Further Statistics 1 Mark Scheme

Question	Scheme	Marks	AOs
1(a)	H_0: There is no association between language and gender	B1	1.2
		(1)	
(b)	$\frac{54\times 85}{150} = 30.6$ *	B1*cso	1.1b
		(1)	
(c)	Expected frequencies (Language: French, Spanish, Mandarin; Gender: Male 26.43…, 23.4, 15.16…; Female 34.56…, [30.6], 19.83…)	M1	2.1
	$\chi^2 = \sum\frac{(O-E)^2}{E} = \frac{(23-26.43)^2}{26.43}+\ldots+\frac{(15-19.83)^2}{19.83}$	M1	1.1b
	Awrt <u>**3.6/3.7**</u>	A1	1.1b
		(3)	
(d)	Degrees of freedom (3 – 1)(2 – 1) → Critical value $\chi^2_{2,0.01} = 9.210$	M1	3.1b
	As $\sum\frac{(O-E)^2}{E} < 9.210$, the null hypothesis is not rejected	A1	2.2b
		(2)	
(e)	Still not rejected since $\sum\frac{(O-E)^2}{E} < \chi^2_{2,0.1} = 4.605$	B1	2.4
		(1)	
		(8 marks)	

Expected frequencies table for (c):

Expected frequencies		Language		
		French	Spanish	Mandarin
Gender	Male	26.43…	23.4	15.16…
	Female	34.56…	[30.6]	19.83…

Notes:

(a)
B1: For correct hypothesis in context

(b)
B1*: For a correct calculation leading to the given answer and no errors seen

(c)
M1: For attempt at $\frac{\text{(Row Total)(Column Total)}}{\text{(Grand Total)}}$ to find expected frequencies

M1: For applying $\sum\frac{(O-E)^2}{E}$

A1: awrt 3.6 or 3.7

(d)
M1: For using degrees of freedom to set up a χ^2 model critical value
A1: For correct comparison and conclusion

(e)
A1ft: For correct conclusion with supporting reason

Question	Scheme	Marks	AOs
2(a)	$-4 = 2 - 5\text{E}(X)$	M1	3.1a
	$\text{E}(X) = 1.2$		
	$-1\times c + 0\times a + 1\times a + 2\times b + 3\times c = 1.2$	M1	1.1b
	$a + 2b + 2c = 1.2$ $\boxed{1}$		
	$\text{P}(Y \geqslant -3) = 0.45$ gives $\text{P}(2 - 5X \geqslant -3) = 0.45$ i.e. $\text{P}(X \leqslant 1) = 0.45$	M1	2.1
	$2a + c = 0.45$ $\boxed{2}$		
	$2a + b + 2c = 1$ $\boxed{3}$	M1	1.1b
	$\begin{pmatrix} 1 & 2 & 2 \\ 2 & 0 & 1 \\ 2 & 1 & 2 \end{pmatrix}\begin{pmatrix} a \\ b \\ c \end{pmatrix} = \begin{pmatrix} 1.2 \\ 0.45 \\ 1 \end{pmatrix} \Rightarrow \begin{pmatrix} a \\ b \\ c \end{pmatrix} = \begin{pmatrix} 1 & 2 & -2 \\ 2 & 2 & -3 \\ -2 & -3 & 4 \end{pmatrix}\begin{pmatrix} 1.2 \\ 0.45 \\ 1 \end{pmatrix}$ <u>or</u>	M1	1.1b
	e.g. $\boxed{3} - \boxed{2} \Rightarrow b + c = 0.55$ sub. $2(b + c)$ into $\boxed{1} \Rightarrow a = 0.1$ etc		
	$a = 0.1$ $b = 0.3$ $c = 0.25$	A1 A1	1.1b 1.1b
		(7)	
(b)	$\text{Var}(Y) = 75 - (-4)^2$ <u>or</u> 59	M1	1.1a
	$[\text{Var}(Y) = 5^2\text{Var}(X)$ implies$]$ $\text{Var}(X) = 2.36$	A1	1.2
		(2)	
(c)	$\text{P}(Y > X) = \text{P}(2 - 5X > X) \rightarrow \text{P}(X < \frac{1}{3})$	M1	3.1a
	$\text{P}(X < \frac{1}{3}) = a + c = 0.35$	A1ft	1.1b
		(2)	
		(11 marks)	

Notes:

(a)

M1: For using given information to find an expression for $\text{E}(X)$ i.e. use of $\text{E}(Y) = 2 - 5\text{E}(X)$

M1: For use of $\sum x\text{P}(X = x) =$ '1.2'

M1: For use of $\text{P}(Y \geqslant -3) = 0.45$ to set up the argument for solving by forming an equation in a and c

M1: For use of $\sum \text{P}(X = x) = 1$

M1: For solving their 3 linear equations (matrix or elimination)

A1: For any 2 of a, b or c correct

A1: For all 3 correct values

Question 2 notes continued:
Another method for part (a) is: **M1:** For using given information to find the probability distribution for Y leading to an expression for E(Y) **M1:** For use of $\sum yP(Y = y) = -4$ **M1:** For use of P($Y \geqslant -3$) = 0.45 to set up the argument for solving by forming an equation in a and c **M1:** For use of $\sum P(Y = y) = 1$ **M1:** For solving their 3 linear equations (matrix or elimination) **A1:** For any 2 of a, b or c correct **A1:** For all 3 correct values
(b) **M1:** For use of Var(Y) = E(Y^2) – [E(Y)]^2 (may be implied by a correct answer) **A1:** For use of Var(aX) = a^2 Var(X) to reach 2.36 or exact equivalent
(c) **M1:** For rearranging to the form P($X < k$) **A1ft:** 0.1' + '025' (provided their a and c and their $a + c$ are all probabilities) **Another method for part (c) is:** **M1:** For comparing distribution of X with distribution of Y to identify $X = -1$ and $X = 0$ **A1ft:** '0.1' + '025' (provided their a and c and their $a + c$ are all probabilities)

Question	Scheme	Marks	AOs
3(a)	$X \sim \text{Po}(2.6)$ $\quad Y \sim \text{Po}(1.2)$		
	P(each hire 2 in 1 hour) $= \text{P}(X=2) \times \text{P}(Y=2) = 0.25104\ldots \times 0.21685\ldots$	M1	3.3
	$= 0.05444\ldots$ awrt **0.0544**	A1	1.1b
		(2)	
(b)	$W = X + Y \rightarrow \quad W \sim \text{Po}(3.8)$	M1	3.4
	$\text{P}(W = 3) = 0.20458\ldots.$ awrt **0.205**	A1	1.1b
		(2)	
(c)	$T \sim \text{Po}((2.6+1.2)\times 2)$	M1	3.3
	$\text{P}(T < 9) = 0.64819\ldots$ awrt **0.648**	A1	1.1b
		(2)	
(d)	**(i)** Mean = np = **2.4**	B1	1.1b
	(ii) Variance = $np(1 - p) = 2.3904$ awrt **2.39**	B1	1.1b
		(2)	
(e)	**(i)** [$D \sim \text{Po}(2.4)$ $\quad \text{P}(D \leqslant 4)$] $= 0.9041\ldots$ awrt **0.904**	B1	1.1b
	(ii) Since n is large and p is small/mean is approximately equal to variance	B1	2.4
		(2)	
			(10 marks)

Notes:

(a)
M1: For $\text{P}(X=2) \times \text{P}(Y=2)$ from $X \sim \text{Po}(2.6)$ and $Y \sim \text{Po}(1.2)$ i.e. correct models (may be implied by correct answer)
A1: **awrt 0.0544**

(b)
M1: For combining Poisson distributions and use of Po('3.8') (may be implied by correct answer)
A1: **awrt 0.205**

(c)
M1: For setting up a new model and attempting mean of Poisson distribution (may be implied by correct answer)
A1: **awrt 0.648**

(d)(i)
B1: For **2.4**

(d)(ii)
B1: For **awrt 2.39**

(e)(i)
B1: For **awrt 0.904**

(e)(ii)
B1: For a correct explanation to support use of Poisson approximation in this case

Question	Scheme	Marks	AOs
4(a)	**(i)** P(X = 1) = 0.34523… awrt **0.345**	B1	1.1b
	(ii) P($X \leqslant$ 4) = 0.98575… awrt **0.986**	B1	1.1b
		(2)	
(b)	$\frac{(0\times10)+1\times16+2\times7+3\times4+4\times2+(5\times0)+6\times1}{40}=1.4*$	B1*cso	1.1b
		(1)	
(c)	r = 40 × '0.34523…' s = 40 × '1 – 0.986…'	M1	3.4
	r = **13.81** s = **0.57**	A1ft	1.1b
		(2)	
(d)	H_0: The Poisson distribution is a suitable model H_1: The Poisson distribution is not a suitable model	B1	3.4
	[Cells are combined when expected frequencies < 5] So combine the last 3 cells	M1	2.1
	$\chi^2 = \sum\frac{(O-E)^2}{E} = \frac{(10-9.86)^2}{9.86} + ... + \frac{(7-(4.51+1.58+0.57))^2}{(4.51+1.58+0.57)}$	M1	1.1b
	awrt **1.1**	A1	1.1b
	Degrees of freedom = 4 – 1 – 1 = 2	B1	3.1b
	(Do not reject H_0 since 1.10 < $\chi^2_{2,(0.05)} = 5.991$). The number of mortgages approved each week follows a Poisson distribution	A1	3.5a
		(6)	
			(11 marks)

Notes:

(a)(i)
B1: awrt 0.345

(a)(ii)
B1: awrt 0.986

(b)
B1*: For a fully correct calculation leading to given answer with no errors seen

(c)
M1: For attempt at r or s (may be implied by correct answers)
A1ft: For both values correct (follow through their answers to part (a))

(d)
B1: For both hypotheses correct (lambda should not be defined so correct use of the model)
M1: For understanding the need to combine cells before calculating the test statistic (may be implied)
M1: For attempt to find the test statistic using $\chi^2 = \sum\frac{(O-E)^2}{E}$
A1: awrt 1.1
B1: For realising that there are 2 degrees of freedom leading to a critical value of $\chi_2^{\,2}(0.05) = 5.991$
A1: Concluding that a Poisson model is suitable for the number of mortgages approved each week