MODEL ANSWERS

Write your name here

Surname	Other names

Pearson Edexcel Level 3 GCE

Centre Number

Candidate Number

Further Mathematics

Advanced Subsidiary
Further Mathematics options
24: Further Statistics 2
(Part of option G only)

Thursday 17 May 2018 – Afternoon

Paper Reference **8FM0-24**

You must have:
Mathematical Formulae and Statistical Tables, calculator

Total Marks

Candidates may use any calculator allowed by the regulations of the Joint Council for Qualifications. Calculators must not have the facility for symbolic algebra manipulation, differentiation and integration, or have retrievable mathematical formulae stored in them.

Instructions

- Use **black** ink or ball-point pen.
- If pencil is used for diagrams/sketches/graphs it must be dark (HB or B).
- **Fill in the boxes** at the top of this page with your name, centre number and candidate number.
- Answer **all** questions and ensure that your answers to parts of questions are clearly labelled.
- Answer the questions in the spaces provided – *there may be more space than you need.*
- You should show sufficient working to make your methods clear. Answers without working may not gain full credit.
- Answers should be given to three significant figures unless otherwise stated.

Information

- A booklet 'Mathematical Formulae and Statistical Tables' is provided.
- The total mark for this part of the examination is 40. There are 4 questions.
- The marks for each question are shown in brackets – *use this as a guide as to how much time to spend on each question.*

Advice

- Read each question carefully before you start to answer it.
- Try to answer every question.
- Check your answers if you have time at the end.

Turn over ▶

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Answer ALL questions. Write your answers in the spaces provided.

1. The scores achieved on a maths test, m, and the scores achieved on a physics test, p, by 16 students are summarised below.

$$\sum m = 392 \quad \sum p = 254 \quad \sum p^2 = 4748 \quad S_{mm} = 1846 \quad S_{mp} = 1115$$

(a) Find the product moment correlation coefficient between m and p

(2)

(b) Find the equation of the linear regression line of p on m

(3)

Figure 1 shows a plot of the residuals.

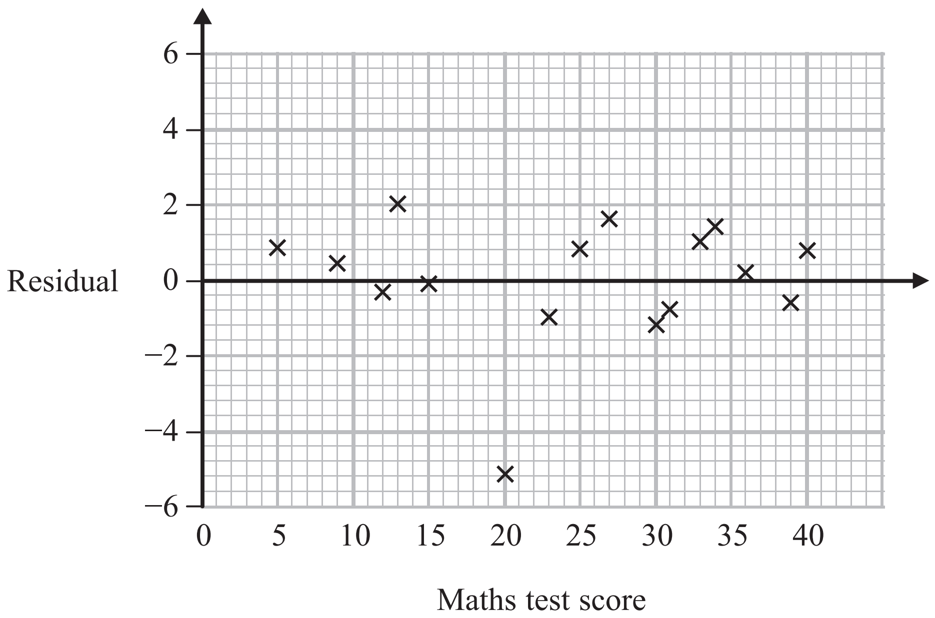


Figure 1

(c) Calculate the residual sum of squares (RSS).

(2)

For the person who scored 30 marks on the maths test,

(d) find the score on the physics test.

(2)

The data for the person who scored 20 on the maths test is removed from the data set.

(e) Suggest a reason why.

(1)

The product moment correlation coefficient between m and p is now recalculated for the remaining 15 students.

(f) Without carrying out any further calculations, suggest how you would expect this recalculated value to compare with your answer to part (a).
Give a reason for your answer.

(1)

Question 1 continued

a) $S_{pp} = \sum p^2 - \left(\frac{\sum p}{n}\right)^2 = 715.75$

$\therefore r = \frac{S_{mp}}{\sqrt{S_{mm} S_{pp}}} = \frac{1115}{\sqrt{(715.75)(1846)}} = \boxed{0.970}$ (3dp)

b) $p = bm + a$

$b = \frac{S_{mp}}{S_{mm}} = \frac{1115}{1846} = 0.6040...$

$a = \frac{\sum p}{n} - b\frac{\sum m}{n} = \frac{254}{16} - \frac{1115}{1846}\left(\frac{392}{16}\right)$

$= 1.0768...$

so $\boxed{p = 1.08 + 0.604m}$ a, b to 3s.f

c) $RSS = S_{pp} - \frac{S_{mp}^2}{S_{mm}} = 715.75 - \frac{1115^2}{1846}$

$= \boxed{42.3}$

d) $p = 1.08 + 0.604(30) = 19.2$

residual $= -1.2$ (from graph)

$\therefore p = 19.2 - 1.2 = \boxed{18}$

e) Residual is far from 0; doesn't fit the trend of data. Seems to be an outlier.

Question 1 continued

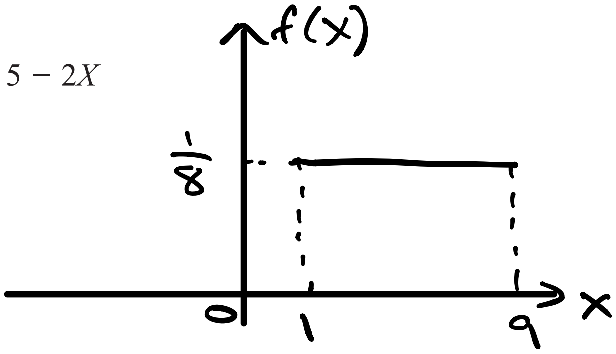
f) Should be closer to 1 since the outlier is gone. Remaining points will be closer to the new regression line.

2. The continuous random variable X has probability density function

$$f(x) = \begin{cases} \frac{1}{8} & 1 \leqslant x \leqslant 9 \\ 0 & \text{otherwise} \end{cases}$$

(a) Write down the name given to this distribution.

(1)



The continuous random variable $Y = 5 - 2X$

(b) Find $P(Y > 0)$

(2)

(c) Find $E(Y)$

(2)

(d) Find $P(Y < 0 \mid X < 7.5)$

(3)

a) Continuous Uniform Distribution.

b) $P(Y>0) = P(5-2X>0) = P(X<\frac{5}{2})$

$\left(\frac{5}{2}-1\right)\left(\frac{1}{8}\right) = \boxed{\frac{3}{16}}$

c) $E(Y) = E(5-2X) = 5-2E(X)$

$E(X) = \frac{1+9}{2} = 5$ //

$\therefore E(Y) = 5-2(5) = \boxed{-5}$

d) $P(Y<0 \mid X<7.5) = \frac{P(Y<0 \cap X<7.5)}{P(X<7.5)}$

$P(Y<0) = P(X>\frac{5}{2})$.

$\therefore P(\text{required}) = \frac{P\left(\frac{5}{2}<X<\frac{15}{2}\right)}{P\left(X<\frac{15}{2}\right)} = \frac{\left(\frac{15}{2}-\frac{5}{2}\right)\left(\frac{1}{8}\right)}{\left(\frac{15}{2}-1\right)\left(\frac{1}{8}\right)}$

$= \boxed{\frac{10}{13}}$

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3. The table below shows the heights cleared, in metres, for each of 6 competitors in a high jump competition.

Competitor	A	B	C	D	E	F
Height (m)	2.05	1.93	2.02	1.96	1.81	2.02

These 6 competitors also took part in a long jump competition and finished in the following order, with C jumping the furthest.

C A F D B E

(a) Calculate Spearman's rank correlation coefficient for these data.
(4)

(b) Stating your hypotheses clearly, test at the 5% level of significance whether or not there is a positive correlation between results in the high jump and results in the long jump.
(4)

The product moment correlation coefficient between the height of the high jump and the length of the long jump for each competitor is found to be 0.678

(c) Use this value to test, at the 5% level of significance, for evidence of positive correlation between results in the high jump and results in the long jump.
(2)

(d) State the condition required for the test in part (c) to be valid.
(1)

(e) Explain what your conclusions in part (b) and part (c) suggest about the relationship between results in the high jump and results in the long jump.
(1)

a)

Competitor	High	Long
A	1	2
B	5	5
C	2.5	1
D	4	4
E	6	6
F	2.5	3

tied ranks.

$\sum h^2 = 90.5$, $\sum l^2 = 91$, $\sum hl = 89$

$$r_s = \frac{S_{xy}}{\sqrt{S_{xx}S_{yy}}} = \frac{89 - \frac{21^2}{6}}{\sqrt{(90.5 - \frac{21^2}{6})(91 - \frac{21^2}{6})}} = \boxed{0.899}$$

P 6 0 2 0 5 A 0 1 0 1 6

Question 3 continued

b) $H_0 : \rho = 0$ critical value : 0.8286

$H_1 : \rho > 0$ $0.899 > 0.8286$

∴ Result is significant. Reject H_0. Evidence suggests a positive correlation between high & long jump.

c) $H_0 : \rho = 0$ critical value : 0.7293

$H_1 : \rho > 0$ $0.678 < 0.7293$

∴ Result is insignificant. Insufficient evidence to conclude a positive correlation between high / long jump.

d) h and l must be jointly normally distributed.

e) Part b did display evidence of a positive correlation, while c did not So the data generally doesn't fit a linear pattern.

4. The continuous random variable X has cumulative distribution function

$$\mathrm{F}(x) = \begin{cases} 0 & x < 3 \\ c - 4.5x^n & 3 \leqslant x \leqslant 9 \\ 1 & x > 9 \end{cases}$$

where c is a positive constant and n is an integer.

(a) Showing all stages of your working, find the value of c and the value of n

(7)

(b) Find the lower quartile of X

(2)

a) $c - 4.5(9)^n = 1$

$c = 4.5(9)^n + 1$ //

$f(x) = \frac{d}{dx}(c - 4.5x^n) = -4.5nx^{n-1}$ //

$$f(x) = \begin{cases} -4.5nx^{n-1}, & 3 \leq x \leq 9 \\ 0, & \text{otherwise} \end{cases}$$

$$\int_3^9 (-4.5nx^{n-1})\,dx = 1$$

$$-4.5\int_3^9 nx^{n-1}\,dx = 1$$

$$-4.5\left[x^n\right]_3^9 = 1$$

$[9^n - 3^n] = \frac{-2}{9}$ //

P 6 0 2 0 5 A 0 1 4 1 6

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Question 4 continued

$$3^{2n} - 3^n + \frac{2}{9} = 0 \;//$$

This is a quadratic in 3^n:

let $y = 3^n$, $\quad y^2 - y + \frac{2}{9} = 0$

$$9y^2 - 9y + 2 = 0$$

$$(3y-1)(3y-2) = 0$$

$$y = \frac{1}{3}, \quad y = \frac{2}{3}$$

$\Rightarrow 3^n = \frac{1}{3}$ $\qquad\qquad \Rightarrow 3^n = \frac{2}{3}$

$\Rightarrow n\ln 3 = \ln\frac{1}{3}$ $\qquad \Rightarrow n\ln 3 = \ln\frac{2}{3}$

$\Rightarrow n = \frac{\ln\frac{1}{3}}{\ln 3} = -1$ $\qquad \Rightarrow n = \frac{\ln\frac{2}{3}}{\ln 3} = -0.37$

n is an integer so $\boxed{n = -1 \;//}$

$\therefore c = 1 + 4.5(9)^{-1} = 1 + \frac{4.5}{9}$

$$c = 1 + \frac{1}{2} = \boxed{\frac{3}{2}}$$

Question 4 continued

b) $F(x) = 0.25$

$\frac{3}{2} - 4.5x^{-1} = 0.25$

$\frac{5}{4} = \frac{4.5}{x} \quad \therefore \; x = \frac{4.5}{\frac{5}{4}} = \boxed{3.6} = Q_1$

(Total for Question 4 is 9 marks)

TOTAL FOR FURTHER STATISTICS 2 IS 40 MARKS