# Mark Scheme (Results)

## Summer 2019

### Pearson Edexcel GCE Further Mathematics
### AS Further Mechanics 2 Paper 8FM0_26

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**General Marking Guidance**

- All candidates must receive the same treatment. Examiners must mark the first candidate in exactly the same way as they mark the last.
- Mark schemes should be applied positively. Candidates must be rewarded for what they have shown they can do rather than penalised for omissions.
- Examiners should mark according to the mark scheme not according to their perception of where the grade boundaries may lie.
- There is no ceiling on achievement. All marks on the mark scheme should be used appropriately.
- All the marks on the mark scheme are designed to be awarded. Examiners should always award full marks if deserved, i.e. if the answer matches the mark scheme. Examiners should also be prepared to award zero marks if the candidate's response is not worthy of credit according to the mark scheme.
- Where some judgement is required, mark schemes will provide the principles by which marks will be awarded and exemplification may be limited.
- When examiners are in doubt regarding the application of the mark scheme to a candidate's response, the team leader must be consulted.
- Crossed out work should be marked UNLESS the candidate has replaced it with an alternative response.

**EDEXCEL GCE MATHEMATICS**
**General Instructions for Marking**

1. The total number of marks for the paper is 40.

2. The Edexcel Mathematics mark schemes use the following types of marks:

- **M** marks: method marks are awarded for 'knowing a method and attempting to apply it', unless otherwise indicated.
- **A** marks: Accuracy marks can only be awarded if the relevant method (M) marks have been earned.
- **B** marks are unconditional accuracy marks (independent of M marks)
- Marks should not be subdivided.

3. Abbreviations

These are some of the traditional marking abbreviations that will appear in the mark schemes.

- bod – benefit of doubt
- ft – follow through
- the symbol $\surd$ will be used for correct ft
- cao – correct answer only
- cso - correct solution only. There must be no errors in this part of the question to obtain this mark
- isw – ignore subsequent working
- awrt – answers which round to
- SC: special case
- oe – or equivalent (and appropriate)
- dep – dependent
- indep – independent
- dp decimal places
- sf significant figures
- $*$ The answer is printed on the paper
- ⌐ The second mark is dependent on gaining the first mark

4. For misreading which does not alter the character of a question or materially simplify it, deduct two from any A or B marks gained, in that part of the question affected.

5. Where a candidate has made multiple responses and indicates which response they wish to submit, examiners should mark this response.
   If there are several attempts at a question which have not been crossed out, examiners should mark the final answer which is the answer that is the most complete.

6. Ignore wrong working or incorrect statements following a correct answer.

7. Mark schemes will firstly show the solution judged to be the most common response expected from candidates. Where appropriate, alternatives answers are provided in the notes. If examiners are not sure if an answer is acceptable, they will check the mark scheme to see if an alternative answer is given for the method used.

## General Principles for Mechanics Marking

*(But note that specific mark schemes may sometimes override these general principles)*

- Rules for M marks: correct no. of terms; dimensionally correct; all terms that need resolving (i.e. multiplied by cos or sin) are resolved.

- Omission or extra g in a resolution is an accuracy error not method error.

- Omission of mass from a resolution is a method error.

- Omission of a length from a moments equation is a method error.

- Omission of units or incorrect units is not (usually) counted as an accuracy error.

- dM indicates a dependent method mark i.e. one that can only be awarded if a previous specified method mark has been awarded.

- Any numerical answer which comes from use of g = 9.8 should be given to 2 or 3 SF.

- Use of g = 9.81 should be penalised once per (complete) question.

  N.B. Over-accuracy or under-accuracy of correct answers should only be penalised *once* per complete question. However, premature approximation should be penalised every time it occurs.

- Marks must be entered in the same order as they appear on the mark scheme.

- In all cases, if the candidate clearly labels their working under a particular part of a question i.e. (a) or (b) or (c),……then that working can only score marks for that part of the question.

- Accept column vectors in all cases.

- Misreads – if a misread does not alter the character of a question or materially simplify it, deduct two from any A or B marks gained, bearing in mind that after a misread, the subsequent A marks affected are treated as A ft

- Mechanics Abbreviations
  M(A) Taking moments about A
  N2L Newton's Second Law (Equation of Motion)
  NEL Newton's Experimental Law (Newton's Law of Impact)
  HL Hooke's Law
  SHM Simple harmonic motion
  PCLM Principle of conservation of linear momentum
  RHS, LHS Right hand side, left hand side

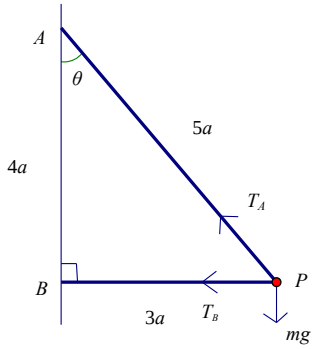
| Question | Scheme | Marks | AOs |
|---|---|---|---|
| **1(a)** | The rods are uniform and the axes of symmetry intersect at midpoint of $AC$. | B1 | 2.4 |
| | | **(1)** | |
| **1(b)** | Use moments: e.g. M($A$): $\left(2aW + aW + 3aW = 4aT_B + aW\right)$ | M1 | 2.1 |
| | e.g. M($A$): $5W.2a\cos 60^o = 4aT_B$ or M($B$): $3a \times 5W = 4aT_A$ | A1 | 1.1b |
| | Resolving vertically: $T_A + T_B = 5W$ | M1 | 2.1 |
| | $\Rightarrow T_A = \frac{15W}{4}, \quad T_B = \frac{5W}{4}$ | A1 | 1.1b |
| | | **(4)** | |
| **1(c)** | $T_A$ will be the larger, so the first to exceed $6W$ so need to use $T_A = 6W$ (e.g. by M($B$) but they may use two equations) to form an equation in $k$ only. | M1 | 3.1a |
| | $6W \times 4a = 5W \times 3a + kW \times 6a + 2kW \times 2a$ | A1 | 1.1b |
| | $24aW = 15aW + 10kaW$ | A1 | 1.1b |
| | $k = 0.9$ | A1 | 1.1b |
| | | **(4)** | |
| | | | **(9 marks)** |

## Notes

| | | |
|---|---|---|
| **(a)** | B1 | Any equivalent clear justification. Needs to mention uniformity and symmetry and the midpoint of $AC$ |
| **(b)** | M1 | Form ANY moments equation. Require all terms. Dimensionally correct. Condone sign errors. |
| | A1 | Correct unsimplified (including trig) equation<br>e.g. M($G$) : $T_A.2a\cos 60^o = T_B.(4a - 2a\cos 60^o)$ or $T_B.(4a\cos^2 30^o)$ |
| | M1 | Form a second equation in $T_A$ and/or $T_B$ e.g. by resolving vertically or a second moments equation, and solve for $T_A$ and $T_B$ |
| | A1 | Both tensions correct. If answers reversed, allow M marks. |
| **(c)** | M1 | Realise that the first to break will be the rope at $A$ and complete method to form an equation in $k$ only (allow uncancelled $W$'s) using $T_A = 6W$ . Require all terms (in all equations used). Dimensionally correct. Condone sign errors.<br>M0 if they use $T_B = 6W$ to find $k$ (this gives $k = 9.5$) |
| | A1 | Unsimplified equation or inequality in $k$ only with at most one error |
| | A1 | Correct unsimplified equation or inequality in $k$ only |
| | A1 | Correct only. Decimal or fraction. |

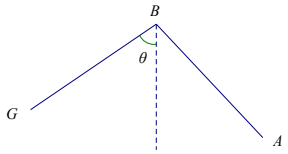
| Question | Scheme | Marks | AOs |
|---|---|---|---|
| **2(a)** | $a=\dfrac{4}{2+v} \Rightarrow \int(2+v)\,\mathrm{d}v=\int 4\mathrm{d}t$ | M1 | 2.1 |
| | $\dfrac{(2+v)^2}{2}=4t+C_1$ | M1 | 1.1b |
| | | A1 | 1.1b |
| | $t=0,\ v=2 \Rightarrow C_1=8$ | M1 | 3.4 |
| | $\dfrac{(2+v)^2}{2}=4t+8$ | A1 | 1.1b |
| | $(2+v)^2=8t+16,\quad v=\sqrt{8t+16}-2$ * | A1* | 2.2a |
| | | **(6)** | |
| **2(a) alt** | $a=\dfrac{4}{2+v} \Rightarrow \int(2+v)\,\mathrm{d}v=\int 4\mathrm{d}t$ | M1 | 2.1 |
| | $2v+\dfrac{v^2}{2}=4t+C_2$ | M1 | 1.1b |
| | | A1 | 1.1b |
| | $t=0,\ v=2 \Rightarrow C_2=6$ | M1 | 3.4 |
| | $2v+\dfrac{v^2}{2}=4t+6$ | A1 | 1.1b |
| | $4v+v^2=8t+12,\quad (v+2)^2=8t+16$<br>$\Rightarrow v=\sqrt{8t+16}-2$ * | A1* | 2.2a |
| | | **(6)** | |
| **(b)** | $v=4 \Rightarrow 36=8t+16 \Rightarrow t=2.5$ | B1 | 1.1b |
| | $\dfrac{\mathrm{d}x}{\mathrm{d}t}=\sqrt{8t+16}-2$ | M1 | 3.3 |
| | $x=k(8t+16)^{\frac{3}{2}}-2t+C$ | M1 | 1.1b |
| | $x=\dfrac{1}{12}(8t+16)^{\frac{3}{2}}-2t+C$ | A1 | 1.1b |
| | $t=0,\ x=0 \Rightarrow \dfrac{64}{12}+C=0,\ C=-\dfrac{16}{3}$ | M1 | 3.4 |
| | $AB=\dfrac{1}{12}(36)^{\frac{3}{2}}-5-\dfrac{16}{3}=\dfrac{23}{3}$ (m) | A1 | 1.1b |
| | | **(6)** | |
| | | | **(12 marks)** |

| Notes | | |
|---|---|---|
| **(a)** | M1 | Form differential equation in $v$ and $t$ and prepare to integrate. |
| | M1 | Integrate to obtain $k(2+v)^2$ or equivalent |
| | A1 | Correct integration. Condone missing constant of integration. |
| | M1 | Use the model to find the value of constant of integration. |
| | A1 | Correct solution in any form |
| | A1* | Obtain **given solution** from correct working. Allow use of quadratic formula. |
| **(a) alt** | M1 | Form differential equation in $v$ and $t$ and prepare to integrate. |
| | M1 | Integrate to obtain $k(2+v)^2$ or equivalent |
| | A1 | Correct integration. Condone missing constant of integration. |
| | M1 | Use the model to find the value of constant of integration. |
| | A1 | Correct solution in any form |
| | A1* | Obtain **given solution** from correct working. |
| **(b)** | B1 | Use the result from (a) to find $t$ when $v = 4$ : seen or implied |
| | M1 | Form differential equation in $x$ and $t$ |
| | M1 | Integrate to obtain terms of the correct form. Condone missing constant of integration. |
| | A1 | Correct integration. Condone missing constant of integration. |
| | M1 | Use boundary conditions in the model to find constant of integration, or as limits on a definite integral.<br>Note this is an independent M mark.<br>M0 if they use $t = 4$ |
| | A1 | Correct answer only. 7.7 (m) or better |

| Question | Scheme | Marks | AOs |
|---|---|---|---|
| **3(a)** |  | | |
| | No vertical motion: $T_A \cos\theta = mg$ | M1 | 1.1b |
| | $T_A = \frac{5mg}{4}$ | A1 | 1.1b |
| | Circular motion: $T_B + T_A \sin\theta = m \times \frac{v^2}{r}$ | M1 | 3.1b |
| | $T_B + \frac{3}{5}T_A = m\frac{v^2}{3a}$ | A1 | 1.1b |
| | $T_B > 0 \left( \Rightarrow v^2 > \frac{9ag}{4} \right)$ | DM1 | 2.1 |
| | $T_B \le \frac{3mg}{2} \Rightarrow m\frac{v^2}{3a} - \frac{3}{4}mg \le \frac{3}{2}mg, \left( m\frac{v^2}{3a} \le \frac{9mg}{4} \right)$ | DM1 | 2.1 |
| | $\Rightarrow \frac{9ag}{4} < v^2 \le \frac{27ag}{4}$ * | A1* | 2.2a |
| | | **(7)** | |
| **(b)** | Use $v^2 = \frac{27ag}{4}$ and $T = \frac{2\pi r}{v}$ oe | M1 | 3.1b |
| | $T = 4\pi\sqrt{\frac{a}{3g}}$ | A1 | 1.1b |
| | | **(2)** | |
| | | | **(9 marks)** |

| Notes | | |
|---|---|---|
| **(a)** | | **N.B.** If they have the same tension in both parts of the string, can score ONLY first M1A1 for a correct equation.<br>**N.B**. If no right angle at *B*, could score max: M1A0M1A0DM1DM1A0 |
| | M1 | One equation in $T_A$ and / or $T_B$ . Dimensionally correct, with all relevant terms.<br>Condone sign errors and sin/cos oe confusion |
| | A1 | Correct equation (no trig) |
| | M1 | Form a second equation in $T_A$ and / or $T_B$ . Dimensionally correct, with all relevant terms. Condone sign errors and sin/cos oe confusion. Allow $mr\omega^2$ |
| | A1 | Correct equation (no trig) |
| | DM1 | Use the model to form one inequality or equation in $v^2$, $a$ and $g$ **only, dependent on both M's** |
| | DM1 | Use the model to form a second inequality or equation in $v^2$, $a$ and $g$ **only dependent on both M's**<br>**Allow use of** $T_B = \frac{3mg}{2}$ or $T_B < \frac{3mg}{2}$ |
| | A1* | Deduce the given answer from correct working. Only available if working with inequalities throughout and fully correct |
| **(b)** | M1 | Correct method to find $T$ in terms of $a$ and ($g$) **only.**<br>They may sub 9.8 for $g$ of course |
| | A1 | Any equivalent form but no fractions within fractions.<br>If they use 9.8 for $g$, the numerical part needs to be to 2 sf or 3sf. i.e $2.3\sqrt{a}$ or $2.32\sqrt{a}$ |

<table>
<tr><th>Question</th><th>Scheme</th><th>Marks</th><th>AOs</th></tr>
<tr><td rowspan="2">4(a)</td><td>In the folding process, each point of the lamina remains the same distance from CD</td><td>B1</td><td>2.4</td></tr>
<tr><td></td><td>(1)</td><td></td></tr>
<tr><td rowspan="6">(b)</td><td>For the folded lamina: $\overline{x} = 2a \quad (= d_2)$ oe</td><td>B1</td><td>1.1b</td></tr>
<tr><td>Distances from EA
<table>
<tr><td>Large triangle (ACE)</td><td>Removed triangle (BCD)</td><td>Added triangle (BCD)</td><td>Folded lamina</td></tr>
<tr><td>$27a^2$</td><td>$12a^2$</td><td>$12a^2$</td><td>$27a^2$</td></tr>
<tr><td>$3a$</td><td>$5a$</td><td>$a$</td><td>$\overline{y}$</td></tr>
</table>
<u>Alternative 1</u><br>
Distances from BD
<table>
<tr><td>Rectangle EDBH</td><td>Triangle BHA</td><td>Triangle DBC</td><td>Folded lamina</td></tr>
<tr><td>$12a^2$</td><td>$3a^2$</td><td>$12a^2$</td><td>$27a^2$</td></tr>
<tr><td>$1.5a$</td><td>$2a$</td><td>$2a$</td><td>$\overline{y}$</td></tr>
</table>
<u>Alternative 2</u><br>
Distances from BD
<table>
<tr><td>Triangle FAB</td><td>Triangle EFC</td><td>2 x Rectangle DGEF</td><td>2 x Triangle BGF</td><td>Folded lamina</td></tr>
<tr><td>$6a^2$</td><td>$3a^2$</td><td>$12a^2$</td><td>$6a^2$</td><td>$27a^2$</td></tr>
<tr><td>$2a$</td><td>$4a$</td><td>$1.5a$</td><td>$a$</td><td>$\overline{y}$</td></tr>
</table></td><td></td><td></td></tr>
<tr><td>Area ratios</td><td>B1</td><td>1.2</td></tr>
<tr><td>Distances from EA</td><td>B1</td><td>1.2</td></tr>
<tr><td>Moments about EA:</td><td>M1</td><td>2.1</td></tr>
<tr><td>$27 \times 3a - 12 \times 5a + 12 \times a = 27\overline{y}$</td><td>A1ft</td><td>1.1b</td></tr>
<tr><td>$\overline{y} = \frac{11a}{9}$</td><td>A1</td><td>1.1b</td></tr>
</table>

| Question | Scheme | Marks | AOs |
|---|---|---|---|
| **(b) cont** |  | | |
| | $\theta = \tan^{-1}\dfrac{4a-\overline{x}}{3a-\overline{y}}\left(=\tan^{-1}\dfrac{9}{8}\right)$ or $(90^o - \theta) = \tan^{-1}$ (reciprocal) | M1 | 1.1b |
| | $\alpha = \tan^{-1}\dfrac{4a-\overline{x}}{3a-\overline{y}} + \tan^{-1}\dfrac{2}{3}$ or oe | M1 | 3.1b |
| | $= 82°$ (nearest degree) | A1 | 1.1b |
| | Alternative for the final 3 marks: | | |
| | $\overrightarrow{BA}.\overrightarrow{BG} = \dfrac{2}{9}\begin{pmatrix}-9\\-8\end{pmatrix}.\begin{pmatrix}2\\-3\end{pmatrix}\left(=\dfrac{4}{3}\right)$ | M1 | 1.1b |
| | $\cos\alpha = \dfrac{\frac{4}{3}}{\frac{2}{9}\sqrt{145}\sqrt{13}}(=0.138...)$ | M1 | 3.1b |
| | $\theta = 82°$ | A1 | 1.1b |
| | | **(9)** | |
| | | | **(10 marks)** |

<table>
<tr><th colspan="3">Notes</th></tr>
<tr><td>(a)</td><td>B1</td><td>Any equivalent explanation e.g. folding doesn't change the mass distribution relative to CD. A calculation to verify is not the same as an explanation.<br>Allow use of 'vertical' for CD.</td></tr>
<tr><td>(b)</td><td>B1</td><td>Seen anywhere</td></tr>
<tr><td></td><td></td><td>Distances from EA
<table>
<tr><td>Large triangle (ACE)</td><td>Removed triangle (BCD)</td><td>Added triangle (BCD)</td><td>Folded lamina</td></tr>
<tr><td>$27a^2$</td><td>$12a^2$</td><td>$12a^2$</td><td>$27a^2$</td></tr>
<tr><td>$3a$</td><td>$5a$</td><td>$a$</td><td>$\overline{y}$</td></tr>
</table>
N.B. B marks only available for viable dissections<br>Other dissections are possible:<br><br><u>Alternative 1</u><br>Distances from BD
<table>
<tr><td>Rectangle EDBH</td><td>Triangle BHA</td><td>Triangle DBC</td><td>Folded lamina</td></tr>
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</table>
EDBH + BHA + DBC, where H is midpoint of AF<br><br><u>Alternative 2</u><br>Distances from BD
<table>
<tr><td>Triangle FAB</td><td>Triangle EFC</td><td>2 x Rectangle DGEF</td><td>2 x Triangle BGF</td><td>Folded lamina</td></tr>
<tr><td>$6a^2$</td><td>$3a^2$</td><td>$12a^2$</td><td>$6a^2$</td><td>$27a^2$</td></tr>
<tr><td>$2a$</td><td>$4a$</td><td>$1.5a$</td><td>$a$</td><td>$\overline{y}$</td></tr>
</table>
FAB + EFC + (2 x DGEF) + (2 x GBF) , where G is midpoint of DB</td></tr>
<tr><td></td><td>B1</td><td>Any equivalent form for the mass (area) ratios</td></tr>
<tr><td></td><td>B1</td><td>Or correct distances from an alternative axis parallel to AE e.g. BD</td></tr>
<tr><td></td><td>M1</td><td>Moments about AE or a parallel axis. Need all terms. Must be dimensionally correct. Condone sign errors.</td></tr>
<tr><td></td><td>A1ft</td><td>Correct unsimplified moments equation ft on their 'table'</td></tr>
<tr><td></td><td>A1</td><td>Correct (for their axis) only</td></tr>
<tr><td></td><td>M1</td><td>Correct use of trigonometry to find a relevant angle</td></tr>
<tr><td></td><td>M1</td><td>Correct strategy for the required angle.</td></tr>
<tr><td></td><td>A1</td><td>Correct answer only</td></tr>
</table>