Please check the examination details below before entering your candidate information

Candidate surname	Other names

Pearson Edexcel Level 3 GCE

Centre Number | Candidate Number

Thursday 16 May 2019

Afternoon | Paper Reference **8FM0-26**

Further Mathematics

Advanced Subsidiary
Further Mathematics options
26: Further Mechanics 2
(Part of option J only)

You must have:
Mathematical Formulae and Statistical Tables (Green), calculator

Total Marks

Candidates may use any calculator allowed by Pearson regulations. Calculators must not have the facility for symbolic algebra manipulation, differentiation and integration, or have retrievable mathematical formulae stored in them.

Instructions

- Use **black** ink or ball-point pen.
- If pencil is used for diagrams/sketches/graphs it must be dark (HB or B).
- **Fill in the boxes** at the top of this page with your name, centre number and candidate number.
- Answer **all** questions and ensure that your answers to parts of questions are clearly labelled.
- Answer the questions in the spaces provided – *there may be more space than you need.*
- You should show sufficient working to make your methods clear. Answers without working may not gain full credit.
- Unless otherwise indicated, whenever a value of g is required, take $g = 9.8$ m s^{-2} and give your answer to either 2 significant figures or 3 significant figures.

Information

- A booklet 'Mathematical Formulae and Statistical Tables' is provided.
- The total mark for this part of the examination is 40. There are 4 questions.
- The marks for **each** question are shown in brackets – *use this as a guide as to how much time to spend on each question.*

Advice

- Read each question carefully before you start to answer it.
- Try to answer every question.
- Check your answers if you have time at the end.

Turn over

1/1/1/1/1

P 6 2 3 1 3 A 0 1 2 0

Pearson

1.

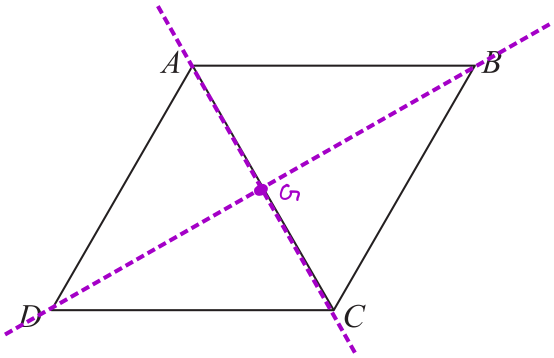


Figure 1

Five identical uniform rods are joined together to form the rigid framework *ABCD* shown in Figure 1. Each rod has weight W and length $4a$. The points A, B, C and D all lie in the same plane.

The centre of mass of the framework is at the point G.

(a) Explain why G is the midpoint of AC.

(1)

The framework is suspended from the ceiling by two vertical light inextensible strings. One string is attached to the framework at A and the other string is attached to the framework at B. The framework hangs freely in equilibrium with AB horizontal.

(b) Find

(i) the tension in the string attached at A,

(ii) the tension in the string attached at B.

(4)

A particle of weight kW is now attached to the framework at D and a particle of weight $2kW$ is now attached to the framework at C. The framework remains in equilibrium with AB horizontal and the strings vertical.

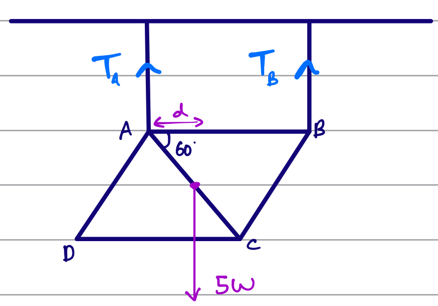
Either string will break if the tension in it exceeds $6W$.

(c) Find the greatest possible value of k.

(4)

a) All the rods have a uniform mass density, so the centre of mass is the intersection of the 2 axes of symmetry

b)



$\triangle ABC$ and $\triangle ADC$ are equilateral

$d = 2a\cos 60° = a$

Taking moments about A: ↻ = ↺

$5W \cdot a = 4a \cdot T_B$

$\Rightarrow T_B = \frac{5}{4}W$

Question 1 continued

Resolving Forces acting on the frame (↑)

$$T_A + T_B = 5W$$

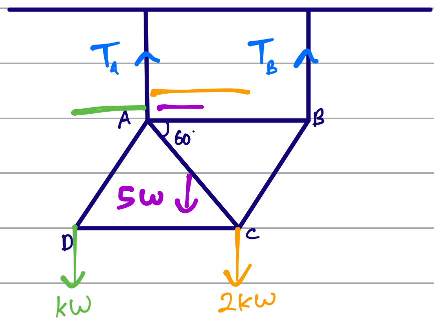
$$T_A = 5W - \frac{5W}{4}$$

$$T_A = \frac{15}{4}W$$

$$\therefore\ T_A = \frac{15}{4}W \quad , \quad T_B = \frac{5}{4}W$$

c) To find the new tensions:

Taking moments about A : ↻ = ↺



$$2a\cos 60° \cdot 5W + 4a\cos 60° \cdot 2kW = 4aT_B + 4a\cos 60° \cdot kW$$

$$4aT_B = 5aW + 4kaW - 2akW$$

$$T_B = \frac{(2k+5)W}{4}$$

By resolving ↑ ⇒ $T_A = (5+3k)W - \frac{(2k+5)}{4}W$

$$= \left(\frac{15}{4} + \frac{5}{2}k\right)W$$

When $T_A = 6W$,

$$\Rightarrow \frac{15}{4} + \frac{5}{2}k = 6$$

$$\frac{5}{2}k = \frac{9}{4}$$

$$k = 0.9$$

When $T_B = 6W$,

$$\Rightarrow \frac{2k+5}{4} = 6$$

$$2k = 19$$

$$k = 9.5$$

∴ As k increases from 0, the first string to snap would be A. This occurs at k = 0.9. So k cannot exceed 0.9.

Question 1 continued

Question 1 continued

(Total for Question 1 is 9 marks)

2. A car moves in a straight line along a horizontal road. The car is modelled as a particle. At time t seconds, where $t \geqslant 0$, the speed of the car is v m s^{-1}

At the instant when $t = 0$, the car passes through the point A with speed 2 m s^{-1}

The acceleration, a m s^{-2}, of the car is modelled by

$$a = \frac{4}{2+v}$$

in the direction of motion of the car.

(a) Use algebraic integration to show that $v = \sqrt{8t+16} - 2$

(6)

At the instant when the car passes through the point B, the speed of the car is 4 m s^{-1}

(b) Use algebraic integration to find the distance AB.

(6)

a) $a = \frac{dv}{dt}$

$\frac{4}{2+v} = \frac{dv}{dt} \Rightarrow 4\,dt = (2+v)\,dv$

$\int 4\,dt = \int 2+v\,dv$

$4t = 2v + \frac{v^2}{2} + c$

At $t=0$, $v=2$

$\Rightarrow 4(0) = 2 \cdot 2 + \frac{2^2}{2} + c$

$c = -2-4 = -6$

$\therefore \frac{v^2}{2} + 2v - 4t - 6 = 0$

$v^2 + 4v - 8t - 12 = 0$

$v = \frac{-4 \pm \sqrt{4^2 + 4 \cdot 1 \cdot (8t+12)}}{2}$

$= -2 \pm \frac{1}{2}\sqrt{16 + 32t + 48}$

$= -2 \pm \sqrt{16 + 8t}$

Question 2 continued

Since acceleration is ALWAYS in the same direction of v, v does not change sign

$\Rightarrow v = \sqrt{8t+16} - 2$

b) When $v = 4$,

$4 = \sqrt{8t+16} - 2$

$\sqrt{8t+16} = 6$

$8t + 16 = 36$

$8t = 20$

$t = 5/2$

distance AB $= \int_0^{5/2} v \, dt$

$= \int_0^{5/2} \sqrt{8t+16} - 2 \, dt$

$= \left[\frac{2}{3}(8t+16)^{3/2} \cdot \frac{1}{8} - 2t \right]_0^{5/2}$

$= \frac{2}{24}\left(8 \cdot \frac{5}{2} + 16\right)^{3/2} - 5$

$- \frac{1}{12}(16)^{3/2} + 0$

$= \frac{1}{12} \cdot 6^3 - 5 - \frac{4^3}{12}$

AB $= 7\frac{2}{3} = \frac{23}{3}$ m

Question 2 continued

Question 2 continued

(Total for Question 2 is 12 marks)

3. A light inextensible string has length $8a$. One end of the string is attached to a fixed point A and the other end of the string is attached to a fixed point B, with A vertically above B and $AB = 4a$. A small ball of mass m is attached to a point P on the string, where $AP = 5a$.

The ball moves in a horizontal circle with constant speed v, with both AP and BP taut.

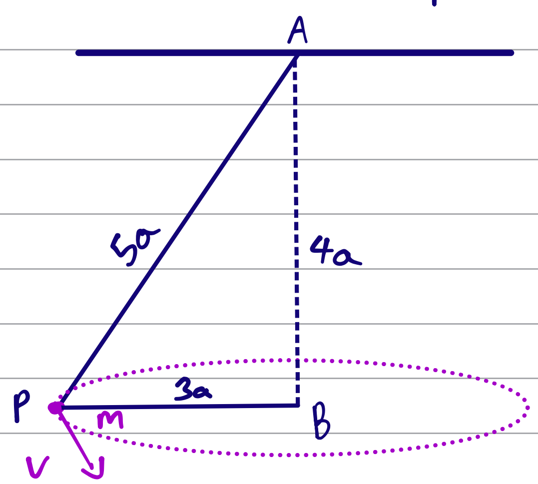
The string will break if the tension in it exceeds $\frac{3mg}{2}$

By modelling the ball as a particle and assuming the string does not break,

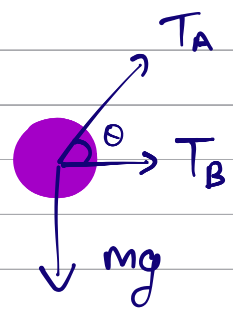
(a) show that $\frac{9ag}{4} < v^2 \leqslant \frac{27ag}{4}$ **(7)**

(b) find the least possible time needed for the ball to make one complete revolution. **(2)**

a) Picture of the set up:



Free-body diagram of ball:



Based on $\triangle APB$,

$\sin\theta = \frac{4}{5}$, $\cos\theta = \frac{3}{5}$, $\tan\theta = \frac{4}{3}$

At constant speed, since there is 0 vertical acceleration,

Resolving (↑)

$T_A \sin\theta = mg$

$T_A = \frac{5}{4}mg$

Horizontally,

$T_A \cos\theta + T_B = \text{centripetal force} = \frac{mv^2}{r}$

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Question 3 continued

$$\frac{3}{5} \times \frac{5}{4} mg + T_B = \frac{mv^2}{3a}$$

$$T_B = \frac{mv^2}{3a} - \frac{3}{4} mg$$

Since $T_B > 0$ [the string is taut]

$$\frac{mv^2}{3a} - \frac{3mg}{4} > 0$$

$$\Rightarrow \frac{mv^2}{3a} > \frac{3mg}{4}$$

$$v^2 > \frac{9ag}{4}$$

Since $T_B \leq \frac{3mg}{2}$ [or else the string would break]

$$\frac{mv^2}{3a} - \frac{3mg}{4} \leq \frac{3mg}{2}$$

$$\frac{mv^2}{3a} \leq \frac{9mg}{4}$$

$$v^2 \leq \frac{27ag}{4}$$

$$\therefore \frac{9ag}{4} < v^2 \leq \frac{27ag}{4}$$

b) The ball completes one revolution in the least time when it is moving the quickest $\Rightarrow V_{max}^2 = \frac{27ag}{4}$

$$V_{max} = \frac{3}{2}\sqrt{3ag} = \frac{2\pi r}{T_{min}}$$

Question 3 continued

$$\therefore \quad \frac{3}{2}\sqrt{3ag} = \frac{6\pi a}{T_{min}}$$

$$\Rightarrow T = \frac{4\pi a}{\sqrt{3ag}} = 4\pi\sqrt{\frac{a}{3g}}$$

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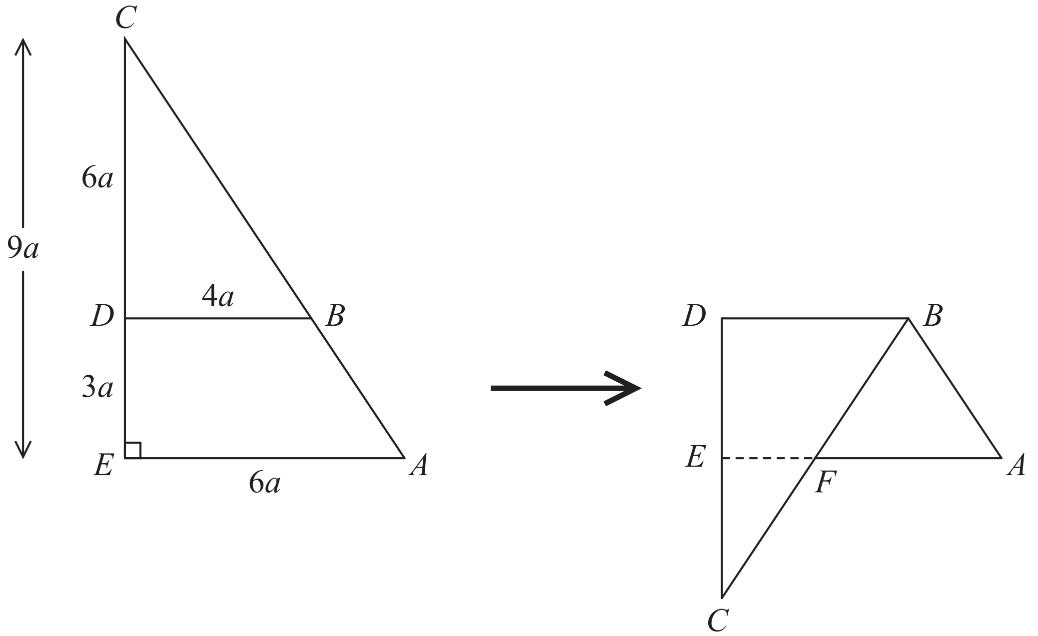
Question 3 continued

(Total for Question 3 is 9 marks)

4.



Triangular lamina   Folded lamina

Figure 2

The uniform triangular lamina *ABCDE* is such that angle $CEA = 90°$, $CE = 9a$ and $EA = 6a$. The point *D* lies on *CE*, with $DE = 3a$. The point *B* on *CA* is such that *DB* is parallel to *EA* and $DB = 4a$. The triangular lamina is folded along the line *DB* to form the folded lamina *ABDECF*, as shown in Figure 2.

The distance of the centre of mass of the triangular lamina from *DC* is d_1

The distance of the centre of mass of the folded lamina from *DC* is d_2

(a) Explain why $d_1 = d_2$

(1)

The folded lamina is freely suspended from *B* and hangs in equilibrium with *BA* inclined at an angle α to the downward vertical through *B*.

(b) Find, to the nearest degree, the size of angle α.

(9)

a) Since the fold was along a line ⊥ to CD, every point on the lamina remains the same distance from CD after the fold.

No movement in ↔ direction

i.e., no mass was moved towards or away from CD, so the centre of mass

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Question 4 continued

b) Centre of mass $= (\bar{x}, \bar{y})$

$\bar{x} = d_1 = d_2 = 2a$

To work out $\bar{y}$, use area ratios and distances from a reference line (we'll choose EA).

	Large Δ (ΔACE)	Removed Δ (ΔBCD)	Added Δ (ΔBCD)	Folded lamina ($\bar{y}$)
Area Ratios →	$27a^2$	$12a^2$	$12a^2$	$27a^2$
Distances from EA →	$3a$	$5a$	a	$\bar{y}$

Use these quantities to "take moments" about EA:

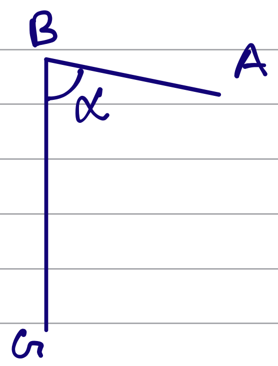
$$27 \times 3a - 12 \times 5a + 12 \times a = 27\bar{y}$$

$$33a = 27\bar{y}$$

$$\bar{y} = \frac{11a}{9}$$

When suspended from B, the centre of mass will be vertically below B

The angle α between any 2 vectors can be found using the dot product:



$$\cos\alpha = \frac{\vec{BA} \cdot \vec{BG}}{|\vec{BA}|\,|\vec{BG}|}$$

$$= \frac{\frac{2}{9}\begin{pmatrix}-9\\-8\end{pmatrix} \cdot \begin{pmatrix}2\\-3\end{pmatrix}}{\frac{2}{9}\sqrt{145}\,\sqrt{13}} = 0.13819603\ldots$$

Acute angle solution: $\alpha = \cos^{-1}(0.138196\ldots) = 82.0565\ldots \approx 82°$

Question 4 continued

Question 4 continued

Question 4 continued

Question 4 continued

Question 4 continued

(Total for Question 4 is 10 marks)

TOTAL FOR FURTHER MECHANICS 2 IS 40 MARKS