Write your name here

Surname MODEL SOLUTIONS	Other names

Pearson Edexcel Level 3 GCE

Centre Number

Candidate Number

Further Mathematics

Advanced Subsidiary
Further Mathematics options
Further Statistics 2

Sample Assessment Material for first teaching September 2017

Time: 50 minutes

Paper Reference

8FM0/2G

You must have:
Mathematical Formulae and Statistical Tables, calculator

Total Marks

Candidates may use any calculator permitted by Pearson regulations. Calculators must not have the facility for algebraic manipulation, differentiation and integration, or have retrievable mathematical formulae stored in them.

Instructions

- Use **black** ink or ball-point pen.
- If pencil is used for diagrams/sketches/graphs it must be dark (HB or B).
- **Fill in the boxes** at the top of this page with your name, centre number and candidate number.
- Answer the questions in the spaces provided – *there may be more space than you need.*
- You should show sufficient working to make your methods clear. Answers without working may not gain full credit.
- Answers should be given to three significant figures unless otherwise stated.

Information

- A booklet 'Mathematical Formulae and Statistical Tables' is provided.
- There are 4 questions in this question paper. The total mark for this paper is 40.
- The marks for **each** question are shown in brackets – *use this as a guide as to how much time to spend on each question.*

Advice

- Read each question carefully before you start to answer it.
- Try to answer every question.
- Check your answers if you have time at the end.

Turn over

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Pearson

Answer ALL questions. Write your answers in the spaces provided.

1. In a gymnastics competition, two judges scored each of 8 competitors on the vault.

Competitor	A	B	C	D	E	F	G	H
Judge 1's rank	8	4	7	6	5	1	3	2
Judge 1's scores	4.6	9.1	8.4	8.8	9.0	9.5	9.2	9.4
Judge 2's scores	7.8	8.8	8.6	8.5	9.1	9.6	9.0	9.3
Judge 2's rank	8	5	6	7	3	1	4	2

(a) Calculate Spearman's rank correlation coefficient for these data.

(4)

(b) Stating your hypotheses clearly, test at the 1% level of significance, whether or not the two judges are generally in agreement.

(4)

(c) Give a reason to support the use of Spearman's rank correlation coefficient in this case.

(1)

The judges also scored the competitors on the beam.

Spearman's rank correlation coefficient for their ranks on the beam was found to be 0.952

(d) Compare the judges' ranks on the vault with their ranks on the beam.

(1)

a)

Competitor	A	B	C	D	E	F	G	H
d^2	0	1	1	1	4	0	1	0

$d^2 = (\text{judge 1 rank} - \text{judge 2 rank})^2$

$\sum d^2 = 8$

$r_s = 1 - \frac{6 \sum d^2}{n(n^2-1)}$

$r_s = 1 - \frac{(6 \times 8)}{8(64-1)}$

$= 0.90476\ldots$

$\boxed{\simeq 0.905}$

Question 1 continued

b) $H_0: \rho_s = 0$ $H_1: \rho_s > 0$

critical value $\rho_s = 0.8333$ from table

$r_s = 0.905$ lies in the critical region

reject H_0

$\therefore$ the two judges are in agreement

c) the data is unlikely to be from a bivariate normal distribution

we are focused on the ranks, not individual scores

d) there is positive correlation in both cases, but r_s is closer to 1 for the beam scores, indicating the judges are more in agreement for the beam

(Total for Question 1 is 10 marks)

2. The continuous random variable X has probability density function

$$f(x) = \begin{cases} \frac{1}{18}(11 - 2x) & 1 \leqslant x \leqslant 4 \\ 0 & \text{otherwise} \end{cases}$$

(a) Find $P(X < 3)$

(2)

(b) State, giving a reason, whether the upper quartile of X is greater than 3, less than 3 or equal to 3

(1)

Given that $E(X) = \frac{9}{4}$

(c) use algebraic integration to find $\text{Var}(X)$

(3)

The cumulative distribution function of X is given by

$$F(x) = \begin{cases} 0 & x < 1 \\ \frac{1}{18}(11x - x^2 + c) & 1 \leqslant x \leqslant 4 \\ 1 & x > 4 \end{cases}$$

(d) Show that $c = -10$

(2)

(e) Find the median of X, giving your answer to 3 significant figures.

(3)

a) $P(X<3) = \int_1^3 \frac{1}{18}(11-2x)\,dx$

continuous p.d.f.
→ integrate over range

$= \left[\frac{1}{18}(11x - x^2)\right]_1^3$

$= \frac{1}{18}(24 - 10)$

$\boxed{= \frac{7}{9}}$ ← upper quartile

b) $P(X<3)$ is > 0.75, so the upper quartile is less than 3

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Question 2 continued

c) $E(X^2) = \int_1^4 x^2 f(x)$

$= \int_1^4 \frac{1}{18} x^2 (11 - 2x)\,dx$

$= \int_1^4 \frac{11x^2}{18} - \frac{1}{9}x^3\,dx = \left[\frac{11x^3}{18\times 3} - \frac{x^4}{9\times 4}\right]_1^4$

$= \left[\frac{11x^3}{54} - \frac{x^4}{36}\right]_1^4$

$= \frac{160}{27} - \frac{19}{108}$

$= \frac{23}{4}$

$\mathrm{Var}(X) = E(X^2) - (E(X))^2$

$= \frac{23}{4} - \left(\frac{9}{4}\right)^2$

$= \boxed{\frac{11}{16}}$

d) $F(4) = 1$

$\frac{1}{18}(11(4) - 4^2 + c) = 1$

$F(1) = 0$

$\frac{1}{18}(11(1) - 1^2 + c) = 0$

$\rightarrow c = -10$

e) $F(\text{median}) = 0.5$
$\hookrightarrow m$

$\frac{1}{18}(11m - m^2 - 10) = 0.5 \rightarrow 11m - m^2 - 10 = 9$

$\rightarrow m^2 - 11m + 19 = 0$

quadratic formula: $a = 1, b = -11, c = 19$

$m = \frac{11 \pm \sqrt{11^2 - 4(19)}}{2} = 2.1458\ldots$ or $8.8541\ldots$

$\simeq \boxed{2.15}$

but $1 \leq x \leq 4$ so n/a

Question 2 continued

Question 2 continued

(Total for Question 2 is 11 marks)

3. A scientist wants to develop a model to describe the relationship between the average daily temperature, x °C, and a household's daily energy consumption, y kWh, in winter.

A random sample of the average temperature and energy consumption are taken from 10 winter days and are summarised below.

$$\sum x = 12 \quad \sum x^2 = 24.76 \quad \sum y = 251 \quad \sum y^2 = 6341 \quad \sum xy = 284.8$$

$$S_{xx} = 10.36 \qquad S_{yy} = 40.9$$

(a) Find the product moment correlation coefficient between y and x. **(2)**

(b) Find the equation of the regression line of y on x in the form $y = a + bx$ **(3)**

(c) Use your equation to estimate the daily energy consumption when the average daily temperature is 2 °C **(1)**

(d) Calculate the residual sum of squares (RSS). **(2)**

The table shows the residual for each value of x.

x	−0.4	−0.2	0.3	0.8	1.1	1.4	1.8	2.1	2.5	2.6
Residual	−0.63	−0.32	−0.52	−0.73	0.74	2.22	1.84	0.32	f	−1.88

(e) Find the value of f. **(2)**

(f) By considering the signs of the residuals, explain whether or not the linear regression model is a suitable model for these data. **(1)**

a) $r = \frac{n\sum xy - \sum x \sum y}{n\sqrt{S_{xx} S_{yy}}} = \frac{284.8 - \frac{(251 \times 12)}{10}}{\sqrt{10.36 \times 40.9}}$

$= -0.79671\ldots \boxed{\simeq -0.797}$

b) $b = \frac{-16.4}{10.36}$ ← S_{xy}

$a = \frac{251}{10} - b\frac{12}{10} \Rightarrow \boxed{y = 27.0 - 1.58x}$

$a = 26.9996 \simeq 27.0$

Question 3 continued

c) $y = 27.0 - 1.58(2)$

$\boxed{= 23.84}$

d) $RSS = 40.9 - \frac{(-16.4)^2}{10.36}$

$= 14.938\ldots \quad \boxed{\simeq 14.9}$

e) $\sum$ residuals $= 0$

so $-0.63 + (-0.32) + (-0.52) + \ldots + f + (-1.88) = 0$

$\boxed{\therefore f = -1.04}$ (4 above, 6 below)

f) the residuals aren't scattered randomly above & below 0, as is necessary for a linear model, so such a model might not suit the data

Question 3 continued

Question 3 continued

(Total for Question 3 is 11 marks)

4. The continuous random variable X is uniformly distributed over the interval [–3, 5].

(a) Sketch the probability density function f(x) of X.

(2)

(b) Find the value of k such that $P(X < 2[k - X]) = 0.25$

(3)

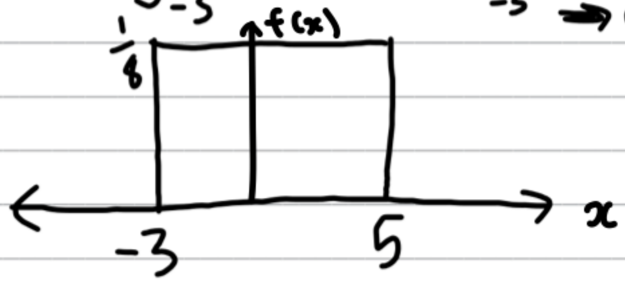
(c) Use algebraic integration to show that $E(X^3) = 17$

(3)

a) continuous & uniform

$\int P(x) = 1 \Rightarrow \int_{-3}^{5} c\,dx = [cx]_{-3}^{5} = 8c$

$\Rightarrow c = \frac{1}{8}$



(they ask for a sketch, so mark on the important details, i.e. bounds, height)

b) $P(X < 2(k - X)) = P(X < \frac{2}{3}k)$

$\frac{\frac{2}{3}k - (-3)}{5 - (-3)} = 0.25$ distance from -3 to $\frac{2}{3}k$ is $\frac{1}{4}$ distance from -3 to 5 on graph

$\frac{2}{3}k = (0.25)8 - 3$

$\Rightarrow \boxed{k = -\frac{3}{2}}$

c) $E(X^3) = \int_{-3}^{5} \frac{1}{5-(-3)} x^3\,dx$

$= \left[\frac{1}{32}x^4\right]_{-3}^{5} = \frac{1}{32}(5^4 - (-3)^4) = \boxed{17}$

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Question 4 continued

Question 4 continued

(Total for Question 4 is 8 marks)

TOTAL IS 40 MARKS