MODEL ANSWERS

Write your name here

Surname	Other names

Pearson Edexcel Level 3 GCE

Centre Number | Candidate Number

Further Mathematics

Advanced
Further Mathematics Option 2
Paper 4: Further Mechanics 2

Sample Assessment Material for first teaching September 2017

Time: 1 hour 30 minutes

Paper Reference **9FM0/4F**

You must have:
Mathematical Formulae and Statistical Tables, calculator

Total Marks

Candidates may use any calculator permitted by Pearson regulations. Calculators must not have the facility for algebraic manipulation, differentiation and integration, or have retrievable mathematical formulae stored in them.

Instructions

- Use **black** ink or ball-point pen.
- If pencil is used for diagrams/sketches/graphs it must be dark (HB or B).
- **Fill in the boxes** at the top of this page with your name, centre number and candidate number.
- Answer **all** questions and ensure that your answers to parts of questions are clearly labelled.
- Answer the questions in the spaces provided – *there may be more space than you need.*
- You should show sufficient working to make your methods clear. Answers without working may not gain full credit.
- Answers should be given to three significant figures unless otherwise stated.

Information

- A booklet 'Mathematical Formulae and Statistical Tables' is provided.
- There are 7 questions in this question paper. The total mark for this paper is 75.
- The marks for **each** question are shown in brackets – *use this as a guide as to how much time to spend on each question.*

Advice

- Read each question carefully before you start to answer it.
- Try to answer every question.
- Check your answers if you have time at the end.

Turn over ▶

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Answer ALL questions. Write your answers in the spaces provided.

Unless otherwise indicated, whenever a numerical value of g is required, take $g = 9.8\,\text{m s}^{-2}$ and give your answer to either 2 significant figures or 3 significant figures.

1. A flag pole is 15 m long.

The flag pole is non-uniform so that, at a distance x metres from its base, the mass per unit length of the flag pole, $m\,\text{kg m}^{-1}$ is given by the formula $m = 10\left(1 - \dfrac{x}{25}\right)$.

The flag pole is modelled as a rod.

(a) Show that the mass of the flag pole is 105 kg.

(3)

(b) Find the distance of the centre of mass of the flag pole from its base.

(4)

a) total mass $= \int_0^{15} 10\left(1 - \frac{x}{25}\right) dx$

$= 10\left[x - \frac{x^2}{50}\right]_0^{15}$

$= 10\left[15 - \frac{15^2}{50}\right]$

$= 105\,\text{kg}$

b) $\bar{x}\,\Sigma m_i = \Sigma m_i x_i$

$\Sigma m_i = 105\,\text{kg}$

$\Sigma m_i x_i = 10\left(1 - \frac{x}{25}\right) \times x$

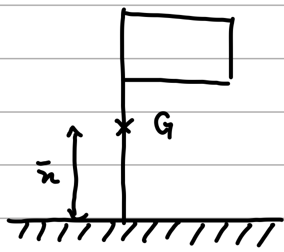
$\therefore 105\bar{x} = \sum_{x=0}^{15} 10x\left(1 - \frac{x}{25}\right)$

$\lim_{x \to 0} \sum_{x=0}^{15} 10x\left(1 - \frac{x}{25}\right)$

$= 10\int_0^{15}\left(x - \frac{x^2}{25}\right) dx$

$= 10\left[\frac{x^2}{2} - \frac{x^3}{75}\right]_0^{15}$

$= 675$



$\therefore 105\bar{x} = 675$

so $\bar{x} = \frac{675}{105}$

$\boxed{= \frac{45}{7}\,\text{m}}$

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Question 1 continued

(Total for Question 1 is 7 marks)

2.

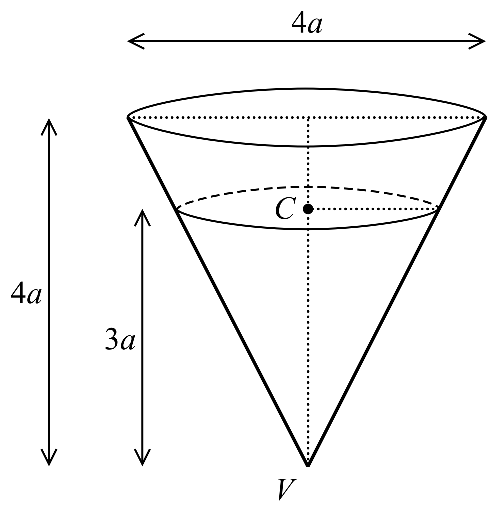


Figure 1

A hollow right circular cone, of base diameter $4a$ and height $4a$ is fixed with its axis vertical and vertex V downwards, as shown in Figure 1.

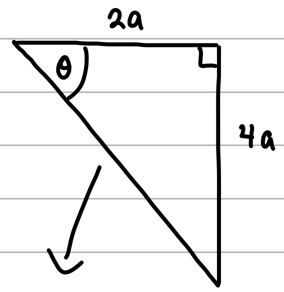
A particle of mass m moves in a horizontal circle with centre C on the rough inner surface of the cone with constant angular speed ω.

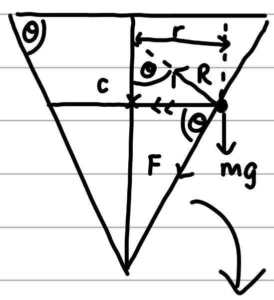
The height of C above V is $3a$.

The coefficient of friction between the particle and the inner surface of the cone is $\frac{1}{4}$.

Find, in terms of a and g, the greatest possible value of ω.

(8)

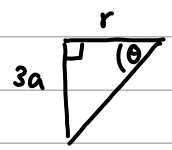




$\tan\theta = \frac{4a}{2a}$

$= 2$

$\sin\theta = \frac{2}{\sqrt{5}}$, $\cos\theta = \frac{1}{\sqrt{5}}$



$\tan\theta = \frac{3a}{r} = 2$

$\therefore r = \frac{3a}{2}$

$\overset{+}{\leftarrow}$

N2L (particle) : $R\sin\theta + F\cos\theta = m\left(\frac{3a}{2}\right)w^2$

note that $F_{max} = \frac{1}{4}R$. when F is max then w will also be max . .

so using $F = \frac{1}{4}R$: $R\left(\frac{2}{\sqrt{5}}\right) + \frac{1}{4}R\left(\frac{1}{\sqrt{5}}\right) = \frac{3ma}{2}w^2$

$\frac{9\sqrt{5}}{20}R = \frac{3ma}{2}w^2$

$\therefore w^2 = \left(\frac{3\sqrt{5}}{10}\right)\left(\frac{R}{ma}\right)$

Question 2 continued

$R(\updownarrow)$: $R\cos\theta = F\sin\theta + mg$

$R\left(\frac{1}{\sqrt{5}}\right) = \frac{1}{4}R\left(\frac{2}{\sqrt{5}}\right) + mg$

$R\left(\frac{1}{\sqrt{5}} - \frac{2}{4\sqrt{5}}\right) = mg$

$\therefore\ R = 2\sqrt{5}\ mg$

so $w^2 = \frac{3\sqrt{5}}{10} \times 2\sqrt{5}\ \frac{\cancel{m}g}{\cancel{m}a}$

$= \frac{3g}{a}$

hence $\boxed{w_{max} = \sqrt{\frac{3g}{a}}}$

(Total for Question 2 is 8 marks)

3.

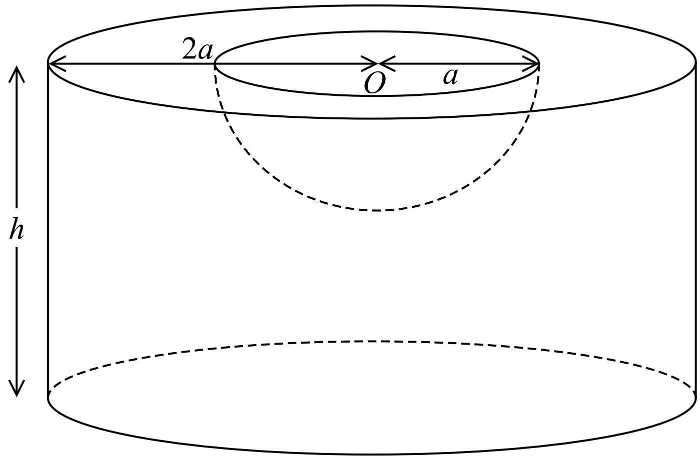


Figure 2

A uniform solid cylinder has radius $2a$ and height h ($h > a$).

A solid hemisphere of radius a is removed from the cylinder to form the vessel V.

The plane face of the hemisphere coincides with the upper plane face of the cylinder.

The centre O of the hemisphere is also the centre of the upper plane face of the cylinder, as shown in Figure 2.

(a) Show that the centre of mass of V is $\dfrac{3(8h^2 - a^2)}{8(6h - a)}$ from O.

(5)

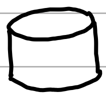
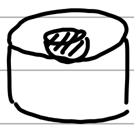
The vessel V is placed on a rough plane which is inclined at an angle ϕ to the horizontal.

The lower plane circular face of V is in contact with the inclined plane.

Given that $h = 5a$, the plane is sufficiently rough to prevent V from slipping and V is on the point of toppling,

(b) find, to three significant figures, the size of the angle ϕ.

(4)

a)	shape	mass (volume)	distance of c.o.m. from O
(+)		$\pi(2a)^2 h$ $= 4\pi a^2 h$	$\frac{h}{2}$
(−)		$\frac{2}{3}\pi a^3$	$\frac{3}{8}a$
(=)		$\pi a^2\left[-\frac{2}{3}a + 4h\right]$	$\bar{y}$

Question 3 continued

taking moments about a diameter through O:

$$4\cancel{\pi}\cancel{a^2}h\left(\frac{h}{2}\right) - \frac{2}{3}\cancel{\pi}a^{\cancel{3}}\left(\frac{3a}{8}\right) = \cancel{\pi}\cancel{a^2}\left(4h - \frac{2}{3}a\right)\bar{y}$$

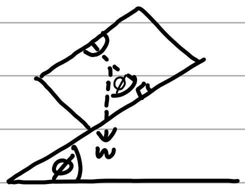
$$2h^2 - \frac{a^2}{4} = \left(4h - \frac{2}{3}a\right)\bar{y}$$

$$\therefore \bar{y} = \frac{2h^2 - \frac{a^2}{4}}{4h - \frac{2}{3}a} \quad \begin{matrix}\times 12 \\ \times 12\end{matrix}$$

$$\bar{y} = \frac{24h^2 - 3a^2}{48h - 8a}$$

$$= \frac{3(8h^2 - a^2)}{8(6h - a)}$$

b)



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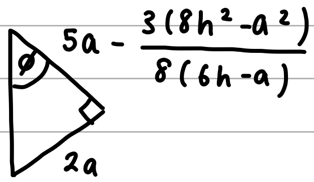

$$5a - \frac{3(8h^2 - a^2)}{8(6h - a)} = 5a - \frac{3(199a^2)}{232a}$$

$$= \frac{563}{232}a$$

$$\therefore \tan\phi = \frac{2\cancel{a}}{\frac{563}{232}\cancel{a}} = \frac{464}{563}$$

$$\text{so } \phi = \tan^{-1}\left(\frac{464}{563}\right)$$

$$= \boxed{39.5°}$$

Question 3 continued

Question 3 continued

(Total for Question 3 is 9 marks)

4. A car of mass 500 kg moves along a straight horizontal road.

The engine of the car produces a constant driving force of 1800 N.

The car accelerates from rest from the fixed point O at time $t = 0$ and at time t seconds the car is x metres from O, moving with speed v m s^{-1}.

When the speed of the car is v m s^{-1}, the resistance to the motion of the car has magnitude $2v^2$ N.

At time T seconds, the car is at the point A, moving with speed 10 m s^{-1}.

(a) Show that $T = \frac{25}{6}\ln 2$ **(6)**

(b) Show that the distance from O to A is $125\ln\frac{9}{8}$ m. **(5)**

a) $2v^2 \leftarrow \boxed{500} \rightarrow 1800$

$\xrightarrow{\quad} +$

N2L (car) : $1800 - 2v^2 = 500a$

$1800 - 2v^2 = 500\frac{dv}{dt}$

$\left(\frac{500}{1800-2v^2}\right)\frac{dv}{dt} = 1$

$250\int \frac{1}{900-v^2}\,dv = t + c$

by partial fractions

$\frac{1}{900-v^2} = \frac{1}{(30+v)(30-v)} = \frac{A}{30+v} + \frac{B}{30-v}$

$1 = A(30-v) + B(30+v)$

$v = 30$: $1 = 60B$ $\therefore B = \frac{1}{60}$ //

$v = 0$: $1 = 30A + \frac{1}{60}(30)$

$\frac{1}{2} = 30A$ $\therefore A = \frac{1}{60}$ //

so $\frac{1}{900-v^2} = \frac{\frac{1}{60}}{30+v} + \frac{\frac{1}{60}}{30-v}$

$\Rightarrow 250\int \frac{\frac{1}{60}}{30+v} + \frac{\frac{1}{60}}{30-v}\,dv = t + c$

$\Rightarrow \frac{250}{60}\left[\ln|30+v| - \ln|30-v|\right] = t + c$

$\Rightarrow \frac{25}{6}\ln\left|\frac{30+v}{30-v}\right| = t + c$

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Question 4 continued

when $t=0$ and $v=0$, $\frac{25}{6}\ln\left|\frac{30}{30}\right| = 0 + c$

$\therefore c = 0$

so $\frac{25}{6}\ln\left|\frac{30+v}{30-v}\right| = t$

$v=10$: $t=T = \frac{25}{6}\ln\left|\frac{30+10}{30-10}\right|$

$= \frac{25}{6}\ln\left|\frac{40}{20}\right|$

$\boxed{= \frac{25}{6}\ln 2}$

b) from a... $1800 - 2v^2 = 500v\frac{dv}{dx}$

$\left(\frac{500v}{1800-2v^2}\right)\frac{dv}{dx} = 1$

$\therefore 250\int\left(\frac{v}{900-v^2}\right)dv = x + c$

by pattern:

$250\left[-\frac{1}{2}\ln|900-v^2|\right] = x + c$

$-125\ln|900-v^2| = x + c$

$t=0$, $x=0$, $v=0$: $-125\ln 900 = c$

$\therefore -125\ln|900-v^2| = x - 125\ln 900$

$125\ln\left|\frac{900}{900-v^2}\right| = x$ //

at A, $v=10$: $x = 125\ln\left(\frac{900}{900-100}\right)$

$x = OA = 125\ln\frac{900}{800}$

$= 125\ln\frac{9}{8}$

(Total for Question 4 is 11 marks)

5.

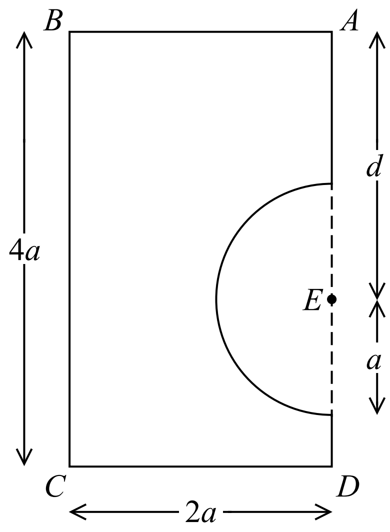


Figure 3

A shop sign is modelled as a uniform rectangular lamina *ABCD* with a semicircular lamina removed.

The semicircle has radius a, $BC = 4a$ and $CD = 2a$.

The centre of the semicircle is at the point E on AD such that $AE = d$, as shown in Figure 3.

(a) Show that the centre of mass of the sign is $\dfrac{44a}{3(16-\pi)}$ from AD.

(4)

The sign is suspended using vertical ropes attached to the sign at A and at B and hangs in equilibrium with AB horizontal.

The weight of the sign is W and the ropes are modelled as light inextensible strings.

(b) Find, in terms of W and π, the tension in the rope attached at B.

(2)

The rope attached at B breaks and the sign hangs freely in equilibrium suspended from A, with AD at an angle α to the downward vertical.

Given that $\tan \alpha = \dfrac{11}{18}$

(c) find d in terms of a and π.

(6)

a)

shape	mass (area)	distance of c.o.m. from AD
(+)	$8a^2$	a
(−)	$\dfrac{\pi a^2}{2}$	$\dfrac{2r\sin\alpha}{3\alpha} = \dfrac{4a}{3\pi}$
(=)	$a^2\left(8-\dfrac{\pi}{2}\right)$	$\bar{x}$

Question 5 continued

taking moments about side AD :

$$8\cancel{a^2}(a) - \frac{\pi \cancel{a^2}}{2}\left(\frac{4a}{3\cancel{\pi}}\right) = \cancel{a^2}\left(8 - \frac{\pi}{2}\right)(\bar{x})$$

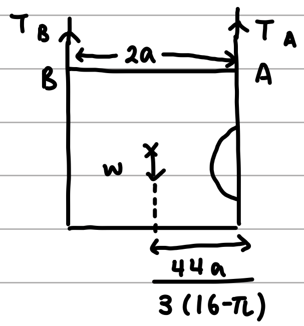
$$\bar{x} = \frac{8a - \frac{4a}{6}}{8 - \frac{\pi}{2}}$$

$$= \frac{\frac{22a}{3}}{8 - \frac{\pi}{2}} \quad \begin{matrix}\times 6 \\ \times 6\end{matrix}$$

$$= \frac{44a}{48 - 3\pi}$$

$$= \frac{44a}{3(16-\pi)}$$

b)

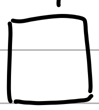
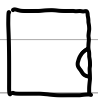


$$M(A) : w\left(\frac{44a}{3(16-\pi)}\right) = T_B\,(2a)$$

$$\therefore T_B = \frac{44\cancel{a}w}{3(16-\pi)\times 2\cancel{a}}$$

$$= \boxed{\frac{22w}{3(16-\pi)}}$$

c)

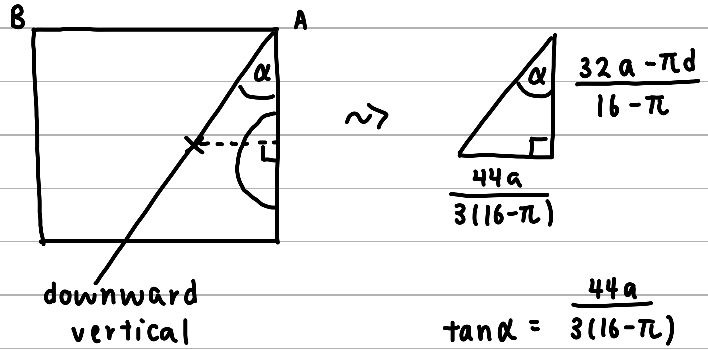
shape		mass	distance of c.o.m. from AB
(+)		$8a^2$	$2a$
(−)		$\frac{\pi a^2}{2}$	d
(=)		$a^2\left(8 - \frac{\pi}{2}\right)$	$\bar{x}$

moments about AB : $8(2a) - \frac{\pi}{2}(d) = \left(8 - \frac{\pi}{2}\right)\bar{x}$

$$\bar{x} = \frac{32a - \pi d}{16 - \pi}$$

Question 5 continued

new scenario :



$$\tan\alpha = \frac{\frac{44a}{3(16-\pi)}}{\frac{32a-\pi d}{16-\pi}} = \frac{11}{18}$$

$$\therefore \frac{11}{18} = \frac{\frac{44a}{3}}{32a-\pi d}$$

$$\Rightarrow 32a - \pi d = \frac{18}{11}\left(\frac{44a}{3}\right)$$

$$\Rightarrow d = \frac{32a - \frac{18(44a)}{33}}{\pi}$$

$$\boxed{= \frac{8a}{\pi}}$$

Question 5 continued

(Total for Question 5 is 12 marks)

6. A small bead B of mass m is threaded on a circular hoop.

The hoop has centre O and radius a and is fixed in a vertical plane.

The bead is projected with speed $\sqrt{\frac{7}{2}ga}$ from the lowest point of the hoop.

The hoop is modelled as being smooth.

When the angle between OB and the downward vertical is θ, the speed of B is v.

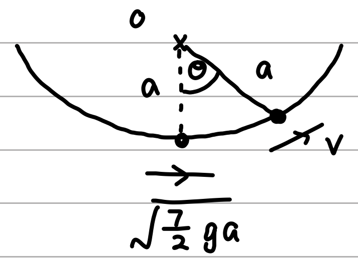
(a) Show that $v^2 = ga\left(\frac{3}{2} + 2\cos\theta\right)$ **(3)**

(b) Find the size of θ at the instant when the contact force between B and the hoop is first zero. **(5)**

(c) Give a reason why your answer to part (b) is not likely to be the actual value of θ. **(1)**

(d) Find the magnitude and direction of the acceleration of B at the instant when B is first at instantaneous rest. **(5)**

a)



initially: $KE = \frac{1}{2}m\left(\frac{7ag}{2}\right)$

$= \frac{7amg}{4}$

$GPE = 0$

at angle θ to d.v: $KE = \frac{1}{2}mv^2$

$GPE = mga(1-\cos\theta)$

C.O.E : $\frac{7a\cancel{m}g}{4} = \frac{\cancel{m}v^2}{2} + \cancel{m}ag(1-\cos\theta)$

$v^2 + 2ag - 2ag\cos\theta = \frac{7ag}{2}$

$v^2 = \frac{3ag}{2} + 2ag\cos\theta$

$\therefore v^2 = ga\left(\frac{3}{2} + 2\cos\theta\right)$

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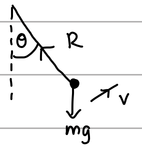
Question 6 continued

b)



N2L ↖⁺ (B) : $R - mg\cos\theta = \frac{mv^2}{a}$

$R = 0$: $-g\cos\theta = \frac{v^2}{a}$

so $v^2 = -ag\cos\theta$

but $v^2 = ag\left(\frac{3}{2} + 2\cos\theta\right)$

so $-\cancel{ag}\cos\theta = \cancel{ag}\left(\frac{3}{2} + 2\cos\theta\right)$

$-\cos\theta = \frac{3}{2} + 2\cos\theta$

$\therefore 3\cos\theta = \frac{-3}{2}$

$\cos\theta = \frac{-1}{2}$

$\theta = \cos^{-1}\left(\frac{-1}{2}\right) = \boxed{120^\circ}$

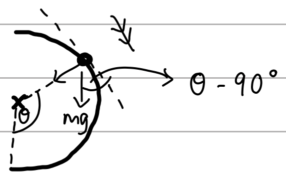
c) air resistance hasn't been accounted for

d) $v = 0$: $\frac{3}{2} + 2\cos\theta = 0$

$\cos\theta = \frac{-3}{4}$

$\theta = 138.6 > 120$

so...



Rf (tangential to hoop) : $\cancel{m}g\cos(\theta - 90) = \cancel{m}a$

(R does not act in this plane)

$\therefore a = g\cos(\theta - 90)$

$a = g\cos\left[\cos^{-1}\left(-\frac{3}{4}\right) - 90\right]$

$\boxed{a \simeq 6.48 \text{ ms}^{-2}}$

at an angle $(\theta - 90)$ to d.v

$\theta - 90 = \cos^{-1}\left(\frac{-3}{4}\right) - 90$

$= \boxed{48.6^\circ \text{ to downward vertical}}$

Question 6 continued

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Question 6 continued

(Total for Question 6 is 14 marks)

7. Two points A and B are 6 m apart on a smooth horizontal surface.

A light elastic string of natural length 2 m and modulus of elasticity 20 N, has one end attached to the point A.

A second light elastic string of natural length 2 m and modulus of elasticity 50 N, has one end attached to the point B.

A particle P of mass 3.5 kg is attached to the free end of each string.

The particle P is held at the point on AB which is 2 m from B and then released from rest.

In the subsequent motion both strings remain taut.

(a) Show that P moves with simple harmonic motion about its equilibrium position.

(7)

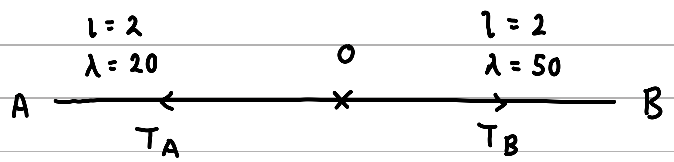
(b) Find the maximum speed of P.

(2)

(c) Find the length of time within each oscillation for which P is closer to A than to B.

(5)

a)



we must find equilibrium point O.

$T_A = T_B$

$$\frac{20(OA-2)}{2} = \frac{50(6-OA-2)}{2}$$

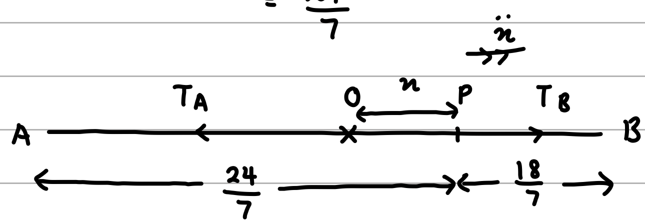
$10(OA) - 20 = 25(4 - OA)$

$10(OA) - 20 = 100 - 25(OA)$

$10(OA) + 25(OA) = 120$

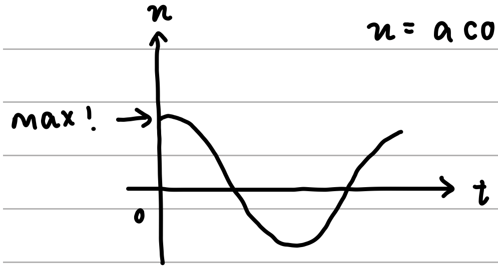
$$OA = \frac{120}{25+10}$$

$$= \frac{24}{7}$$



$\ddot{x}$ is +ve in the direction OB since P starts at an endpoint of the oscillations, so $x = a\cos\omega t$ hence x is max at $t=0$. This must mean x is increasing in the direction OB so $\ddot{x}$ is also increasing in the same direction.

Question 7 continued



$x = a\cos wt$

$\overrightarrow{+}$ N2L (P) : $T_B - T_A = 3.5\ddot{x}$

$$\frac{50}{2}\left(\frac{18}{7} - x - 2\right) - \frac{20}{2}\left(\frac{24}{7} + x - 2\right) = 3.5\ddot{x}$$

$$25\left(\frac{4}{7} - x\right) - 10\left(\frac{10}{7} + x\right) = 3.5\ddot{x}$$

$$\cancel{\frac{100}{7}} - 25x - \cancel{\frac{100}{7}} - 10x = 3.5\ddot{x}$$

$$\frac{-35x}{3.5} = \ddot{x} = -10x$$

$\therefore$ P moves with S.H.M (about O)

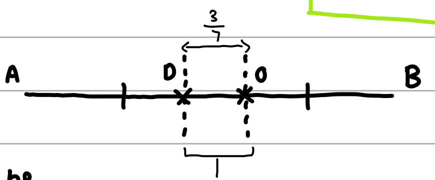
b) $V_{max} = aw$ $\qquad a = \frac{18}{7} - 2$

$\qquad\qquad\qquad\qquad = \frac{4}{7}$

and $w = \sqrt{10}$

so $V_{max} = \boxed{\frac{4\sqrt{10}}{7}\ \text{ms}^{-1}}$

c)



let D be midpoint of AB

P must not be in this region to be closer to A than B (ie $x < -\frac{3}{7}$)

$$\left(OD = \underset{OA}{\frac{24}{7}} - \underset{\frac{AB}{2}}{3} = \frac{3}{7}\right)$$

$$x = \frac{4}{7}\cos(t\sqrt{10})$$

$$-\frac{3}{7} = \frac{4}{7}\cos(t\sqrt{10})$$

$$-\frac{3}{4} = \cos(t\sqrt{10})$$

$$t\sqrt{10} = \cos^{-1}\left(\frac{-3}{4}\right)$$

$$= 2.419$$

Question 7 continued

so $t\sqrt{10} = 2.419 \ , \ 2\pi - 2.419$

$\hookrightarrow t = 0.76, 1.22$

these are the times when P reaches D during the first oscillation

so time required $= 1.22 - 0.76$

$\simeq \boxed{0.46s}$

Question 7 continued

(Total for Question 7 is 14 marks)

TOTAL FOR PAPER IS 75 MARKS