Mark Scheme (Result)

November 2021

Pearson Edexcel GCE Further Mathematics
Advanced Level in Further Mathematics
Paper 9FM0/3B

Edexcel and BTEC Qualifications

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General Marking Guidance

- All candidates must receive the same treatment. Examiners must mark the first candidate in exactly the same way as they mark the last.
- Mark schemes should be applied positively. Candidates must be rewarded for what they have shown they can do rather than penalised for omissions.
- Examiners should mark according to the mark scheme not according to their perception of where the grade boundaries may lie.
- There is no ceiling on achievement. All marks on the mark scheme should be used appropriately.
- All the marks on the mark scheme are designed to be awarded. Examiners should always award full marks if deserved, i.e. if the answer matches the mark scheme. Examiners should also be prepared to award zero marks if the candidate's response is not worthy of credit according to the mark scheme.
- Where some judgement is required, mark schemes will provide the principles by which marks will be awarded and exemplification may be limited.
- When examiners are in doubt regarding the application of the mark scheme to a candidate's response, the team leader must be consulted.
- Crossed out work should be marked UNLESS the candidate has replaced it with an alternative response.

EDEXCEL GCE MATHEMATICS

General Instructions for Marking

1. The total number of marks for the paper is 75.

2. The Edexcel Mathematics mark schemes use the following types of marks:

- M marks: method marks are awarded for 'knowing a method and attempting to apply it', unless otherwise indicated.
- A marks: Accuracy marks can only be awarded if the relevant method (M) marks have been earned.
- B marks are unconditional accuracy marks (independent of M marks)
- Marks should not be subdivided.

3. Abbreviations

These are some of the traditional marking abbreviations that will appear in the mark schemes.

- bod – benefit of doubt
- ft – follow through
- the symbol √ will be used for correct ft
- cao – correct answer only
- cso - correct solution only. There must be no errors in this part of the question to obtain this mark
- isw – ignore subsequent working
- awrt – answers which round to
- SC: special case
- oe – or equivalent (and appropriate)
- dep – dependent
- indep – independent
- dp decimal places
- sf significant figures
- ∗ The answer is printed on the paper
- ⌈ The second mark is dependent on gaining the first mark

4. All A marks are 'correct answer only' (cao.), unless shown, for example, as A1 ft to indicate that previous wrong working is to be followed through. After a misread however, the subsequent A marks affected are treated as A ft, but manifestly absurd answers should never be awarded A marks.

Paper 3B/ 2021: Statistics 1 Mark scheme

Question	Scheme	Marks	AOs
1(a)	$x = 4\times 43 - 47 - 34 - 36\ = 55$*	B1*	3.4
		(1)	
(b)	$\nu = 4 - 1 = 3$ since the only constraint is that the totals agree	B1	2.4
		(1)	
(c)	H_0: The die is unbiased	B1	2.1
	H_1: The die is biased		
	Test Statistic $= \frac{(47-43)^2}{43} + \frac{(34-43)^2}{43} + \frac{(36-43)^2}{43} + \frac{(55-43)^2}{43}$	M1	1.1b
	$= 6.744\ldots$	A1	1.1b
	$\chi^2_{(3,0.05)} = 7.815$	B1	1.1b
	Not in the critical region since 7.815 > "6.74…" therefore insufficient evidence to reject H_0 Inconclusive test - consistent with the **die** being unbiased.	A1	3.5a
		(5)	
			(7 marks)

Notes:

(a)	**B1*:**	Using the uniform model to show the missing observed value eg $x = \frac{43 - 0.25\times(47+34+36)}{0.25} = 55$
(b)	**B1:**	$4 - 1 = 3$ (may be in words) and explanation of what the constraint is
(c)	**B1:**	Both hypotheses correct. eg The data fits a discrete uniform distribution
	M1:	Attempting to find $\sum\frac{(O-E)^2}{E}$ or $\sum\frac{O^2}{E} - N$ May be implied by awrt 6.74 or p value of 0.0805…
	A1:	awrt 6.74 or $\frac{290}{43}$ oe May be implied by p value of 0.0805…
	B1:	awrt 7.82 (Calc 7.8147…)
	A1:	Drawing correct inference in context. Need the word die or tetrahedral

Question	Scheme	Marks	AOs
2(a)	$C \sim$ Poisson (3.75)	M1	3.3
	$\mathrm{P}(C \geqslant 2) =$ 0.88829…*. awrt 0.8883*	A1*cso	1.1b
		(2)	
(b)	$D \sim$ B(6, "0.888")	M1	3.3
	$\mathrm{P}(D \leqslant 3) = 0.02163...$ awrt 0.0216 / 0.0215	A1	1.1b
		(2)	
(c)	$\mathrm{P}(C = 8) = 0.02281...$	B1	1.1b
	$E \sim$ B(150, "0.02281..") $\Rightarrow$ mean $= 150 \times "0.02281..." [= 3.4215\ldots]$	M1	3.3
	$E \sim$ Po("3.4215…") $\Rightarrow \mathrm{P}(E \geq 3) = \left[1 - \mathrm{P}(E \leqslant 2)\right]$	M1	3.4
	$=$ 0.664 *	A1*cso	2.1
		(4)	
(d)	The number of periods is large and the probability of receiving 8 calls in 30-minutes is small.	B1 (1)	2.4
(e)	H_0: $\lambda = 30$ H_1: $\lambda \neq 30$	B1	2.5
		(1)	
(f)	$X \sim$ Po(30)	B1	3.3
	$\mathrm{P}(X \geqslant 40) = 1 - \mathrm{P}(X \leqslant 39)$	M1	1.1b
	$= 0.04625\ldots$	A1	1.1b
	0.046… > 0.025 or no evidence to reject H_0 There is insufficient evidence at the 5% level of significance that the number of **calls** received is different on a Saturday	A1 (4)	2.2b
			(14 marks)

Notes:

(a)	**M1:**	For calculating the mean and setting up the correct model. Poisson may be implied by 0.8883 or better or 1 – awrt 0.1117 but must see 3.75 or 1.25×3
	A1*cso:	$\mathrm{P}(C \geqslant 2) =$ awrt 0.8883 or 1 – awrt 0.1117 = 0.888 Must see $\mathrm{P}(C \geqslant 2)$ oe
(b)	**M1:**	Setting up a new model using their answer to (a) Implied by correct answer
	A1:	awrt 0.0216 or awrt 0.0215
(c)	**B1:**	awrt 0.0228
	M1:	Setting up a new model B(150, "0.0228") and using np (working seen if incorrect)
	M1:	Using the model Po(their np) Must be clearly stated and $\mathrm{P}(E \geqslant 3)$ oe seen
	A1*cso:	Only award if the previous 3 marks have been awarded and 0.664 is stated. NB Use of B(150 0.02281) gives 0.668
(d)	**B1:**	Idea that n = 150 (number of periods selected) is large and p is 0.022… (exactly 8 calls in the time period) is small.
(e)	**B1:**	Both hypotheses correct using λ or μ allow 1.25 or 3.75
(f)	**B1:**	Realising Po(30) needs to be used. NB Implied by correct answer or P(X = 40) = 0.0139…
	M1:	Writing or using $1 - \mathrm{P}(X \leqslant 39)$ or if CR method for $\mathrm{P}\left(X \geqslant 42\right) = 0.0221\ldots$
	A1:	0.04… or awrt 0.05 or CR $X \geqslant 42$ oe must be CR and not probability
	A1:	A fully correct solution and correct inference in context. Calls required If put this prob but then give Cr X >= 40 M1A1A0

Question	Scheme	Marks	AOs
3	$\overline{X} \approx \mathrm{N}(256,...)$ oe	M1	3.1a
	$\overline{X} \approx \mathrm{N}(256, 0.9216)$	A1	1.1b
	$\mathrm{P}(\overline{X} > 257) = \mathrm{P}\left(Z > \frac{257-256}{\sqrt{"0.9216"}}\right)[= \text{awrt } 1.04]$	dM1	3.4
	$p = 0.1492...$	A1	1.1b
		(4)	
		(4 marks)	

Notes:

	M1:	For realising the need to use the CLT with correct mean
	A1:	For a correct normal stated
	dM1:	Dep on previous Method mark. Use of the normal model to find $\mathrm{P}(\overline{X} > 257)$ If final answer is incorrect then we need to see the standardisation using their σ.
	A1:	awrt 0.149 (0.14878… from calculator)
		NB Allow awrt 0.148 if a continuity correction is used.

<table>
<tr><th>Question</th><th>Scheme</th><th>Marks</th><th>AOs</th></tr>
<tr><td>4(a)</td><td>4E(N) + 2 =14.8 or E(N) = 3.2</td><td>M1</td><td>3.1a</td></tr>
<tr><td></td><td>$0.2+0.1+0.75+4b+5c=3.2$</td><td>M1</td><td>1.1b</td></tr>
<tr><td></td><td>$\frac{c}{0.25+b+c}=0.5$ or $0.25=c-b$</td><td>M1</td><td>3.1a</td></tr>
<tr><td></td><td>$b=0.1$ and $c=0.35$</td><td></td><td></td></tr>
<tr><td></td><td>$\mathrm{E}(N^2)=1\times0.2+4\times0.05+9\times0.25+16\times"0.1"+25\times"0.35"\left[=13\right]$</td><td>M1</td><td>1.1b</td></tr>
<tr><td></td><td>$\mathrm{Var}(N)="13"-"3.2"^2$</td><td>dM1</td><td>1.1b</td></tr>
<tr><td></td><td>= 2.76 *</td><td>A1*</td><td>2.1</td></tr>
<tr><td></td><td></td><td>(6)</td><td></td></tr>
<tr><td>(b)</td><td><table><tr><td>fee</td><td>0</td><td>50</td><td>70</td><td>90</td><td>100</td><td>100</td></tr><tr><td>$\mathrm{P}(N=n)$</td><td>a</td><td>0.2</td><td>0.05</td><td>0.25</td><td>b</td><td>c</td></tr></table></td><td>M1</td><td>3.3</td></tr>
<tr><td></td><td>$50\times0.2+70\times0.05+90\times0.25+100\times"0.1"+100\times"0.35"$</td><td>M1</td><td>1.1b</td></tr>
<tr><td></td><td>= 81p</td><td>A1</td><td>1.1b</td></tr>
<tr><td></td><td></td><td>(3)</td><td></td></tr>
<tr><td>(c)</td><td>Poisson distribution will assign substantial probability to $N>5$</td><td>B1</td><td>3.5b</td></tr>
<tr><td></td><td></td><td>(1)</td><td></td></tr>
<tr><td colspan="4" align="right">(10 marks)</td></tr>
</table>

Notes

(a)	**M1:**	For using the given information to find E(N)
		ALT $a+b+c=0.5$ oe
	M1:	For use of $\sum n\mathrm{P}(N=n)="3.2"$ At least 3 terms correct
		ALT $\sum(4n+2)\mathrm{P}(N=n)=14.8 \Rightarrow 2a+1.2+0.5+3.5+18b+22c=14.8$ At least 3 terms correct
	M1:	Forming an equation in *b* and *c* using conditional probability
	M1:	For using $\sum n^2\mathrm{P}(N=n)$ Allow with the letters *b* and *c*
	dM1:	Dependent on previous method mark. Correct method to find Var(N)
	A1*:	All previous marks must be awarded and 2.76 stated
(b)	**M1:**	Setting up a new model with the correct fees. At least 3 terms correct. Allow 0.5, 0.7, 0.9, 1
	M1:	Correct method for calculating E(fee) Allow with the letters *b* and *c*
	A1:	81[p] No units needed. Allow 0.81 if fees are in pounds
(c)	**B1:**	A correct limitation.

Question	Scheme	Marks	AOs
5(a)	P(at least 3 whites) = $\left(1-0.07\right)^3$ **or** $1-0.07-0.93\times0.07-0.93^2\times0.07$	M1	1.1b
	= 0.8043… awrt 0.804	A1	1.1b
		(2)	
(b)	P(2nd red on 9th draw) = $\begin{pmatrix}8\\1\end{pmatrix}0.93^7\times0.07^2$	M1	3.3
	= 0.02358… awrt 0.0236	A1	1.1b
		(2)	
(c)	$\frac{n}{p}=4400$ and $\frac{n(1-p)}{p^2}=660^2$	M1 A1	3.1b 1.1b
	$1-p=99p$ oe	M1	1.1b
	$p=0.01$	A1	1.1b
		(4)	
(d)	H_0: p = 0.07 H_1: p < 0.07	B1	2.5
	J ~ Geo(0.07)	M1	3.3
	$\mathrm{P}(J \geq c) < 0.1 \Rightarrow (1-0.07)^{c-1} < 0.1$	M1	3.4
	$c-1>\frac{\log 0.1}{\log 0.93}$	M1	1.1b
	$c>32.72...$ $\therefore$ CR $J \geq 33$	A1	1.1b
		(5)	
(e)	34 is in the Critical region	M1	1.1b
	There is evidence to suggest that Jerry**'s bag** contains a smaller **proportion** of **red counters** than Asha**'s** bag.	A1	2.2b
		(2)	
(f)	Power of test = $\mathrm{P}(J \geq 33 \mid p=0.011)$	M1	2.1
	= $(1-0.011)^{32}$ oe	M1	1.1b
	= 0.7019…*	A1*	1.1b
		(3)	
		(18 marks)	

Notes:		
(a)	**M1:**	A correct method to find $P(X \geqslant 3)$
	A1:	awrt 0.804
(b)	**M1:**	For selecting the appropriate model negative binomial or binomial with an extra trial
	A1:	awrt 0.0236
(c)	**M1:**	Forming an equation for the mean and variance. At least one correct.
	A1:	Both equations correct
		Allow M1 A1 if both equations correct with the same number subst for *n*
	M1:	Solving the 2 equations leading to $1-p=99p$ oe Allow $p-p^2=99p^2$ ft their 4400 and 660 Allow $1-p=0.15p$
	A1:	0.01
(d)	**M1:**	Both hypotheses correct using correct notation allow eg $p > 0.93$
	M1:	Realising the need to use Geo(0.07) ft their Hypotheses
	M1:	Using the model to find $P(J \geqslant c)$ Condone $(1-0.07)^c < 0.1$ ft their 0.07 $\neq 0.93$ **ALT** $P(J \geqslant 32) = 0.1[054...\}]$ or $P(J \geqslant 33) = 0.09[8...]$ Implied by correct CR
	M1:	For a valid method to solve the inequality or $P(J \geqslant 32) = 0.1[054]$ and $P(J \geqslant 33) = 0.09[81]$ Implied by correct CR
	A1:	Correct CR(any letter) A0 if given as a probability statement. Must be integer
(e)	**M1:**	Comparing 34 with their CR eg $34 > 33$ $34 \geqslant 33$ or $P(J \geqslant 34) = 0.09[12]$
	A1:	Fully correct conclusion in context. Allow Jerry's belief is true. Allow probability for proportion
(f)	**M1:**	Realising they need to find P(their CR in (d)) Allow 1– $P(J \leqslant 32)$
	M1:	For a Correct method. Allow 1 – 0.2981… May be implied by 0.7019… If the CR is incorrect $(1-0.011)^{"CR"-1}$ or 1 – $\{1-(1-0.011)^{"CR"-1}\}$ must be seen
	A1*:	Only award if both method marks awarded.

Question	Scheme	Marks	AOs
6(a)	$G_X(1) = 1$	M1	2.1
	$k \times 3^5 = 1 \quad \therefore k = \dfrac{1}{243}$ *	A1*cso (2)	1.1b
(b)	$P(X=2)$ is coefficient of t^2 so $G_X(t) = k\left(... + {}^5C_2\left(2t\right)^2 + ...\right)$	M1	1.1b
	$P(X=2) = \dfrac{40}{243}$	A1 (2)	1.1b
(c)	$G_W(t) = \dfrac{t^3}{243}\left(1+2\left(t^2\right)\right)^5$	M1	3.1a
	$G_W(t) = \dfrac{t^3}{243}\left(1+2t^2\right)^5$	A1 (2)	1.1b
(d)	$G_U(t) \; = \dfrac{1}{243}(1+2t)^5 \times \dfrac{t(1+2t)^2}{9}$	M1	3.1a
	$= \dfrac{t(1+2t)^7}{2187}$	A1 (2)	1.1b
(e)	$G_U{}'(t) = \dfrac{14t(1+2t)^6}{2187} + \dfrac{(1+2t)^7}{2187}$	M1	2.1
	$G_U{}'(1) = \dfrac{17}{3}$	A1ft	1.1b
	$G_U{}''(t) = \dfrac{168t(1+2t)^5}{2187} + \dfrac{14(1+2t)^6}{2187} + \dfrac{14(1+2t)^6}{2187}$	M1	2.1
	$G_U{}''(1) = 28$	A1	1.1b
	$\text{Var}(U) = "28" + "\dfrac{17}{3}" - \left("\dfrac{17}{3}"\right)^2$	M1	2.1
	$= \dfrac{14}{9}$	A1 (6)	1.1b
ALT(e)	$G_X{}''(t) = A(1+2t)^3$	M1	
	$G_X{}'(1) = \dfrac{10}{3}$ and $G_X{}''(1) = \dfrac{80}{9}$	A1ft	
	$G_Y{}''(t) = H(8+24t)$	M1	
	$G_Y{}'(1) = \dfrac{7}{3}$ and $G_Y{}''(1) = \dfrac{32}{9}$	A1	
	Using $G_U{}''(1) + G_U{}'(1) - \left(G_U{}'(1)\right)^2$ to find Var(X), Var Y and Var U	M1	
	$\dfrac{14}{9}$ or awrt1.56	A1	
			(14 marks)

Notes:

(a)	**M1:**	Stating $G_X(1) = 1$ eg $G_X(1) = k(1 + 2)^5 = 1$ $k(1 + 2)^5 = 1$ Allow Verification $\frac{1}{243}\times 3^5 = 1$
	A1*:	Fully correct proof with no errors Substituting $t = 1$ Verification need therefore $G_X(1) = 1$
(b)	**M1:**	Attempting to find the coefficient of t^2
	A1:	$\frac{40}{243}$ or awrt 0.165
(c)	**M1:**	Realising the need to multiply through by t^3 or subst t^2 for t
	A1:	$\frac{t^3}{243}\left(1+2t^2\right)^5$ oe eg $\frac{t^3}{243}\left(1+10t^2+40t^4+80t^6+80t^8+32t^{10}\right)$
(d)	**M1:**	Realising the need to use $G_U(t) = G_X(t)\times G_Y(t)$
	A1:	$\frac{t(1+2t)^7}{2187}$ oe
(e)	**M1:**	For an attempt to differentiate G (u) e.g $G_U'(t) = At(1+2t)^6 + B(1+2t)^7$ ft their part(d) if in the form $kt(1+2t)^n$ where $n \geqslant 5$
	A1ft:	$\frac{17}{3}$ or awrt 5.67
	M1:	For attempting second derivative eg $G_U''(t) = Ct(1+2t)^5 + D(1+2t)^6$ ft their part(d) if in the form $kt(1+2t)^n$ where $n \geqslant 5$
	A1	28
	M1:	Using $G_U''(1) + G_U'(1) - \left(G_U'(1)\right)^2$ ft their values
	A1:	$\frac{14}{9}$ or awrt1.56

Question	Scheme	Marks	AOs
7(a)	Size of the test = 0.01	B1	1.2
		(1)	
(b)(i)	Let CR be $\bar{L} < k$		
	$\frac{k-15}{\frac{0.2}{\sqrt{n}}} = -2.3263$	M1	3.4
	$k = 15 - \frac{0.46526}{\sqrt{n}}$	A1	1.1b
	$\frac{"15-\frac{0.46526}{\sqrt{n}}"-14.9}{\frac{0.2}{\sqrt{n}}} > 1.6449$	M1d A1ft	3.4 1.1b
	$\frac{0.79424}{\sqrt{n}} < 0.1 \quad \sqrt{n} > 7.9424$ oe	M1d	1.1b
	$n = 64$	A1cso	2.1
		(6)	
(ii)	The probability of a Type II error would decrease.	B1	2.2a
		(1)	
		(8 marks)	

Notes		
(a)	**B1:**	0.01
(b)(i)	**M1:**	Finding the CR using the Normal distribution must have $1.5 < \lvert z \rvert < 3.5$
	A1:	A correct equation in the form $k = \ldots$ and for use of awrt 2.326 (implied by awrt 0.46526 or awrt 0.46527)
	M1d:	Dependent on previous M being awarded. Standardising using their k and equating to a z value $1.5 < \lvert z \rvert < 3$ to form an equation to able n to be found. May use = rather than $>$
	A1ft:	Ft their k for a correct equation with awrt 1.645
	M1d:	Dependent on previous M being awarded. Isolating $\sqrt{n}$ or squaring both sides leading to a value for n. Condone $n = 7.9424$
	A1cso:	64 with correct working
(ii)	**B1:**	Suitable comment

ALT (b)(i)	$\dfrac{k-14.9}{\dfrac{0.2}{\sqrt{n}}}=1.6449$	M1	3.4
	$k=14.9+\dfrac{0.32898}{\sqrt{n}}$	A1	1.1b
	$\dfrac{"14.9+\dfrac{0.32898}{\sqrt{n}}"-15}{\dfrac{0.2}{\sqrt{n}}}>-2.3263$	M1d A1ft	3.4 1.1b
	$\dfrac{0.79424}{\sqrt{n}}<0.1 \quad \sqrt{n}>7.9424$ oe	M1d	1.1b
	$n = 64$	A1cso	2.1
		(6)	