Write your name here

Surname	Other names

Pearson Edexcel Level 3 GCE

Centre Number | | | | | |

Candidate Number | | | | |

Further Mathematics

Advanced Subsidiary
Further Mathematics options
Further Mechanics 2

Sample Assessment Material for first teaching September 2017
Time: 50 minutes

Paper Reference
8FM0/2J

You must have:
Mathematical Formulae and Statistical Tables, calculator

Total Marks

Candidates may use any calculator permitted by Pearson regulations. Calculators must not have the facility for algebraic manipulation, differentiation and integration, or have retrievable mathematical formulae stored in them.

Instructions

- Use **black** ink or ball-point pen.
- If pencil is used for diagrams/sketches/graphs it must be dark (HB or B).
- **Fill in the boxes** at the top of this page with your name, centre number and candidate number.
- Answer the questions in the spaces provided – *there may be more space than you need.*
- You should show sufficient working to make your methods clear. Answers without working may not gain full credit.
- Answers should be given to three significant figures unless otherwise stated.

Information

- A booklet 'Mathematical Formulae and Statistical Tables' is provided.
- There are 3 questions in this question paper. The total mark for this paper is 40.
- The marks for **each** question are shown in brackets – *use this as a guide as to how much time to spend on each question.*

Advice

- Read each question carefully before you start to answer it.
- Try to answer every question.
- Check your answers if you have time at the end.

Turn over ▶

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Answer ALL questions. Write your answers in the spaces provided.

Unless otherwise indicated, whenever a numerical value of g is required, take $g = 9.8\,\text{m s}^{-2}$ and give your answer to either 2 significant figures or 3 significant figures.

1. A particle P moves on the x-axis. At time t seconds the velocity of P is $v\,\text{m s}^{-1}$ in the direction of x increasing, where

$$v = (t - 2)(3t - 10), \qquad t \geqslant 0$$

When $t = 0$, P is at the origin O.

(a) Find the acceleration of P at time t seconds.

(2)

(b) Find the total distance travelled by P in the first 2 seconds of its motion.

(3)

(c) Show that P never returns to O, explaining your reasoning.

(3)

a) $a = \frac{dv}{dt}$

$v = (t-2)(3t-10)$

$= 3t^2 - 6t - 10t + 20$

$= 3t^2 - 16t + 20$

$\frac{dv}{dt} = 6t - 16$

$a = 6t - 16$

b) $s = \int v\,dt$

$s = \int_0^2 (3t^2 - 16t + 20)\,dt$

$= \left[\frac{3t^3}{3} - \frac{16t^2}{2} + 20t\right]_0^2$

$= [8 - 8(2)^2 + 20(2)] - 0$

$= 16\text{m}$

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Question 1 continued

c) P is at O when $s=0$

$$s = t^3 - 8t^2 + 20t$$

$$t^3 - 8t^2 + 20t = 0$$

$$t(t^2 - 8t + 20) = 0$$

$t = 0$ $\qquad$ $t^2 - 8t + 20 = 0$

when $t > 0$, $s = t^2 - 8t + 20$

$$s = (t-4)^2 - 16 + 20$$

$$s = (t-4)^2 + 4$$

s is always >0 for all values of t

$\therefore$ hence P never returns to O.

(Total for Question 1 is 8 marks)

2. A light inextensible string has length $7a$. One end of the string is attached to a fixed point A and the other end of the string is attached to a fixed point B, with A vertically above B and $AB = 5a$. A particle of mass m is attached to a point P on the string where $AP = 4a$. The particle moves in a horizontal circle with constant angular speed ω, with both AP and BP taut.

(a) Show that

(i) the tension in AP is $\frac{4m}{25}(9a\omega^2 + 5g)$

(ii) the tension in BP is $\frac{3m}{25}(16a\omega^2 - 5g)$.

(10)

The string will break if the tension in it reaches a magnitude of $4mg$.

The time for the particle to make one revolution is S.

(b) Show that

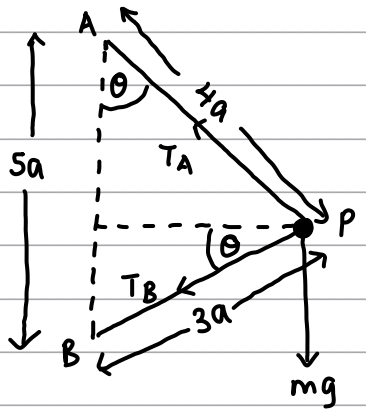
$$3\pi\sqrt{\frac{a}{5g}} < S < 8\pi\sqrt{\frac{a}{5g}}$$

(5)

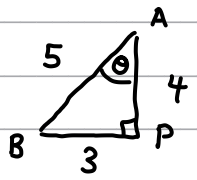
(c) State how in your calculations you have used the assumption that the string is light.

(1)

a)



∠ APB must be 90°



$\sin\theta = \frac{3}{5}$

$\cos\theta = \frac{4}{5}$

R(↕): $T_A\cos\theta = mg + T_B\sin\theta$

$\frac{4}{5}T_A = mg + \frac{3}{5}T_B$ — ①

+ ←

N2L (P): $T_A\sin\theta + T_B\cos\theta = m(r)w^2$

$r = 4a\sin\theta = 4a \times \frac{3}{5} = \frac{12a}{5}$

$\therefore T_A\left(\frac{3}{5}\right) + T_B\left(\frac{4}{5}\right) = \frac{12maw^2}{5}$

$3T_A + 4T_B = 12maw^2$ — ②

Question 2 continued

① × 5 : $4T_A = 5mg + 3T_B$

$T_B = \frac{4T_A - 5mg}{3}$

② : $3T_A + \frac{4}{3}(4T_A - 5mg) = 12maw^2$

$(3 + \frac{16}{3})T_A - \frac{20mg}{3} = 12maw^2$

$\frac{25}{3}T_A = m(12aw^2 + \frac{20g}{3})$

$T_A = \frac{m}{25}(36aw^2 + 20g)$

$\boxed{T_A = \frac{4m}{25}(9aw^2 + 5g)}$

ii) $T_B = \frac{1}{3}(4T_A - 5mg)$

$= \frac{1}{3}\left[\frac{16m}{25}(9aw^2 + 5g) - 5mg\right]$

$= \frac{1}{3}\left(\frac{144}{25}maw^2 + \frac{16mg}{5} - 5mg\right)$

$= \frac{1}{3}\left(\frac{144}{25}maw^2 - \frac{9}{5}mg\right)$

$= \frac{1}{3}\left[\frac{9m}{25}(16aw^2 - 5g)\right]$

$\boxed{T_B = \frac{3m}{25}(16aw^2 - 5g)}$

b) $T_{AP} < 4mg$: $\frac{4m}{25}(9aw^2 + 5g) < 4mg$

$9aw^2 + 5g < 25g$

$9aw^2 < 20g$

$w^2 < \frac{20g}{9a}$

$w < \sqrt{\frac{20g}{9a}}$

$T = \frac{2\pi}{w}$ so $T > 2\pi \times \frac{1}{w}$

sign flips as we take reciprocal of w

$\therefore (T =)\ S > 2\pi\sqrt{\frac{9a}{20g}}$

Question 2 continued

$S > 6\pi\sqrt{\frac{a}{5\times 4g}}$

$S > \frac{6\pi}{2}\sqrt{\frac{a}{5g}}$

$\therefore S > 3\pi\sqrt{\frac{a}{5g}}$

and also $T_B > 0$ for this circular motion to continue

$\Rightarrow \frac{3m}{25}(16a\omega^2 - 5g) > 0$

$16a\omega^2 - 5g > 0$

$\omega^2 > \frac{5g}{16a}$

$\omega > \frac{1}{4}\sqrt{\frac{5g}{a}}$

so $T < \frac{2\pi}{\omega}$

$T < 2\pi \times \frac{1}{\frac{1}{4}\sqrt{\frac{5g}{a}}}$

$T < (2\pi \times 4)\sqrt{\frac{a}{5g}}$

so $\boxed{S < 8\pi\sqrt{\frac{a}{5g}}}$

putting the inequalities together :

$\boxed{3\pi\sqrt{\frac{a}{5g}} < S < 8\pi\sqrt{\frac{a}{5g}}}$

c) tension is constant in both parts of the string (ie AP / BP)

(Total for Question 2 is 16 marks)

3.

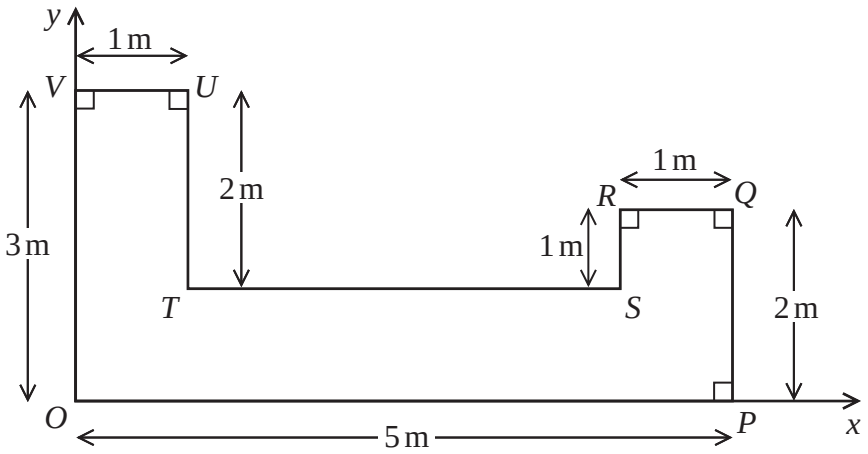


Figure 1

Figure 1 shows the shape and dimensions of a template *OPQRSTUV* made from thin uniform metal.
OP = 5 m, *PQ* = 2 m, *QR* = 1 m, *RS* = 1 m, *TU* = 2 m, *UV* = 1 m, *VO* = 3 m.
Figure 1 also shows a coordinate system with *O* as origin and the *x*-axis and *y*-axis along *OP* and *OV* respectively. The unit of length on both axes is the metre.

The centre of mass of the template has coordinates $(\bar{x}, \bar{y})$.

(a) (i) Show that $\bar{y} = 1$

(ii) Find the value of $\bar{x}$.

(7)

A new design requires the template to have its centre of mass at the point (2.5, 1). In order to achieve this, two circular discs, each of radius *r* metres, are removed from the template which is shown in Figure 1, to form a new template *L*. The centre of the first disc is (0.5, 0.5) and the centre of the second disc is (0.5, *a*) where *a* is a constant.

(b) Find the value of *r*.

(4)

(c) (i) Explain how symmetry can be used to find the value of *a*.

(ii) Find the value of *a*.

(2)

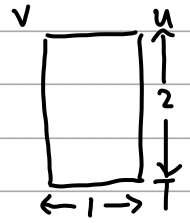
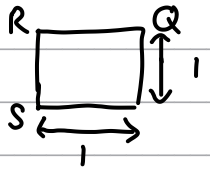
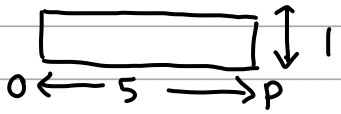
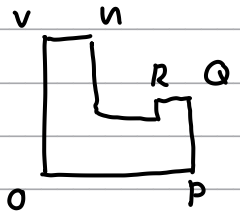
The template *L* is now freely suspended from the point *U* and hangs in equilibrium.

(d) Find the size of the angle between the line *TU* and the horizontal.

(3)

Question 3 continued

a)

	shape	area	c.o.m coordinates (origin O)
(+)		2	(0.5, 2)
(+)		1	(4.5, 1.5)
(+)		5	(2.5, 0.5)
(=)		8	$(\bar{x}, \bar{y})$

moments about O

$$2m\begin{pmatrix}0.5\\2\end{pmatrix} + m\begin{pmatrix}4.5\\1.5\end{pmatrix} + 5m\begin{pmatrix}2.5\\0.5\end{pmatrix} = 8m\begin{pmatrix}\bar{x}\\\bar{y}\end{pmatrix}$$

$$2(0.5) + 4.5 + 5(2.5) = 8\bar{x}$$

$$18 = 8\bar{x}$$

$$\bar{x} = \frac{18}{8} = \frac{9}{4}$$

$$2(2) + 1.5 + 5(0.5) = 8\bar{y}$$

$$8 = 8\bar{y}$$

$$\bar{y} = 1$$

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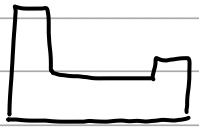
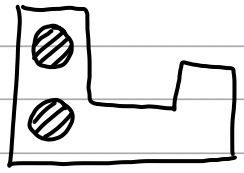
Question 3 continued

b)

shape	area	c.o.m.
(+) 	8	$(\frac{9}{4}, 1)$
(−) 	πr^2	$(\frac{1}{2}, \frac{1}{2})$
(−) 	πr^2	$(\frac{1}{2}, a)$
(=) 	$8-2\pi r^2$	$(\frac{5}{2}, 1)$

moments about O (for x)

$$8m(\tfrac{9}{4}) + \pi r^2 m(\tfrac{1}{2}) + \pi r^2 m(\tfrac{1}{2}) = (8-2\pi r^2)m(\tfrac{5}{2})$$

$$18 - \pi r^2 = 20 - 5\pi r^2$$

$$4\pi r^2 = 2$$

$$\therefore r^2 = \frac{2}{4\pi}$$

$$= \frac{1}{2\pi}$$

$$r = \sqrt{\frac{1}{2\pi}}$$

$$= 0.399$$

ci) $\bar{y} = 1$ for original template so the holes must be equidistant from $(\bar{y} = 1)$

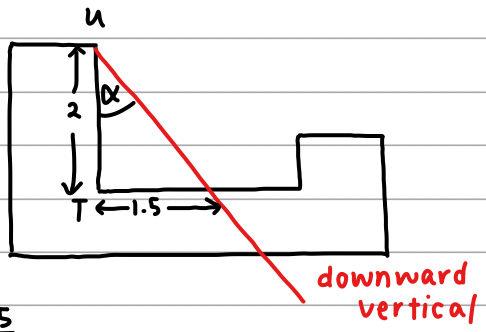
ii) $1 - 0.5 = 0.5$

$1 + 0.5 = 1.5$

$a = 1.5$

Question 3 continued

d)



$\tan\alpha = \frac{1.5}{2}$

$\alpha = \tan^{-1}\left(\frac{1.5}{2}\right)$

$= 36.87°$

so angle with horizontal $= 90 - 36.87$

$= \boxed{53.1°}$

(Total for Question 3 is 16 marks)

TOTAL IS 40 MARKS