Pearson Edexcel

Mark Scheme

Summer 2023

Pearson Edexcel GCE

A Level Further Mathematics (9FM0)

Paper 3C

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General Marking Guidance

- All candidates must receive the same treatment. Examiners must mark the first candidate in exactly the same way as they mark the last.
- Mark schemes should be applied positively. Candidates must be rewarded for what they have shown they can do rather than penalised for omissions.
- Examiners should mark according to the mark scheme not according to their perception of where the grade boundaries may lie.
- There is no ceiling on achievement. All marks on the mark scheme should be used appropriately.
- All the marks on the mark scheme are designed to be awarded. Examiners should always award full marks if deserved, i.e. if the answer matches the mark scheme. Examiners should also be prepared to award zero marks if the candidate's response is not worthy of credit according to the mark scheme.
- Where some judgement is required, mark schemes will provide the principles by which marks will be awarded and exemplification may be limited.
- When examiners are in doubt regarding the application of the mark scheme to a candidate's response, the team leader must be consulted.
- Crossed out work should be marked UNLESS the candidate has replaced it with an alternative response.

EDEXCEL GCE MATHEMATICS

General Instructions for Marking

1. The total number of marks for the paper is 75.

2. The Edexcel Mathematics mark schemes use the following types of marks:

- **M** marks: method marks are awarded for 'knowing a method and attempting to apply it', unless otherwise indicated.
- **A** marks: Accuracy marks can only be awarded if the relevant method (M) marks have been earned.
- **B** marks are unconditional accuracy marks (independent of M marks)
- Marks should not be subdivided.

3. Abbreviations

These are some of the traditional marking abbreviations that will appear in the mark schemes.

- bod – benefit of doubt
- ft – follow through
- the symbol $\sqrt{}$ will be used for correct ft
- cao – correct answer only
- cso - correct solution only. There must be no errors in this part of the question to obtain this mark
- isw – ignore subsequent working
- awrt – answers which round to
- SC: special case
- oe – or equivalent (and appropriate)
- dep – dependent
- indep – independent
- dp decimal places
- sf significant figures
- * The answer is printed on the paper
- ⎾ The second mark is dependent on gaining the first mark

4. For misreading which does not alter the character of a question or materially simplify it, deduct two from any A or B marks gained, in that part of the question affected.

5. Where a candidate has made multiple responses <u>and indicates which response they wish to submit</u>, examiners should mark this response.
If there are several attempts at a question <u>which have not been crossed out</u>, examiners should mark the final answer which is the answer that is the <u>most complete</u>.

6. Ignore wrong working or incorrect statements following a correct answer.

7. Mark schemes will firstly show the solution judged to be the most common response expected from candidates. Where appropriate, alternatives answers are provided in the notes. If examiners are not sure if an answer is acceptable, they will check the mark scheme to see if an alternative answer is given for the method used.

Question	Scheme	Marks	AOs
1(a)	Impulse-momentum:	M1	3.1a
	$(-6\mathbf{i}+42\mathbf{j}) = 2\{\mathbf{v}-(-4\mathbf{i}+3\mathbf{j})\}$	A1	1.1b
	Find magnitude of their **v:** $\sqrt{(-7)^2+24^2}$	M1	1.1b

	$25\ (\text{m s}^{-1})$	A1	1.1b
		(4)	
1(b)	Use scalar product $\cos\alpha = \dfrac{(-4\times -7)+(3\times 24)}{\sqrt{(-4)^2+3^2}\times\sqrt{(-7)^2+24^2}}$	M1	3.1a
	$\alpha = 37$ or better	A1	1.1b
		(2)	
1(b)alt 1	Use cosine rule in a vector triangle: $\cos\alpha = \dfrac{\{(-4)^2+3^2\}+\{(-7)^2+24^2\}-(3^2+(-21)^2)}{2\times 5\times\sqrt{(-7)^2+24^2}}$	M1	3.1a
	$\alpha = 37$ or better	A1	1.1b
		(2)	
1(b)alt 2	Use inverse tan: Eg $\alpha = \tan^{-1}\left(\dfrac{24}{7}\right)-\tan^{-1}\left(\dfrac{3}{4}\right)$ $\alpha = 90-\tan^{-1}\left(\dfrac{7}{24}\right)-\tan^{-1}\left(\dfrac{3}{4}\right)$	M1	3.1a
	$\alpha = 37$ or better	A1	1.1b
		(2)	
			(6 marks)

Notes:

(a)	
M1	Dimensionally correct, mass $\times$ velocity. Must be subtracting momenta but condone subtracting in the wrong order. M0 if g is included.
A1	Correct unsimplified equation.
M1	Correct application of Pythagoras to find the magnitude of their v. M0 for an incorrect speed if there is no evidence of Pythagoras being used on their velocity.
A1	Correct answer following the correct velocity.
(b)	
M1	Complete method to find the required angle. Correct use of scalar product with their **v**. The formula must be correct, $\cos\alpha = \dfrac{\mathbf{u}\cdot\mathbf{v}}{\lvert\mathbf{u}\rvert\lvert\mathbf{v}\rvert}$ M0 if the fraction is up the wrong way. Do not ISW.
A1	cao in degrees
(b)alt1	

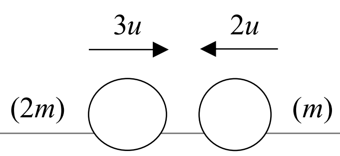
M1	Complete method to find the required angle. Correct use of cosine rule on (**v** – **u**) or (**u** – **v**) vector triangle for their **v**. M0 if using (**v** + **u**). Do not ISW.
A1	cao in degrees
(b)alt2	
M1	Complete method to find the required angle. Correct use of inverse tan formulae for their **v.** Do not ISW. M0 for $\tan^{-1}\left(\frac{3}{4}\right)$ alone which also gives the value 36.869…
A1	cao in degrees

Question	Scheme	Marks	AOs
2(a)	$F = \dfrac{16000}{v}$	M1	3.3
	Equation of motion: $F - 400 = 0$	M1	3.1b
	$U = 40$	A1	1.1b

		(3)	
2(b)	$F = \dfrac{16000}{\left(\dfrac{20}{3}\right)}$	M1	3.3
	Equation of motion for system or car or trailer:	M1	3.1b
	$F - 700 = 1600a$ or $F - 400 - T = 1000a$ or $T - 300 = 600a$	A1	1.1b
	Second equation of motion	A1	1.1b
	$T = 940$ or 938 or 937.5 or $\dfrac{1875}{2}$ oe (N)	A1	1.1b
		(5)	
			(8 marks)

Notes:

(a)	
M1	Correct use of $P = Fv$. The expression $\dfrac{16000}{v}$ may be on a diagram or embedded in their $F = ma$. Condone use of 16 000 or 16 for the method mark.
M1	Correct unsimplified equation of motion with $a = 0$ or equilibrium equation. F does not need to be substituted.
A1	cao
(b)	
M1	Correct use of $P = Fv$ with $v = \dfrac{20}{3}$. This expression may be on the diagram or embedded in their $F = ma$. Condone use of 16 000 or 16 for the method mark.
M1	An equation of motion for the whole system or car or trailer. Must have all terms and be dimensionally correct. Condone sign errors. M0 if $a = 0$ is used. NB: Full marks in (b) can be scored if consistent extra g's (must be present in both 'ma' terms in a complete solution). Otherwise penalise as A error.
A1	One correct unsimplified equation.
A1	Two correct unsimplified equations. Note: $a = \dfrac{17}{16}$ but does not need to be seen.
A1	Correct answer

Question	Scheme	Marks	AOs
3(a)	**If it helps the candidate, ignore their diagram.** 		

	$\overrightarrow{\quad} v_P \quad \overrightarrow{\quad} v_Q$		
	CLM:	M1	3.1a
	$2m\times 3u - m\times 2u = 2mv_P + mv_Q \qquad (4u = 2v_P + v_Q)$	A1	1.1b
	Impact Law:	M1	3.4
	$5ue = -v_P + v_Q$	A1	1.1b
	Attempts to solve for v_Q	dM1	2.1
	$v_Q = \dfrac{(4+10e)u}{3}$ *	A1*	2.2a
		(6)	
3(b)	$v_P = \dfrac{(4-5e)u}{3}$ oe	M1	1.1b
	Correct rebound speed or velocity of Q seen $\pm\dfrac{f(4+10e)u}{3}$	B1	3.4
	States a correct inequality Eg 2nd collision if • $\dfrac{f(4+10e)u}{3} > -\dfrac{(4-5e)u}{3}$ • $\dfrac{f(4+10e)u}{3} > \dfrac{(5e-4)u}{3}$ • $\dfrac{(4-5e)u}{3} > -\dfrac{f(4+10e)u}{3}$ • $-\dfrac{(5e-4)u}{3} > -\dfrac{f(4+10e)u}{3}$	M1	3.1a
	$(1\geq) f > \dfrac{5e-4}{4+10e}$	A1	1.1b
		(4)	
			(10 marks)

Notes:

(a)	
M1	CLM used. Dimensionally correct, mass × velocity. All terms required. Condone sign errors. Condone consistent g's or cancelled m's.
A1	Correct unsimplified equation
M1	NEL used correctly with e appearing on the correct side of the equation. Condone sign errors, must have the correct number of terms.
A1	Correct unsimplified equation. Direction of v_Q and v_P must be consistent with their CLM equation.

dM1	Use their correctly formed equations to solve for v_Q At least one line of working should be seen.
A1*	Correct given answer correctly obtained $v_Q = \frac{(4+10e)u}{3}$ * Also accept: $\frac{1}{3}(4+10e)u$ $\frac{u(4+10e)}{3}$ $\frac{u}{3}(4+10e)$ Do not accept the 4 and 10e reversed eg $\frac{(10e+4)u}{3}$ is A0*
(b)	
M1	Attempt to solve for v_P. If v_P is found in (a) it must be used in (b) to score this mark. Note that if P is assumed to reverse direction in (a) then $v_P = \frac{(5e-4)u}{3}$ oe
B1	Correct expression seen for speed or velocity of Q after rebound $\pm\frac{f(4+10e)u}{3}$. This may appear on a diagram.
M1	Correct unsimplified inequality seen. The inequality must be correct, accepts cancelled u's and/or 3's
A1	Correct inequality, do not ISW. Allow $1 \geq f$ to be omitted but do not allow the strict inequality $1 > f$.

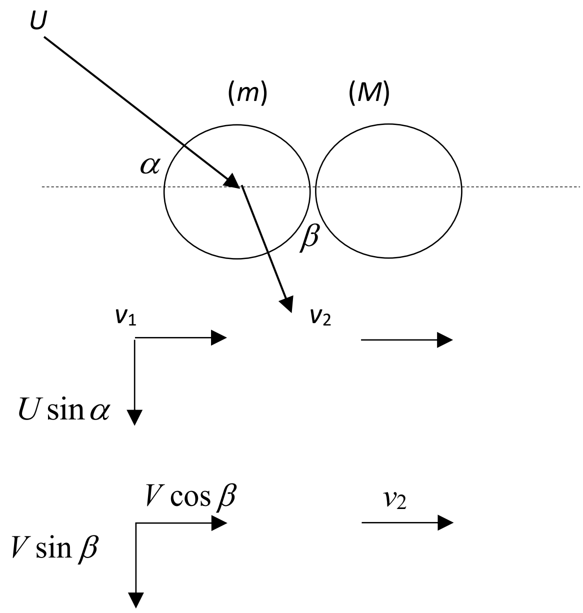
Question	Scheme	Marks	AOs
4(a)	$T = \frac{4mge}{2a}$	B1	3.3
	$T = mg$	M1	3.1a
	$e = \frac{1}{2}a$	A1	1.1b
	$OE = \frac{5a}{2}$	A1	1.1b
		(4)	
4(b)	GPE term, $\pm mga$	B1	3.4
	Work done against resistance, $\pm \frac{1}{4}mga$	B1	3.4
	Use of EPE formula once.	M1	3.4
	$\pm \frac{4mg}{2\times 2a}\left\{(2a)^2 - a^2\right\}$	A1	1.1b
	Work energy equation:	M1	3.1a
	$\frac{1}{4}mga = \frac{4mg}{2\times 2a}\left\{(2a)^2 - a^2\right\} - mga - \frac{1}{2}mv^2$	A1	1.1b
	$v = \sqrt{\frac{7ag}{2}}$ oe	A1	1.1b
		(7)	
4(c)	$mg - T - \frac{1}{4}mg = 0$	M1	3.1a
	$mg - \frac{4mgx}{2a} - \frac{1}{4}mg = 0$	A1	1.1b
	$x = \frac{3a}{8}$	A1	1.1b
	$OB = \frac{19a}{8}$ oe	A1	1.1b
		(4)	
		(15 marks)	

Notes:

(a)	

B1	Hooke's Law seen with *4mg* and *2a* substituted.
M1	Resolving vertically. Correct number of terms.
A1	cao for extension.
A1	cao for *OE*. Note that if the extension is ($OE - 2a$) in their equation, *OE* can be found directly and both A's can be earned together.
(b)	
B1	GPE term seen, ignore sign.
B1	Work term seen $\frac{mga}{4}$, ignore sign. Allow B1 for the case where WD = $\frac{5mga}{4}$. This is a special case where the work done against resistance is included within the term. $\frac{5mga}{4}$ = WD against resistance + WD against weight.
M1	Use of EPE formula. Accept EPE in the form $\frac{\lambda x^2}{ka}$
A1	Difference between two correct EPE terms seen, unsimplified.
M1	Work-energy equation is formed with all relevant terms and no extras.: KE, GPE, 2EPE, WD. Condone sign errors. M0: For work-energy equation with WD = $\frac{5mga}{4}$ **and** a GPE term. This is because weight is considered twice and so the equation contains an extra term.
A1	Correct unsimplified equation
A1	Correct answer in terms of *a* and *g*, do not allow 9.8 for *g* $v=\sqrt{\frac{7ag}{2}}$, $v=\frac{1}{2}\sqrt{14ag}$
(c)	
M1	Vertical equilibrium equation or equation of motion with a = 0. Condone sign errors. Correct no. of terms - all 3 forces must be included although $\left(mg \pm \frac{mg}{4}\right)$ may already be simplified. Hooke's Law does not need to be substituted but M0 if the equilibrium position from (a) is used.
A1	Correct equation in one unknown.
A1	cao
A1	cao Note that if the extension is ($OB - 2a$) in their equation, *OB* can be found directly and both A's can be earned together.
4(c) Alt 1	**Using differentiation with a Work - energy equation from the point of release**

M1	Forming work-energy equation with the usual rules: all relevant terms to be included and of the correct form and no extra terms. $\frac{1}{2}mv^2 = mgh - \frac{4mg(h-2a)^2}{2(2a)} - \frac{mgh}{4}$
A1	Correct equation for v^2 and h (may use a different letter)
A1	Correct equation after differentiating v^2 or v with respect to h and setting it equal to zero. $\frac{\mathrm{d}}{\mathrm{dh}}\left(v^2\right) = 0 \quad \to \quad \frac{3g}{2} = \frac{4g(h-2a)}{a}$ oe
A1	Correct answer $OB = \frac{19a}{8}$

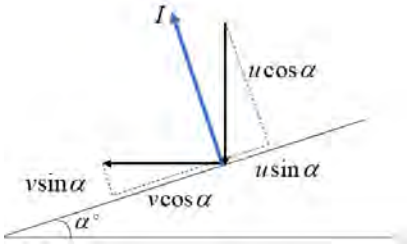
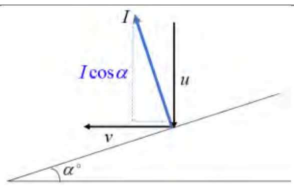
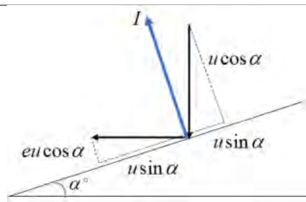
Question	Scheme	Marks	AOs
5(a)	**If it helps the candidate, ignore their diagram.** 		
	$U\sin\alpha$ seen as velocity component of S, perpendicular to line of centres after impact.	B1	3.4
	CLM along line of centres	M1	3.1b
	$mU\cos\alpha = mv_1 + Mv_2$	A1	1.1b
	NEL used along line of centres	M1	3.3
	$eU\cos\alpha = -v_1 + v_2$	A1	1.1b
	$\tan\beta = \dfrac{U\sin\alpha}{v_1}$	dM1	2.1
	Solve to produce an equation for $\tan\beta$ in m, M, e and α	dM1	1.1b
	$\tan\beta = \dfrac{(m+M)\tan\alpha}{(m-eM)}$ *	A1*	1.1b
		(8)	
5(a) alt1	$U\sin\alpha$ seen as velocity cpt of S, perpendicular to line of centres after impact.	B1	3.4
	CLM along line of centres	M1	3.1b
	$mU\cos\alpha = mV\cos\beta + Mv_2$	A1	1.1b
	NEL used along line of centres	M1	3.3
	$eU\cos\alpha = -V\cos\beta + v_2$	A1	1.1b

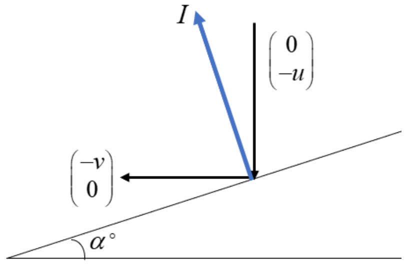
	$\tan\beta = \dfrac{U\sin\alpha}{V\cos\beta}$ or $V\sin\beta = U\sin\alpha$	dM1	2.1
	Solve to produce an equation for $\tan\beta$ in *m*, *M*, *e* and α	dM1	1.1b
	$\tan\beta = \dfrac{(m+M)\tan\alpha}{(m-eM)}$ *	A1*	1.1b
		(8)	
5(b)	Use the given condition to find the direction of *S* after impact. Eg • $m = eM \Rightarrow \tan\beta = \infty$ or $\tan\beta$ is undefined so $\beta = 90^\circ$ oe • $m = eM \Rightarrow v_1 = 0$ so velocity component of *S* parallel to line of centres is zero.	M1	3.1b
	Conclusion: After the collision, *S* moves perpendicular to the line of centres and the other sphere moves parallel to the line of centres i.e. they move at right angles oe *	A1*	2.4
		(2)	
			(10 marks)

Notes:

(a)	
B1	$U\sin\alpha$ or $U\cos(90-\alpha)$ used as the perpendicular velocity component of *S* after impact. Must be seen in working for (a) or on a velocity diagram.
M1	CLM along the line of centres. Dimensionally correct, correct no. of terms, condone sin/cos confusion and sign errors.
A1	Correct equation.
M1	NEL used correctly along the line of centres with *e* appearing on the correct side of the equation. Condone sin/cos confusion as long as it is consistent with their CLM. Condone sign errors but must have the correct number of terms.
A1	Correct equation (the signs of v_1 and v_2 must be consistent with their CLM)
dM1	Use of the fact that *S* moves at β to the line of centres after the collision. Use of their components after the collision to form an equation in β. Dependent on both previous M's.
dM1	Eliminate v_1 to produce an equation for $\tan\beta$ in *m*, *M*, *e* and α. Dependent on first two M's in (b) Note: $v_1 = u\cos\alpha\left(\dfrac{m-eM}{m+M}\right)$
A1*	Given answer correctly obtained. Must match printed answer EXACTLY.
5(a) alt1	

B1	$U\sin\alpha$ or $U\cos(90-\alpha)$ used as the perpendicular velocity component of *S* after impact. Must be seen in (a) or on a velocity diagram.
M1	CLM along the line of centres. Dimensionally correct, correct no. of terms, condone sin/cos confusion and sign errors.
A1	Correct equation
M1	NEL used correctly along the line of centres with *e* appearing on the correct side of the equation. Condone sin/cos confusion as long as it is consistent with their CLM. Condone sign errors but must have the correct number of terms.
A1	Correct equation (signs and sin/cos must be consistent with their CLM)
dM1	Use of the fact that *S* moves at β to the line of centres after the collision. Use of their components after the collision to form an equation β. Dependent on both previous M's.
dM1	Eliminate $V\cos\beta$ to produce an equation for $\tan\beta$ in *m*, *M*, *e* and α. Dependent on first two M's in (b) Note: $V\cos\beta = u\cos\alpha\left(\dfrac{m-eM}{m+M}\right)$
A1*	Given answer correctly obtained. Must match printed answer EXACTLY.
(b)	
M1	Use of given condition to deduce that $\beta = 90^\circ$ or that velocity component parallel to line of centres is zero.
A1*	Correct explanation using given information. Must refer correctly to the direction of both particles, eg perpendicular, at right angles, parallel and perpendicular to the line of centres, Do not accept horizontally and vertically since the surface is defined as horizontal.

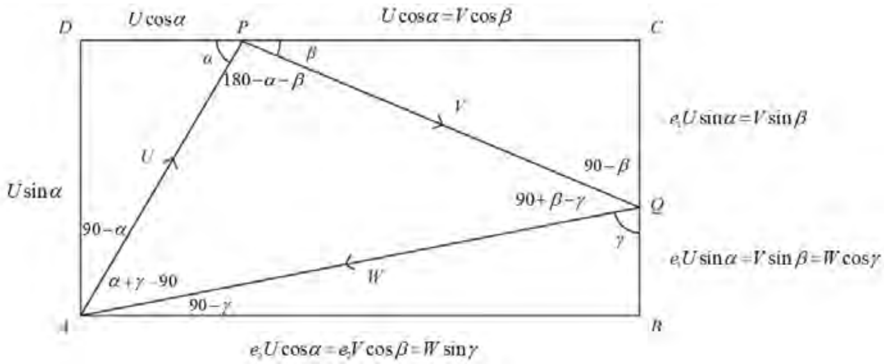
Question	Scheme	Marks	AOs
6(a)			
	CLM along the plane:	M1	3.1a
	$(m)u\sin\alpha = (m)v\cos\alpha$	A1	1.1b
	Impulse-momentum perp to the plane:	M1	3.1a
	$I = m(v\sin\alpha - (-u\cos\alpha))$	A1	1.1b
	$I = m(\frac{u\sin^2\alpha}{\cos\alpha} + u\cos\alpha) = \frac{mu}{\cos\alpha}(\sin^2\alpha + \cos^2\alpha) = mu\sec\alpha$ *	A1*	2.2a
		(5)	
6(a) alt1		M1	3.1a
	Impulse-momentum vertically.	M1	3.1a
	$I\cos\alpha = m(0 - -u)$	A1 A1	1.1b 1.1b
	$I = mu\sec\alpha$ *	A1*	2.2a
		(5)	
6(a) alt 2	Introduce and use an expression for e 		
	CLM along the plane:	M1	3.1a
	$u\sin\alpha$ unchanged	A1	1.1b
	Finds an expression for e together with Impulse-momentum perpendicular to the plane $\tan\alpha = \frac{eu\cos\alpha}{u\sin\alpha} \Rightarrow e = \tan^2\alpha$ and $I = m(eu\cos\alpha - (-u\cos\alpha))$	M1	3.1a
	$I = m(u\cos\alpha\tan^2\alpha - (-u\cos\alpha))$	A1	1.1b
	$I = m(\frac{u\sin^2\alpha}{\cos\alpha} + u\cos\alpha) = \frac{mu}{\cos\alpha}(\sin^2\alpha + \cos^2\alpha) = mu\sec\alpha$ *	A1*	2.2a
		(5)	

6(a) alt 3	Use a vector approach and magnitude of impulse 		
	CLM along the plane:	M1	3.1a
	$(m)u\sin\alpha = (m)v\cos\alpha$ (this leads to $v = u\tan\alpha$)	A1	1.1b
	Impulse-momentum as a vector equation followed by Pythagoras to find the magnitude. $I = m\begin{pmatrix}-v\\u\end{pmatrix}$ and $\lvert I\rvert = m\sqrt{v^2+u^2}$	M1	3.1a
	$\lvert I\rvert = m\sqrt{u^2\tan^2\alpha+u^2}$	A1	1.1b
	$I = m\sqrt{u^2(1+\tan^2\alpha)} = m\sqrt{u^2\sec^2\alpha} = mu\sec\alpha$ *	A1*	2.2a
		(5)	
6(b)	NEL: $eu\cos\alpha = v\sin\alpha$	M1	3.4
	Squaring and adding their expressions for v sinα and v cosα.	M1	1.1b
	$v^2 = u^2(\sin^2\alpha+e^2\cos^2\alpha)$ *	A1*	1.1b
		(3)	
6(c)	KE loss = $\frac{1}{2}mu^2 - \frac{1}{2}mu^2(\sin^2\alpha+e^2\cos^2\alpha)$.	M1	2.1
	Use $\sin^2\alpha+\cos^2\alpha=1$ to give KE loss = $\frac{1}{2}mu^2(1-e^2)\cos^2\alpha$ *	A1*	1.1b
		(2)	
6(d)	Use $\tan^2\alpha = e$ oe to eliminate α in given expression from (c)	M1	3.1a
	KE Loss = $\frac{1}{2}mu^2(1-e)$ or $\frac{1}{2}mu^2\frac{1}{1+e}(1-e^2)$	A1	1.1b
		(2)	
			(12 marks)

Notes:

(a)	
M1	Correct no. of terms, dimensionally correct, mass × velocity, condone sin/cos confusion.
A1	Correct equation
M1	Dimensionally correct. Must be subtracting, but condone subtracting in the wrong order and sin/cos confusion

A1	Correct unsimplified equation
A1*	Given answer correctly obtained. Must be EXACT factorisation.
(b)	
M1	Attempt at NEL
M1	Squaring and adding their expressions for $v \sin\alpha$ and $v \cos\alpha$ to obtain v^2.
A1*	Given answer correctly obtained. Must be EXACT.
(c)	
M1	Expression for difference of KE in terms of m, u, α and e
A1*	Given answer correctly obtained. Factorisation must be EXACT.
(d)	
M1	Complete method to eliminate α e.g. using $\tan^2\alpha = e$ to eliminate α Any trig identity used must be correct eg $\sec^2\alpha = 1 + e$ or $\cos^2\alpha = \frac{1}{1+e}$
A1	Correct answer.

Question	Scheme	Marks	AOs
	Note: The diagram below is an aide for marking. In reality, the velocity components cannot be represented by the side lengths of the snooker table. The magnitude of PC is not the magnitude of $U\cos\alpha$ 		
7(a)	$(V\sin\beta =)e_1U\sin\alpha$	B1	3.4
	$(V\cos\beta =)U\cos\alpha$	B1	3.4
	Eliminate U and V from two equations	M1	1.1b
	$\tan\beta = e_1\tan\alpha$ *	A1*	2.2a
		(4)	
7(b)	Form a correct equation for γ β and e_2 $\tan\gamma = e_2\tan(90^\circ-\beta)$ $\tan\gamma = e_2\cot\beta$ $\cot\gamma = \dfrac{\tan\beta}{e_2}$	B1	1.1b
	$\tan\gamma = e_2 \times \dfrac{1}{\tan\beta} = e_2 \times \dfrac{1}{e_1\tan\alpha}$	M1	3.1b
	$e_1\tan\alpha = e_2\cot\gamma$ *	A1*	2.2a
		(3)	
7(c)	(angle APQ + angle AQP) = $(180^\circ-\alpha-\beta)+\{180^\circ-(90^\circ-\beta)-\gamma\}$ = $270-\alpha-\gamma$ Otherwise: • angle PAQ = $\alpha+\gamma-90$	M1	1.1b
	To return to A, (angle APQ + angle AQP) < 180°, since APQ is a triangle Otherwise: • angle $PAQ > 0$	M1	3.1b
	$270^\circ-\alpha-\gamma<180^\circ$ => $\alpha>90^\circ-\gamma$ oe	A1	1.1b
	$\tan\alpha > \tan(90^\circ-\gamma)$ oe See notes for completion using addition formulae.	M1	1.1b

	$\frac{e_2 \cot \gamma}{e_1} > \cot \gamma$	M1	1.1b
	$e_2 > e_1$ *	A1*	2.2a
		(6)	
7(d)	From (b), $\alpha = 90^\circ - \gamma$, so it moves parallel to *AP* oe Eg parallel to the initial velocity	B1	2.4
			(14 marks)

Notes:

(a)	
B1	$e_1 U \sin \alpha$ seen from a relevant equation or on a diagram.
B1	$U \cos \alpha$ seen in a relevant equation or on a diagram.
M1	A clear method using **two equations** to eliminate *U* and *V*.
A1*	GIVEN answer correctly obtained. Must include two equations showing how to reach **both** $\tan \beta$ **and** $e_1 \tan \alpha$. It is not sufficient to use the side lengths of the snooker eg using $\tan \beta = \frac{CQ}{PC}$ oe is not sufficient. Accept $\tan \beta = e_1 \tan \alpha$ or $e_1 \tan \alpha = \tan \beta$
(b)	This part states 'hence' so β must be used.
B1	Form a correct expression for $\tan\gamma$ or $\cot\gamma$ in terms of e_2 and β or $(90 - \beta)$. May quote result from (a) or obtain again.
M1	**Use result from (a)** to eliminate $\tan\beta$ and form an equation in α, γ, e_1, e_2
A1*	Given answer correctly obtained. The solution must include the replacement of $\tan\beta$ and rearrangement to the correct form. Accept $e_1 \tan \alpha = e_2 \cot \gamma$ or $e_2 \cot \gamma = e_1 \tan \alpha$
(c)	
M1	Clear attempt to find angle sum (condone slips) or another relevant starting point eg an expression for angle *PAQ*
M1	Clear statement to form an inequality eg • the correct angle sum < 180 is acceptable • angle *PAQ* > 0
A1	Correct simplified inequality in correct form
M1	Correct method to form an inequality in tan or cot
M1	Using part (b) to eliminate the angles

A1*	Given answer correctly obtained
7(c) alt	**Use of trig identity**
M1	(angle *APQ* + angle *AQP*) = $(180^{\circ} - \alpha - \beta) + \{180^{\circ} - (90^{\circ} - \beta) - \gamma)\}$ = 270 – α – γ
M1	To return to *A*, (angle *APQ* + angle *AQP*) < 180°, since *APQ* is a triangle
A1	$\tan(\alpha+\gamma) = \dfrac{\tan\alpha + \tan\gamma}{1 - \tan\alpha\tan\gamma}$ and $\tan\alpha = \dfrac{e_2\cot\gamma}{e_1}$ or $\tan\alpha = \dfrac{e_2}{e_1\tan\gamma}$ Leads to $\tan(\alpha+\gamma) = \dfrac{e_2 + e_1\tan^2\gamma}{e_1\tan\gamma - e_2\tan\gamma}$ oe
M1	$180 > (\alpha+\gamma) > 90 \Rightarrow \tan(\alpha+\gamma) < 0 \Rightarrow \dfrac{e_2 + e_1\tan^2\gamma}{e_1\tan\gamma - e_2\tan\gamma} < 0$ Condone if ‘180 >’ is not stated again.
M1	Since numerator > 0 $e_1\tan\gamma - e_2\tan\gamma < 0$
A1	$e_2 > e_1$ *
(d)	
B1	Use the given information in (b) to make any equivalent statement with a correct reason and no incorrect statements. • $\alpha = 90^{\circ} - \gamma$, so it moves parallel to *AP* • $\alpha = 90^{\circ} - \gamma$, so it moves parallel to the initial velocity Do not accept ‘it moves parallel to the initial speed’.

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