit of doubt
   - ft – follow through
   - the symbol $\sqrt{}$ will be used for correct ft
   - cao – correct answer only
   - cso - correct solution only. There must be no errors in this part of the question to obtain this mark
   - isw – ignore subsequent working
   - awrt – answers which round to
   - SC: special case
   - o.e. – or equivalent (and appropriate)
   - d... or dep – dependent
   - indep – independent
   - dp decimal places
   - sf significant figures
   - ∗ The answer is printed on the paper or ag- answer given
   - $\sqsubset$ or d... The second mark is dependent on gaining the first mark

4. All A marks are 'correct answer only' (cao), unless shown, for example, as A1ft to indicate that previous wrong working is to be followed through. After a misread however, the subsequent A marks affected are treated as A ft, but manifestly absurd answers should never be awarded A marks.

5. For misreading which does not alter the character of a question or materially simplify it, deduct two from any A or B marks gained, in that part of the question affected.

6. If a candidate makes more than one attempt at any question:
    - If all but one attempt is crossed out, mark the attempt which is NOT crossed out.
    - If either all attempts are crossed out or none are crossed out, mark all the attempts and score the highest single attempt.

7. Ignore wrong working or incorrect statements following a correct answer.

General Principles for Further Pure Mathematics Marking

(*But note that specific mark schemes may sometimes override these general principles*)

Method mark for solving 3 term quadratic:

1. Factorisation

$(x^2 + bx + c) = (x + p)(x + q),\ \text{where} |pq| = |c|$, leading to $x = \ldots$

$(ax^2 + bx + c) = (mx + p)(nx + q),\ \text{where} |pq| = |c| \text{ and } |mn| = |a|$, leading to $x = \ldots$

2. Formula

Attempt to use the correct formula (with values for *a*, *b* and *c*).

3. Completing the square

Solving $x^2 + bx + c = 0$: $\left(x \pm \frac{b}{2}\right)^2 \pm q \pm c = 0,\ \ q \neq 0$, leading to $x = \ldots$

Method marks for differentiation and integration:

1. Differentiation

Power of at least one term decreased by 1. ($x^n \rightarrow x^{n-1}$)

2. Integration

Power of at least one term increased by 1. ($x^n \rightarrow x^{n+1}$)

Use of a formula

Where a method involves using a formula that has been learnt, the advice given in recent examiners' reports is that the formula should be quoted first.

Normal marking procedure is as follows:

Method mark for quoting a correct formula and attempting to use it, even if there are small errors in the substitution of values.

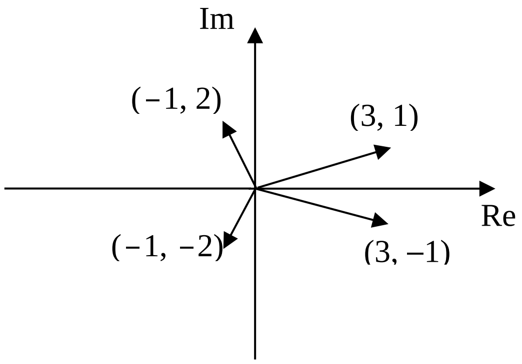
Where the formula is not quoted, the method mark can be gained by implication from correct working with values but may be lost if there is any mistake in the working.

Exact answers

Examiners' reports have emphasised that where, for example, an exact answer is asked for, or working with surds is clearly required, marks will normally be lost if the candidate resorts to using rounded decimals.

Question Number	Scheme	Notes	Marks
1(a)	$\mathrm{f}(x)=x^3-\dfrac{10\sqrt{x}-4x}{x^2} \qquad x>0$		
	$\mathrm{f}(1.4)=-0.435673...$ $\mathrm{f}(1.5)=0.598356...$	Attempts both f(1.4) and f(1.5)	M1
	Sign change (positive, negative) (and $\mathrm{f}(x)$ is continuous) therefore (a **root**) α is between $x=1.4$ and $x=1.5$	Both f(1.4) = awrt −0.4 and f(1.5) = awrt 0.6, sign change and conclusion. For 'sign change' indication that f(1.4) < 0 and f(1.5) > 0 is sufficient. Also f(1.4) f(1.5) < 0 is sufficient. 'Therefore root' (without mention of the interval) is a sufficient conclusion. Mention of 'continuous' is not required.	A1
			(2)
(b)	$\left(\mathrm{f}(x)=x^3-\dfrac{10\sqrt{x}-4x}{x^2}=x^3-10x^{-\frac{3}{2}}+4x^{-1}\right)$		
	$\mathrm{f}'(x)=3x^2+15x^{-\frac{5}{2}}-4x^{-2}$ (or equivalent, see below)	$x^n \to x^{n-1}$ for one term	M1
		2 correct terms simplified or unsimplified	A1
		All correct simplified or unsimplified	A1
			(3)
(c)	$(x_1)=1.4-\dfrac{\mathrm{f}(1.4)}{\mathrm{f}'(1.4)}$ $\left(=1.4-\frac{-0.43567...}{10.30720...}\right)=...$	Correct application of N-R leading to an answer. <u>Values</u> of f(1.4) and f '(1.4) need not be seen before their final answer.	M1
	= 1.442	cao (must be corrected to 3 d.p.) isw if x_2, etc. have been found, but the answer for 'one use of N-R' must be seen as 1.442 to score this mark.	A1
			(2)
			Total 7
(b)	Equivalent unsimplified versions are acceptable, e.g. (using quotient rule); $3x^2-\dfrac{\left(5x^{\frac{3}{2}}-4x^2\right)-20x^{\frac{3}{2}}+8x^2}{x^4}$	The 'two correct terms' still applies for the first A1. Here a 'term' would be, for example, the $x^{-\frac{5}{2}}$ terms in unsimplified form. Isw after a correct unsimplified form.	
(b)(c)	A common error in (b) is to have $+4x^{-2}$ instead of $-4x^{-2}$, giving 1.430 in (c). This, if otherwise correct, would score (b) 110 and (c) 10		

Question Number	Scheme	Notes	Marks
2	$5x^2-2x+3=0$		
(a)	$\alpha+\beta=\frac{2}{5},\quad \alpha\beta=\frac{3}{5}$	Both correct	B1
			(1)
(b)(i)	$\alpha^2+\beta^2=(\alpha+\beta)^2-2\alpha\beta$	Uses a correct identity	M1
	$=\left(\frac{2}{5}\right)^2-2\left(\frac{3}{5}\right)=-\frac{26}{25}$	Correct value (allow -1.04), even after $\alpha+\beta=-\frac{2}{5}$ in (a)	A1
(ii)	$\alpha^3+\beta^3=(\alpha+\beta)^3-3\alpha\beta(\alpha+\beta)$ or $\alpha^3+\beta^3=(\alpha+\beta)(\alpha^2-\alpha\beta+\beta^2)$	Uses a correct identity	M1
	$=\left(\frac{2}{5}\right)^3-3\left(\frac{3}{5}\right)\left(\frac{2}{5}\right)=-\frac{82}{125}$	Correct value (allow -0.656)	A1
			(4)
(c)	$\text{Sum}=\alpha+\beta+\alpha^2+\beta^2=\frac{2}{5}-\frac{26}{25}\left(=-\frac{16}{25}\right)$	Attempts value of sum	M1
	$\text{Product}=\alpha\beta+\alpha^3+\beta^3+(\alpha\beta)^2=\frac{3}{5}-\frac{82}{125}+\left(\frac{3}{5}\right)^2\left(=\frac{38}{125}\right)$	Attempts value of product, using the <u>correct</u> expansion of $(\alpha+\beta^2)(\beta+\alpha^2)$	M1
	$x^2+\frac{16}{25}x+\frac{38}{125}(=0)$	Applies x^2 – (their sum)x + their product Accept unsimplified versions. The '= 0' is not required	M1
	$125x^2+80x+38=0$	Allow any integer multiple. Must be a fully correct equation, including the '= 0' Not just p = 125, q = 80, r = 38	A1
			(4)
			Total 9

Question Number	Scheme	Notes	Marks
3	$\mathrm{f}(z) = z^4 + az^3 + bz^2 + cz + d$		
(a)	$(z =) 3 - \mathrm{i}$ or $(z =) -1 + 2\mathrm{i}$		B1
	$(z =) 3 - \mathrm{i}$ and $(z =) -1 + 2\mathrm{i}$		B1
			(2)
(b)		$3 \pm \mathrm{i}$ correctly plotted with vectors or dots or crosses etc. or $-1 \pm 2\mathrm{i}$ correctly plotted with vectors or dots or crosses etc.	B1
		All 4 correct roots correctly plotted with scaling approximately correct (e.g. (-1, 2) higher than (3, 1), etc.) There should be approximate symmetry about the real axis, but be generous	B1
			(2)
(c)	$z = 3 \pm \mathrm{i} \Rightarrow (z - (3 + \mathrm{i}))(z - (3 - \mathrm{i})) = ...$ or $z = -1 \pm 2\mathrm{i} \Rightarrow (z - (-1 + 2\mathrm{i}))(z - (-1 - 2\mathrm{i})) = ...$	Correct strategy to find at least one quadratic factor. Throughout this part ignore the use of x (or other variable) instead of z	M1
	$z^2 - 6z + 10$ or $z^2 + 2z + 5$	One correct quadratic	A1
	$z^2 - 6z + 10$ and $z^2 + 2z + 5$	Both correct	A1
	$(z^2 - 6z + 10)(z^2 + 2z + 5) = ...$	Attempts product of their two <u>3-term</u> quadratic factors... no 'missing terms' in the expansion	M1
	$a = -4, b = 3, c = -10, d = 50$ or $\mathrm{f}(z) = z^4 - 4z^3 + 3z^2 - 10z + 50$	All correct values or correct quartic	A1
			(5)
			Total 9

(c) **Way 2**	$(z-(3+i))(z-(-1\pm 2i))=\cdots$ or $(z-(3-i))(z-(-1\pm 2i))=\cdots$	Correct strategy to find at least one quadratic factor. Throughout this part ignore the use of *x* (or other variable) instead of *z*	M1
	$z^2+z(-2+i)+(-1-7i)$ (i) or $z^2+z(-2-i)+(-1+7i)$ (ii) or $z^2+z(-2+3i)+(-5-5i)$ (iii) or $z^2+z(-2-3i)+(-5+5i)$ (iv)	One correct quadratic	A1
	(i) and (ii) correct or (iii) and (iv) correct	A correct pair	A1
	e,g $[z^2+z(-2+i)+(-1-7i)]\times$ $[z^2+z(-2-i)+(-1+7i)]=$...	Attempts product of their two <u>3-term</u> quadratic factors... no 'missing terms' in the expansion	M1
	$a=-4, b=3, c=-10, d=50$ or $\mathrm{f}(z)=z^4-4z^3+3z^2-10z+50$	All correct values or correct quartic	A1

(c) **Way 3**	$\sum\propto=(3+i)+(3-i)+(-1-2i)+(-1+2i)=..$ or $\alpha\beta\gamma\delta=(3+i)(3-i)(-1-2i)(-1+2i)=..$	Attempts one of these	M1
	$a=-4$ or $d=50$		A1
	Both $a=-4$ and $d=50$		A1
	$\sum\alpha\beta=\ldots$ and $\sum\alpha\beta\gamma=\ldots$	Attempts $\sum\alpha\beta$ (all 6 terms) and $\sum\alpha\beta\gamma$ (all 4 terms)	M1
	$a=-4, b=3, c=-10, d=50$ or $\mathrm{f}(z)=z^4-4z^3+3z^2-10z+50$	All correct values or correct quartic	A1

(c) **Way 4**	$(3+i)^4+a(3+i)^3+b(3+i)^2+c(3+i)+d=0$ (......) + (......) i = 0	Substitutes one of the roots into the given quartic and fully multiplies out	M1
	$(28+18a+8b+3c+d)+i(96+26a+6b+c)$ or $(-7+11a-3b-c+d)+i(-24+2a+4b-2c)$	One correct expansion	A1
	$(28+18a+8b+3c+d)+i(96+26a+6b+c)$ and $(-7+11a-3b-c+d)+i(-24+2a+4b-2c)$	Obtains a second correct expansion using another root.	A1
	$(28+18a+8b+3c+d)=0$, etc leading to $a=$, $b=$, $c=$, $d=$	Solves 4 simultaneous equation to find values of *a*, *b*, *c* and *d*	M1
	$a=-4, b=3, c=-10, d=50$ or $\mathrm{f}(z)=z^4-4z^3+3z^2-10z+50$	All correct values or correct quartic	A1

Question Number	Scheme	Notes	Marks
4(a)	$(2r-1)^2 = 4r^2 - 4r + 1$	Correct expansion	B1
	$\sum_{r=1}^{n}(4r^2 - 4r + 1) = 4\times\frac{1}{6}n(n+1)(2n+1) - 4\times\frac{1}{2}n(n+1) + n$ M1: Attempt to use at least one of the standard results correctly A1: Correct expression		M1A1
	$= \frac{1}{3}n\left[2(n+1)(2n+1) - 6(n+1) + 3\right]$	Attempt to factorise $\frac{1}{3}n(\ldots.)$ Condone one slip but there must have been + n, not +1 in their expression for the sum	M1
	$= \frac{1}{3}n\left[4n^2 - 1\right]$*	Correct proof with no errors. There should be an intermediate step showing the expansion of $(n+1)(2n+1)$, or equivalent	A1*
	Condone poor or incorrect use of notation, e.g. Σ used at every step of the proof		
			(5)
(b)	$2r - 1 = 499 \Rightarrow r = 250$	Identifies the correct upper limit (may be implied)	B1
	$2r - 1 = 201 \Rightarrow r = 101$	Identifies the correct lower limit (may be implied)	B1
	$\sum_{r=101}^{250}(2r-1)^2 = \frac{1}{3}\times250\left(4\times250^2 - 1\right) - \frac{1}{3}\times100\left(4\times100^2 - 1\right)$ Uses the result from part (a) together with their upper limit and their lower limit – 1. A common mistake is to assume 500 and 200 are the limits, and in this case the mark is scored if 199 is used		M1
	= 19 499 950	Cao	A1
			(4)
			Total 9

Question Number	Scheme	Notes	Marks
5(a)	$xy=64 \Rightarrow y=64x^{-1} \Rightarrow \frac{\mathrm{d}y}{\mathrm{d}x}=-64x^{-2}$ or $xy=64 \Rightarrow x\frac{\mathrm{d}y}{\mathrm{d}x}+y=0 \Rightarrow \frac{\mathrm{d}y}{\mathrm{d}x}=-\frac{y}{x}$ or $x=8p, y=\frac{8}{p} \Rightarrow \frac{\mathrm{d}y}{\mathrm{d}x}=\frac{-8p^{-2}}{8}$	Correct $\frac{\mathrm{d}y}{\mathrm{d}x}$ This can be in any form, simplified or unsimplified. The parameter could be a different variable, e.g. t	B1
	$m_T=-\frac{64}{64p^2} \Rightarrow m_N=p^2$ or $m_T=-\frac{8/p}{8p} \Rightarrow m_N=p^2$ or $m_T=-p^{-2} \Rightarrow m_N=p^2$	Correct use of the perpendicular gradient rule and the point P to obtain the normal gradient	M1
		Correct normal gradient of p^2	A1
	$y-\frac{8}{p}=p^2(x-8p)$ or $y=p^2x+c, \frac{8}{p}=p^2\times 8p+c \Rightarrow c=...$	Correct straight line method for normal	M1
	$p^3x-py=8(p^4-1)$*	cso. (No errors, but possibly direct from the version in line 1 above)	A1*
			(5)
(b)	$p^3x-py=8(p^4-1), xy=64 \Rightarrow$ $p^3x-p\frac{64}{x}=8(p^4-1)$ or $p^3\frac{64}{y}-py=8(p^4-1)$	Uses both equations to obtain an equation in one variable	M1
	$p^3x^2+8(1-p^4)x-64p=0$ or $py^2+8(p^4-1)y-64p^3=0$	Correct quadratic. Must have the x^2 or y^2 term, but the x or y terms need not be combined. The terms do not need to be 'all on one side', and the coefficients could involve fractions, e.g. $p^2x^2+\frac{8x}{p}-8p^3x=64$	A1
	$(x-8p)(p^3x+8)=0 \Rightarrow x=...$ or $(py-8)(y+8p^3)=0 \Rightarrow y=...$	Solves their 3TQ (usual rules) to obtain the <u>other</u> value of x or y. The <u>other</u> value must be picked out as a solution. This could be done by algebraic division... (see below)	dM1
	$x=-\frac{8}{p^3}$ $y=-8p^3$ or $\left(-\frac{8}{p^3},-8p^3\right)$	Correct coordinates (ignore coordinates of P if they are also given as an answer). $-8p^{-3}$ may be seen rather than $-\frac{8}{p^3}$	A1
			(4)
			Total 9

5(b)	Rather than solving the 3TQ for the dM1, algebraic division can be used. To score the mark the division should follow the usual rules for solution by factorisation, so in the first case, e.g. if the quadratic is correct, the quotient should be $\pm p^3x \pm 8$, then this must lead to the other value $x_2 = \cdots$

5(b)	Note that another way to find the other value for the dM1 is to use the 'sum of roots' = $-\frac{b}{a}$, e.g. $8p + x_2 = \dfrac{-8(1-p^4)}{p^3} \qquad x_2 = \cdots$

(b) Way 2	$p^3x - py = 8(p^4-1), \left(8q, \frac{8}{q}\right) \Rightarrow$ $p^3 8q - p\frac{8}{q} = 8(p^4-1)$	Uses the given normal equation and the parametric form for Q to form an equation in p and q	M1
	$p^3q^2 - p = qp^4 - q$	Correct quadratic. Must have the q^2 term, but the q terms need not be combined. The terms do not need to be 'all on one side', and the coefficients could involve fractions.	A1
	$(p-q)(p^3q+1) = 0 \Rightarrow q = \ldots$	Solves their 3TQ (usual rules) to obtain the value of q. This could be done by algebraic division (condition as for main scheme)	dM1
	$q = -\frac{1}{p^3} \Rightarrow x = -\frac{8}{p^3} \quad y = -8p^3$ or $\left(-\frac{8}{p^3}, -8p^3\right)$	Correct coordinates (ignore coordinates of P if they are also given as an answer). $-8p^{-3}$ may be seen rather than $-\frac{8}{p^3}$	A1
			(4)

Question Number	Scheme	Notes	Marks
6(i)(a)	Stretch scale factor 3 parallel to the y-axis	Stretch (<u>not</u> enlargement)	B1
		Scale factor 3 parallel to the y-axis. Allow, e.g. '3 times y values', 'y increased by 3 factor', or similar. Allow, e.g. 'direction of y', 'along y', 'vertical', or similar. Ignore any mention of the origin. If additional transformations are included, send to Review	B1
			(2)
(b)	$\begin{pmatrix} \frac{\sqrt{2}}{2} & \frac{\sqrt{2}}{2} \\ -\frac{\sqrt{2}}{2} & \frac{\sqrt{2}}{2} \end{pmatrix}$	Correct matrix. $\frac{1}{\sqrt{2}}$ may be seen rather than $\frac{\sqrt{2}}{2}$	B1
			(1)
(c)	$\begin{pmatrix} \frac{\sqrt{2}}{2} & \frac{\sqrt{2}}{2} \\ -\frac{\sqrt{2}}{2} & \frac{\sqrt{2}}{2} \end{pmatrix}\begin{pmatrix} 1 & 0 \\ 0 & 3 \end{pmatrix}$	Attempt to multiply the right way round, i.e. **BA**, not **AB** At least two correct terms (for their matrix **B**) are needed to indicate a correct multiplication attempt	M1
	$\begin{pmatrix} \frac{\sqrt{2}}{2} & \frac{3\sqrt{2}}{2} \\ -\frac{\sqrt{2}}{2} & \frac{3\sqrt{2}}{2} \end{pmatrix}$ or equiv. e.g. $\frac{1}{\sqrt{2}}\begin{pmatrix} 1 & 3 \\ -1 & 3 \end{pmatrix}$	Correct matrix	A1
			(2)
(ii)	Trapezium area$=\frac{1}{2}(5+2)(k+8)$	Correct method for the area of the trapezium	M1
	$\begin{vmatrix} 5 & 1 \\ -2 & 3 \end{vmatrix} = 5\times 3-(-2)\times 1=17$	Correct method for the determinant	M1
		17 (Allow $\pm$ 17)	A1
	$\frac{1}{2}(5+2)(k+8)\times 17=510 \Rightarrow k=...$	Multiplies their trapezium area by their determinant, sets equal to 510 and solves for k. Or equivalently: Equates their trapezium area to (510 ÷ determinant) and solves for k	M1
	$k=\frac{4}{7}$	$\frac{4}{7}$ or exact equivalent. If additional answers such as $-\frac{4}{7}$ are given and not rejected, this is A0	A1
			(5)

(ii) **Way 2**	$\begin{pmatrix} 5 & 1 \\ -2 & 3 \end{pmatrix}\begin{pmatrix} -2 & -2 & 5 & 5 \\ 0 & k & 8 & 0 \end{pmatrix}$	Multiplies correct matrices to find the coordinates for T'	2nd M
	$= \begin{pmatrix} -10 & -10+k & 33 & 25 \\ 4 & 4+3k & 14 & -10 \end{pmatrix}$	Correct coordinates (can be left in matrix form)	A1
	$\frac{1}{2}[-10(4+3k)+14(-10+k)-330+100-4(-10+k)-33(4+3k)-350-100]$	Correct method for area of T' ('shoelace rule' with or without a modulus), using their coordinates for T'	1st M
	$\pm\frac{1}{2}(952+119k)=510$, $k = ...$	Sets area of T' equal to 510 and solves for k	M1
	$k = \frac{4}{7}$	$\frac{4}{7}$ or exact equivalent. If additional answers such as $-\frac{4}{7}$ are given and not rejected, this is A0	A1
			Total 10

Question Number	Scheme	Notes	Marks
7(a) Way 1	$3x-4y+48=0 \Rightarrow x=\dfrac{4y-48}{3}$ $y^2=4ax \Rightarrow y^2=4a\left(\dfrac{4y-48}{3}\right)$ or $3x-4y+48=0 \Rightarrow y=\dfrac{3x+48}{4}$ $y^2=4ax \Rightarrow \left(\dfrac{3x+48}{4}\right)^2=4ax$ or $x=\dfrac{y^2}{4a} \Rightarrow \dfrac{3y^2}{4a}-4y+48=0$	Uses both equations to obtain an equation in one variable.	M1
	$3y^2-16ay+192a=0$ or $9x^2+(288-64a)x+2304=0$ or $3x-8\sqrt{a}\sqrt{x}+48=0$	Correct 3TQ (Coefficients could be 'fractional') (This could be a quadratic in $\sqrt{x}$)	A1
	Equal roots: $(16a)^2=4\times 3\times 192a$ or $(288-64a)^2=4\times 9\times 2304$ $\Rightarrow a=...$	Uses "$b^2=4ac$" to find a value for a	M1
	$a=9$ *	cso	A1*
	Beware the use of the given result a = 9, but there may be cases where 'working backwards' deserves merit (if in doubt, send to Review).		
			(4)

(a) Way 2	$y^2 = 4ax \Rightarrow 2y\frac{\mathrm{d}y}{\mathrm{d}x} = 4a$ $3x - 4y + 48 = 0 \Rightarrow \frac{\mathrm{d}y}{\mathrm{d}x} = \frac{3}{4}$ $\Rightarrow 2y \times \frac{3}{4} = 4a$	Uses differentiation to obtain the gradient of C and substitutes the gradient of l to obtain an equation connecting y and a, or connecting x and a, or an equation in t	M1
	$y = 2a^{\frac{1}{2}}x^{\frac{1}{2}} \Rightarrow \frac{\mathrm{d}y}{\mathrm{d}x} = a^{\frac{1}{2}}x^{-\frac{1}{2}} \Rightarrow a^{\frac{1}{2}}x^{-\frac{1}{2}} = \frac{3}{4}$		
	$x = at^2,\ y = 2at \Rightarrow \frac{\mathrm{d}y}{\mathrm{d}x} = \frac{1}{t}$ $\frac{1}{t} = \frac{3}{4}$		
	$y = \frac{8a}{3}$ or $x = \frac{16a}{9}$	Correct y value, or correct x value (possibly implied in subsequent work, particularly if using the parametric equations)	A1
	$y^2 = 4ax \Rightarrow \frac{64a^2}{9} = 4ax \Rightarrow x = \frac{16a}{9}$ $3 \times \frac{16a}{9} - 4 \times \frac{8a}{3} + 48 = 0 \Rightarrow a = ...$	Uses $y^2 = 4ax$ or l to find a value for x (or y) and substitutes their x and y into the other equation to find a value for a If using parameter t, substitutes their value for t into $3(at^2) - 4(2at) + 48 = 0$ and solves to find a value for a	M1
	$x = \frac{4y - 48}{3} = \frac{32a}{9} - 16$ $\frac{64a^2}{9} = 4a\left(\frac{32a}{9} - 16\right) \Rightarrow a = \cdots$		
	$y^2 = 4ax \Rightarrow y^2 = \frac{64a^2}{9} \Rightarrow y = \frac{8a}{3}$ $3 \times \frac{16a}{9} - 4 \times \frac{8a}{3} + 48 = 0 \Rightarrow a = ...$		
	$y = \frac{3x + 48}{4} = \frac{4a}{3} + 12$ $\left(\frac{4a}{3} + 12\right)^2 = 4a\left(\frac{16a}{9}\right) \Rightarrow a = \cdots$		
	$3(at^2) - 4(2at) + 48 = 0$ $3\left(\frac{16a}{9}\right) - 4\left(\frac{8a}{3}\right) + 48 = 0 \Rightarrow a = \cdots$		
	$a = 9$ *	cso	A1*

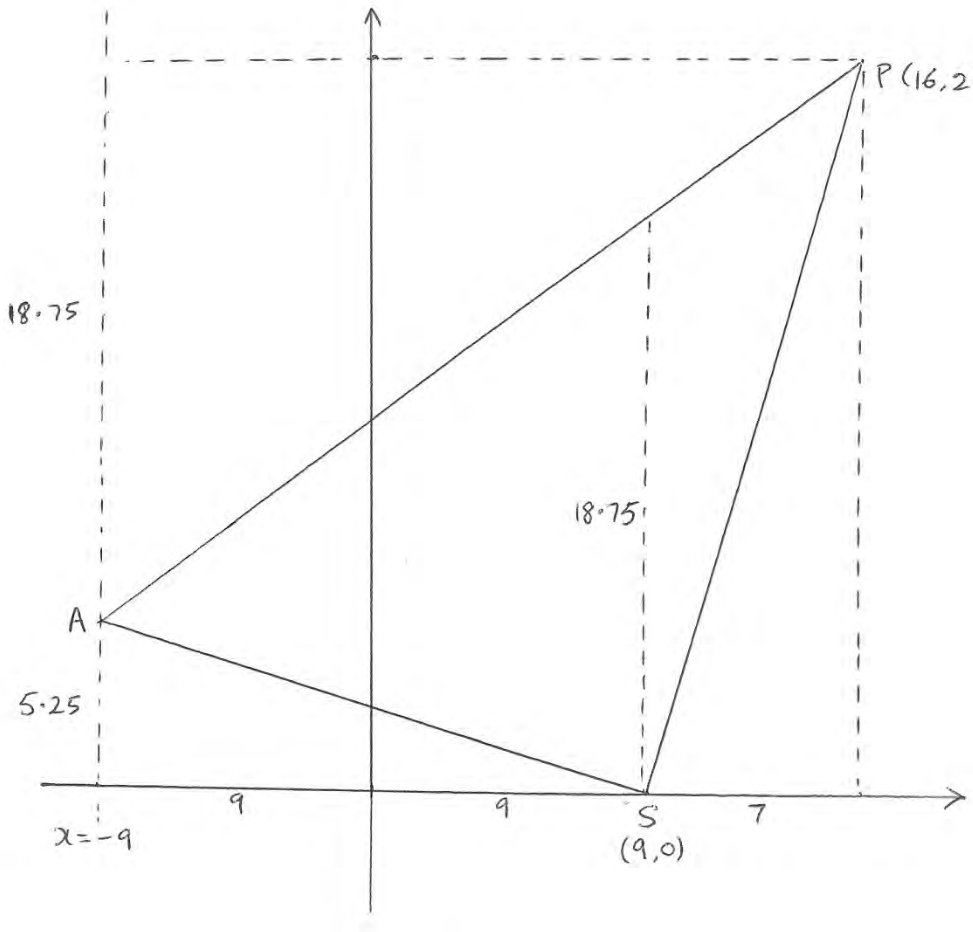
(b)	$a=9\Rightarrow 3y^2-144y+1728=0\Rightarrow y=24$ $9x^2-288x+2304=0\Rightarrow x=16$	Uses a = 9 to solve their 3TQ to obtain the repeated root for x or y.	M1
	$x=16$ and $y=24$	Correct values or coordinates.	A1
			(2)

(b) Way 2 follows (a)Way2	$a=9\Rightarrow x=\cdots$ or $y=\cdots$	Substitutes a = 9 into their expression for x or y, OR substitutes a = 9 into at^2 to find x , or into $2at$ to find y.	M1
	$x=16$ and $y=24$	Correct values or coordinates.	A1

(c) Way 1	Focus is at (9, 0)	Correct focus (could be seen on a sketch or implied in working)	B1
	$x=-9\Rightarrow 3(-9)-4y+48=0\Rightarrow y=5.25$	Correct method with the correct directrix to find the y coordinate of A	M1
	E.g.Trapezium – 2 triangles $=\frac{1}{2}\left(\frac{21}{4}+24\right)\times 25-\frac{1}{2}\times 18\times\frac{21}{4}-\frac{1}{2}\times 7\times 24=\frac{1875}{8}$ Fully correct triangle area method (condone one slip if the intention seems clear)		dM1
	$=\frac{1875}{8}\ (234.375)$	Correct area (exact)	A1
			(4)
			Total 10

(c) Way 2	Focus is at $(9,0)$	Correct focus (could be seen on a sketch or implied in working)	B1
	$x=9\Rightarrow 3(9)-4y+48=0\Rightarrow y=18.75$	Correct method to find the y coordinate when x = 9, but also requires correct directrix at some stage of the solution	M1
	E.g. $A=\frac{1}{2}(18.75\times 18)+\frac{1}{2}(18.75\times(16-9))$	Fully correct triangle area method (condone one slip if the intention seems clear)	dM1
	$=\frac{1875}{8}\ (234.375)$	Correct area (exact)	A1
(c) Way 3	Focus is at (9, 0)	Correct focus (could be seen on a sketch or implied in working)	B1
	$x=-9\Rightarrow 3(-9)-4y+48=0\Rightarrow y=5.25$	Correct method with the correct directrix to find the y coordinate of A	M1
	E.g. $\frac{1}{2}\begin{vmatrix} 9 & -9 & 16 & 9 \\ 0 & 5.25 & 24 & 0 \end{vmatrix}$ $=\frac{1}{2}\lvert 47.25-216-84-216\rvert$	Fully correct area method (condone one slip if the intention seems clear)	dM1
	$=\frac{1875}{8}\ (234.375)$	Correct area (exact)	A1

(c) Way 4	Focus is at (9, 0)	Correct focus (could be seen on a sketch or implied in working)	B1
	$x=-9 \Rightarrow 3(-9)-4y+48=0 \Rightarrow y=5.25$	Correct method with the correct directrix to find the *y* coordinate of *A*	M1
	E.g.Rectangle – 3 triangles $(25\times24)-\frac{1}{2}(18\times5.25)-\frac{1}{2}(7\times24)-\frac{1}{2}(25\times18.75)$ Fully correct triangle area method (condone one slip if the intention seems clear)		dM1
	$=\dfrac{1875}{8}\ (234.375)$	Correct area (exact)	A1



Question Number	Scheme	Notes	Marks
8(i)	$\sum_{r=1}^{n} \frac{2r^2-1}{r^2(r+1)^2} = \frac{n^2}{(n+1)^2}$		
	$\frac{2(1)^2-1}{1^2(1+1)^2} = \frac{1}{4}, \frac{1^2}{(1+1)^2} = \frac{1}{4}$	Getting $\frac{1}{2^2}$ or $\frac{1}{4}$ from each is the minimum	B1
	Assume $\sum_{r=1}^{k} \frac{2r^2-1}{r^2(r+1)^2} = \frac{k^2}{(k+1)^2}$		
	$\sum_{r=1}^{k+1} \frac{2r^2-1}{r^2(r+1)^2} = \frac{k^2}{(k+1)^2} + \frac{2(k+1)^2-1}{(k+1)^2(k+2)^2}$ Assumes the result is true for say $n = k$ and adds the next term		M1
	$\frac{k^2(k+2)^2+2(k+1)^2-1}{(k+1)^2(k+2)^2}$	Attempts common denominator	dM1
		Correct expression	A1
	$\frac{k^4+4k^3+4k^2+2k^2+4k+1}{(k+1)^2(k+2)^2} = \frac{k^4+4k^3+6k^2+4k+1}{(k+1)^2(k+2)^2} = \frac{(k+1)^4}{(k+1)^2(k+2)^2}$		
	$\frac{(k+1)^2}{(k+2)^2}$	Achieves this result with intermediate working and no errors	A1
	If the result is <u>true for $n = k$</u>, then it is <u>true for $n = k+1$</u>. As the result has been shown to be <u>true for $n = 1$</u>, then the result is <u>true for all n</u>.	The underlined features should be seen. Some may appear earlier in the solution.	A1cso
			(6)
(ii)	$f(n) = 12^n + 2\times 5^{n-1}$		
	$f(1) = 12 + 2\times 1 = 14$	This is sufficient	B1
	$f(k+1) = 12^{k+1} + 2\times 5^k$	Attempt f(k + 1)	M1
	$f(k+1) - f(k) = 12^{k+1} + 2\times 5^k - 12^k - 2\times 5^{k-1}$ $f(k+1) - f(k) = 11\times 12^k + 22\times 5^{k-1} + 10\times 5^{k-1} - 24\times 5^{k-1}$	Working with f(k + 1) – f(k)	
	$= 11\times\left(12^k + 2\times 5^{k-1}\right) - 14\times 5^{k-1}$	$11\times\left(12^k + 2\times 5^{k-1}\right)$ or $11f(k)$	A1
		$-14\times 5^{k-1}$	A1
	$f(k+1) = 12f(k) - 14\times 5^{k-1}$	Makes f(k + 1) the subject **Dependent on at least one of the A marks**	dM1
	If the result is <u>true for $n = k$</u>, then it is <u>true for $n = k+1$</u>. As the result has been shown to be <u>true for $n = 1$</u>, then the result is <u>true for all n</u>.	The underlined features should be seen. Some may appear earlier in the solution.	A1cso
			(6)
			Total 12

ALT 1	$\mathrm{f}(1) = 12 + 2 \times 1 = 14$	This is sufficient	B1
	$\mathrm{f}(k+1) = 12^{k+1} + 2 \times 5^{k}$	Attempt f(k+1)	M1
	$\mathrm{f}(k+1) = 12\left(12^{k} + 2 \times 5^{k-1}\right) + 2 \times 5 \times 5^{k-1} - 12 \times 2 \times 5^{k-1}$		
	$\mathrm{f}(k+1) = 12\left(12^{k} + 2 \times 5^{k-1}\right) - 14 \times 5^{k-1}$	$12\left(12^{k} + 2 \times 5^{k-1}\right)$ or $12\mathrm{f}(k)$	A1
		$-14 \times 5^{k-1}$	A1
	$\mathrm{f}(k+1) = 12\mathrm{f}(k) - 14 \times 5^{k-1}$	**Dependent on at least one of the A marks**	dM1
	If the result is <u>true for $n = k$</u>, then it is <u>true for $n = k+1$</u>. As the result has been shown to be <u>true for $n = 1$</u>, then the result is <u>true for all *n*</u>.	The underlined features should be seen. Some may appear earlier in the solution.	A1cso
ALT 2	$\mathrm{f}(1) = 12 + 2 \times 1 = 14$	This is sufficient	B1
	Let $12^{k} + 2 \times 5^{k-1} = 7M$		
	$\mathrm{f}(k+1) = 12^{k+1} + 2 \times 5^{k}$	Attempt f(k+1)	M1
	$\mathrm{f}(k+1) = 12\left(7M - 2 \times 5^{k-1}\right) + 2 \times 5^{k}$	OR: $\mathrm{f}(k+1) = 5(7M) + 7 \times 12^{k}$	
	$\mathrm{f}(k+1) = 84M - 14 \times 5^{k-1}$ OR: $\mathrm{f}(k+1) = 35M + 7 \times 12^{k}$	$84M$ OR: $35M$	A1
		$-14 \times 5^{k-1}$ $+7 \times 12^{k}$	A1
	$\mathrm{f}(k+1) = 12\mathrm{f}(k) - 14 \times 5^{k-1}$ OR: $\mathrm{f}(k+1) = 5\mathrm{f}(k) + 7 \times 12^{k}$	**Dependent on at least one of the A marks**	dM1
	If the result is <u>true for $n = k$</u>, then it is <u>true for $n = k+1$</u>. As the result has been shown to be <u>true for $n = 1$</u>, then the result is <u>true for all *n*</u>.	The underlined features should be seen. Some may appear earlier in the solution.	A1cso
ALT 3	$\mathrm{f}(1) = 12 + 2 \times 1 = 14$	This is sufficient	B1
	$\mathrm{f}(k+1) = 12^{k+1} + 2 \times 5^{k}$	Attempts f(k + 1)	M1
	Working with f(k + 1) – *m*f(k) $\mathrm{f}(k+1) - m\mathrm{f}(k) = 12^{k+1} + 2 \times 5^{k} - m\left(12^{k} + 2 \times 5^{k-1}\right)$ $\mathrm{f}(k+1) - \mathrm{f}(k) = (12-m) \times 12^{k} + 2 \times (12-m) \times 5^{k-1} + 10 \times 5^{k-1} - 24 \times 5^{k-1}$		
	$= (12-m) \times \left(12^{k} + 2 \times 5^{k-1}\right) - 14 \times 5^{k-1}$	$(12-m) \times \left(12^{k} + 2 \times 5^{k-1}\right)$ or $(12-m)\mathrm{f}(k)$	A1
		$-14 \times 5^{k-1}$	A1
	$\mathrm{f}(k+1) = 12\mathrm{f}(k) - 14 \times 5^{k-1}$	Makes f(k + 1) the subject **Dependent on at least one of the A marks**	dM1
	If the result is <u>true for $n = k$</u>, then it is <u>true for $n = k+1$</u>. As the result has been shown to be <u>true for $n = 1$</u>, then the result is <u>true for all *n*</u>.	The underlined features should be seen. Some may appear earlier in the solution.	A1cso

ALT 4	$f(1) = 12 + 2 \times 1 = 14$	This is sufficient	B1
	$f(k+1) = 12^{k+1} + 2 \times 5^k$	Attempts f(k + 1)	M1
	$f(k+1) - 5f(k) = 12^{k+1} + 2 \times 5^k - 5\left(12^k + 2 \times 5^{k-1}\right)$	Working with f(k + 1) – 5f(k)	
	$= 7 \times 12^k + 2 \times 5^k - 2 \times 5^k$	7×12^k	A1
		$2 \times 5^k - 2 \times 5^k$ (or zero)	A1
	$f(k+1) = 5f(k) + 7 \times 12^k$	Makes f(k + 1) the subject **Dependent on at least one of the A marks**	dM1
	If the result is <u>true for $n = k$</u>, then it is <u>true for $n = k+1$</u>. As the result has been shown to be <u>true for $n = 1$</u>, then the result is <u>true for all n</u>.	The underlined features should be seen. Some may appear earlier in the solution.	A1cso

<u>NOTES</u>:

<u>Part (i)</u>
This approach may be seen:
Assume result is true for $n = k$ <u>and</u> $n = k + 1$
Subtract: (sum to (k + 1) terms) minus (sum to k terms)
Show that this is equal to the (k + 1)th term
Please send any such response to Review.

<u>Part (ii)</u>
Apart from the given alternatives, other versions will work and can be marked equivalently.
If in any doubt, send to Review.

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