Please check the examination details below before entering your candidate information

Candidate surname	Other names

Centre Number

Candidate Number

Pearson Edexcel Level 3 GCE

Time 1 hour 30 minutes

Paper reference **9FM0/3C**

Further Mathematics

Advanced
PAPER 3C: Further Mechanics 1

You must have:
Mathematical Formulae and Statistical Tables (Green), calculator

Total Marks

Candidates may use any calculator permitted by Pearson regulations. Calculators must not have the facility for symbolic algebraic manipulation, differentiation and integration, or have retrievable mathematical formulae stored in them.

Instructions
- Use **black** ink or ball-point pen.
- If pencil is used for diagrams/sketches/graphs it must be dark (HB or B).
- **Fill in the boxes** at the top of this page with your name, centre number and candidate number.
- Answer **all** questions and ensure that your answers to parts of questions are clearly labelled.
- Answer the questions in the spaces provided – *there may be more space than you need*.
- You should show sufficient working to make your methods clear. Answers without working may not gain full credit.
- Unless otherwise indicated, whenever a numerical value of g is required, take $g = 9.8\,\text{m}\,\text{s}^{-2}$ and give your answer to either 2 significant figures or 3 significant figures.

Information
- A booklet 'Mathematical Formulae and Statistical Tables' is provided.
- There are 8 questions in this question paper. The total mark for this paper is 75.
- The marks for **each** question are shown in brackets – *use this as a guide as to how much time to spend on each question.*

Advice
- Read each question carefully before you start to answer it.
- Try to answer every question.
- Check your answers if you have time at the end.

Turn over

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P 7 2 0 9 2 R A 0 1 3 2

Pearson

1. A particle A of mass $3m$ and a particle B of mass m are moving along the same straight line on a smooth horizontal surface. The particles are moving in opposite directions towards each other when they collide directly.

Immediately before the collision, the speed of A is ku and the speed of B is u. Immediately after the collision, the speed of A is v and the speed of B is $2v$.

The magnitude of the impulse received by B in the collision is $\frac{3}{2}mu$.

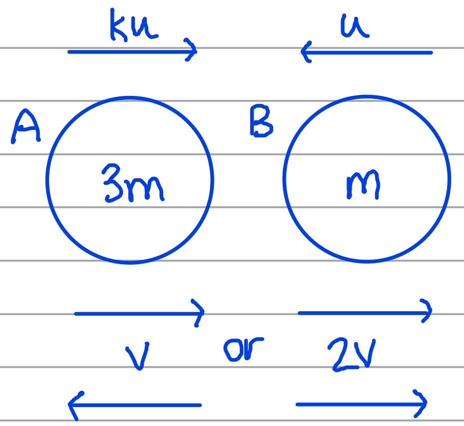
(a) Find v in terms of u only.

(3)

(b) Find the two possible values of k.

(5)

a)



A and B cannot be both moving left as this creates an impossible situation, with B passing through A.

Impulse received by B: $(\rightarrow +)$

$\frac{3}{2}mu = m(2v - (-u))$ (1) (1)

$\frac{3u}{2} = 2v + u$

$2v = \frac{1}{2}u$

$v = \frac{1}{4}u$ (1)

b) If A moves right: CLM $(\rightarrow +)$

(1)

$3kmu - mu = 3mv + 2mv$ (1)

$(3k-1)u = 5v = \frac{5u}{4}$

$3k - 1 = \frac{5}{4} \Rightarrow k = \frac{3}{4}$ (1)

Question 1 continued

If A moves left: CLM $(\rightarrow +)$

$3kmu - mu = -3mv + 2mv$ 1

$(3k-1)u = -v = -\frac{u}{4}$

$3k-1 = -\frac{1}{4}$

$k = \frac{1}{4}$ 1

Question 1 continued

Question 1 continued

(Total for Question 1 is 8 marks)

2.

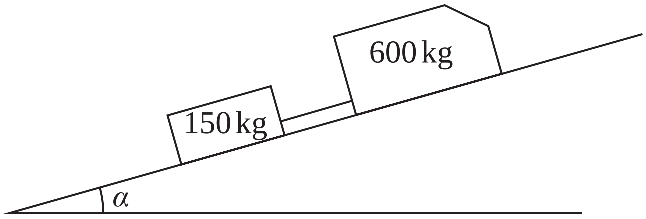


Figure 1

A van of mass 600 kg is moving up a straight road which is inclined at an angle α to the horizontal, where $\sin\alpha = \frac{1}{15}$. The van is towing a trailer of mass 150 kg. The van is attached to the trailer by a towbar which is parallel to the direction of motion of the van and the trailer, as shown in Figure 1.

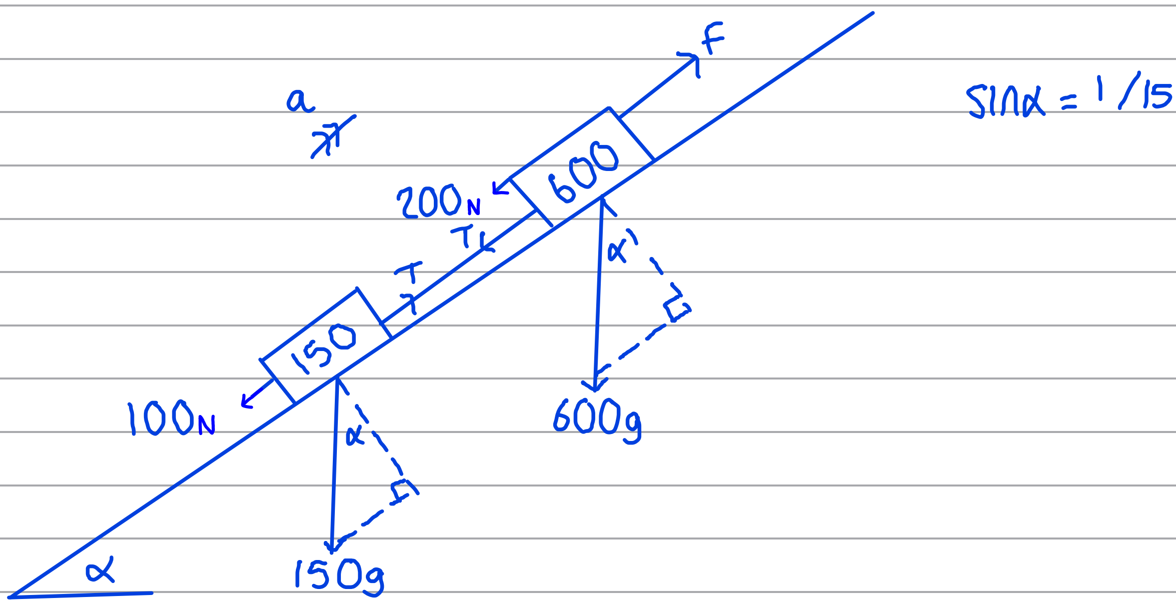
The resistance to the motion of the van from non-gravitational forces is modelled as a constant force of magnitude 200 N.
The resistance to the motion of the trailer from non-gravitational forces is modelled as a constant force of magnitude 100 N.

The towbar is modelled as a light rod.

The engine of the van is working at a constant rate of 12 kW.

Find the tension in the towbar at the instant when the speed of the van is 9 m s^{-1}

(8)



Resolve ↗ for whole system:

$F - (100+200) - (150+600)g\sin\alpha = (150+600)a$ (1) (1) (1)

Tension cancels out ↗ $\quad F - 790 = 750a$

P 7 2 0 9 2 R A 0 6 3 2

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Question 2 continued

using $P = Fv$ where $P = 12{,}000$ and $v = 9$:

$$12000 = 9F \Rightarrow F = \frac{12{,}000}{9} \quad (1)$$

$$\frac{12{,}000}{9} - 790 = 750a$$

$$a = \frac{163}{225} = 0.72\,\text{ms}^{-2} \text{ (2sf)}$$

Resolve ↗ for trailer:

(1)

$$T - 100 - 150g\sin\alpha = 150 \times \frac{163}{225} \quad (1)$$

$$T = \frac{920}{3} \quad (1)$$

$$T = 307\,\text{N (2sf)} \quad (1)$$

Question 2 continued

Question 2 continued

(Total for Question 2 is 8 marks)

3.

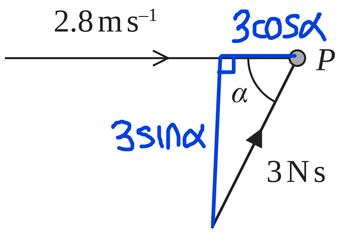


Figure 2

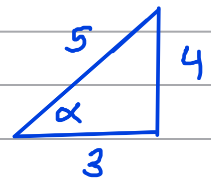
A particle P of mass 0.5 kg is moving in a straight line with speed $2.8\,\mathrm{m\,s^{-1}}$ when it receives an impulse of magnitude 3 N s.
The angle between the direction of motion of P immediately before receiving the impulse and the line of action of the impulse is α, where $\tan\alpha = \frac{4}{3}$, as shown in Figure 2.

Find the speed of P immediately after receiving the impulse.

(5)

convert to vector format: $\underline{u} = \begin{pmatrix} 2.8 \\ 0 \end{pmatrix}$

$\tan\alpha = 4/3$



$\sin\alpha = 4/5$

$\cos\alpha = 3/5$

$\underline{I} = \begin{pmatrix} 3\cos\alpha \\ 3\sin\alpha \end{pmatrix} = \begin{pmatrix} 9/5 \\ 12/5 \end{pmatrix}$

$\underline{I} = m(\underline{v} - \underline{u})$

$\begin{pmatrix} 9/5 \\ 12/5 \end{pmatrix} = 0.5\left(\underline{v} - \begin{pmatrix} 2.8 \\ 0 \end{pmatrix}\right)$ 1 1

$\underline{v} = 2\begin{pmatrix} 9/5 \\ 12/5 \end{pmatrix} + \begin{pmatrix} 2.8 \\ 0 \end{pmatrix}$ 1

$v = \begin{pmatrix} 32/5 \\ 24/5 \end{pmatrix}$

1

speed $= |\underline{v}| = \sqrt{\left(\frac{32}{5}\right)^2 + \left(\frac{24}{5}\right)^2} = 8\,\mathrm{ms^{-1}}$ 1

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Question 3 continued

(Total for Question 3 is 5 marks)

4.

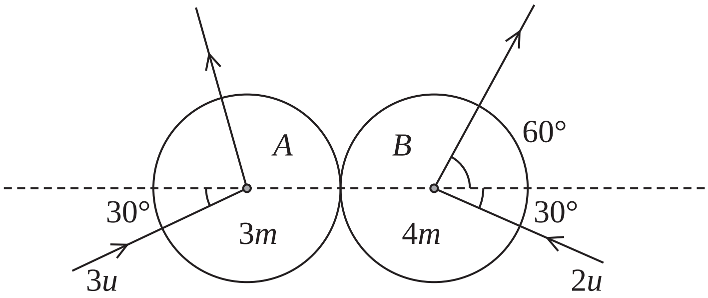


Figure 3

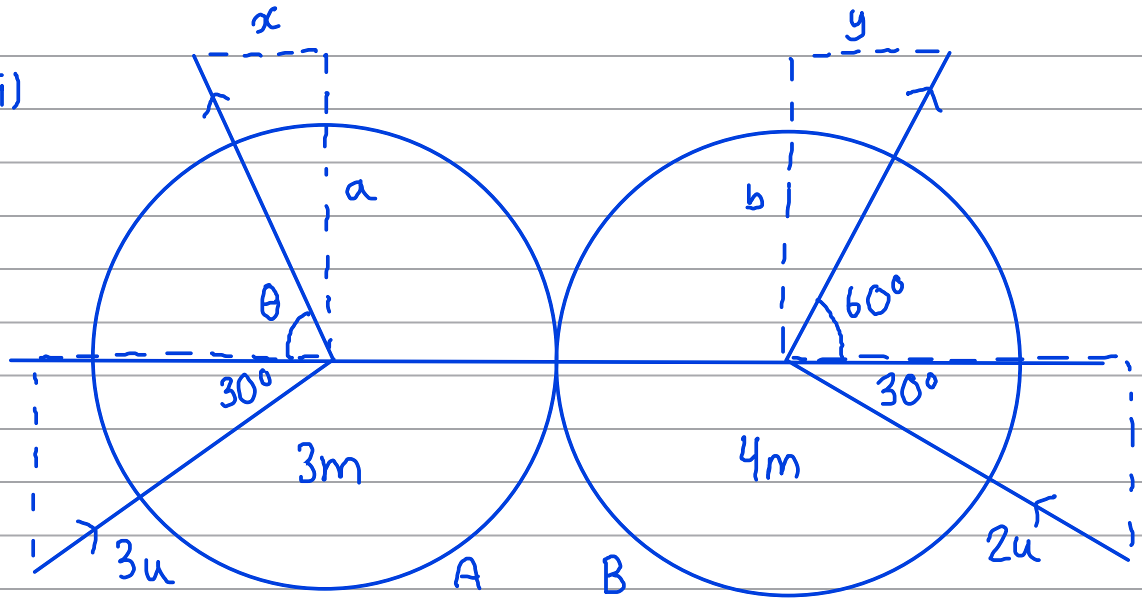
Two smooth uniform spheres, A and B, have equal radii. The mass of A is $3m$ and the mass of B is $4m$. The spheres are moving on a smooth horizontal plane when they collide obliquely. Immediately before they collide, A is moving with speed $3u$ at 30° to the line of centres of the spheres and B is moving with speed $2u$ at 30° to the line of centres of the spheres. The direction of motion of B is turned through an angle of 90° by the collision, as shown in Figure 3.

(i) Find the size of the angle through which the direction of motion of A is turned as a result of the collision.

(ii) Find, in terms of m and u, the magnitude of the impulse received by B in the collision.

(9)

(i)



$\perp$ to LOC:

$a = 3u\sin 30$

$= \frac{3u}{2}$ (1)

$b = 2u\sin 30$

$= u$ (1)

// to LOC:

CLM ($\rightarrow +$):

$3m(3u\cos 30) - 4m(2u\cos 30) = -3mx + 4my$ (1) (1)

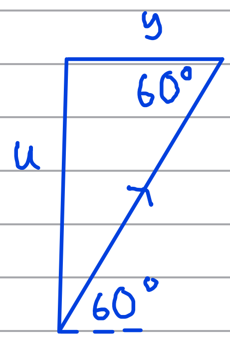
$-3x + 4y = \frac{\sqrt{3}}{2}u$ (1)

Question 4 continued

considering B:

$\tan 60 = \frac{u}{y}$

$\therefore y = \frac{u}{\tan 60} = \frac{u}{\sqrt{3}}$

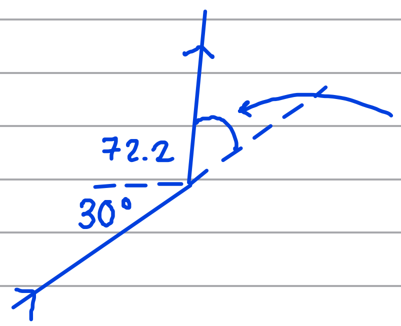


sub into ① : $-3x + \frac{4u}{\sqrt{3}} = \frac{\sqrt{3}}{2}u$

$x = \frac{5\sqrt{3}}{18}u$

$\tan\theta = \frac{a}{x} = \frac{3u/2}{5\sqrt{3}u/18} = \frac{9\sqrt{3}}{5}$

$\theta = 72.2°$ ①



angle deflected $= 180° - 30° - 72.2°$

$= 77.8°$ ①

(ii) Magnitude of impulse $(\rightarrow +)$ $= 4m\left(y - (-2u\cos 30)\right)$ ②

$= 4m\left(\frac{\sqrt{3}u}{3} + \sqrt{3}u\right)$

$= \frac{16\sqrt{3}}{3}mu$ ①

Question 4 continued

Question 4 continued

(Total for Question 4 is 9 marks)

5. Two particles, P and Q, are moving in opposite directions along the same straight line on a smooth horizontal surface when they collide directly.
The mass of P is $3m$ and the mass of Q is $4m$.
Immediately before the collision the speed of P is $2u$ and the speed of Q is u.
The coefficient of restitution between P and Q is e.

(a) Show that the speed of Q immediately after the collision is $\frac{u}{7}(9e+2)$

(6)

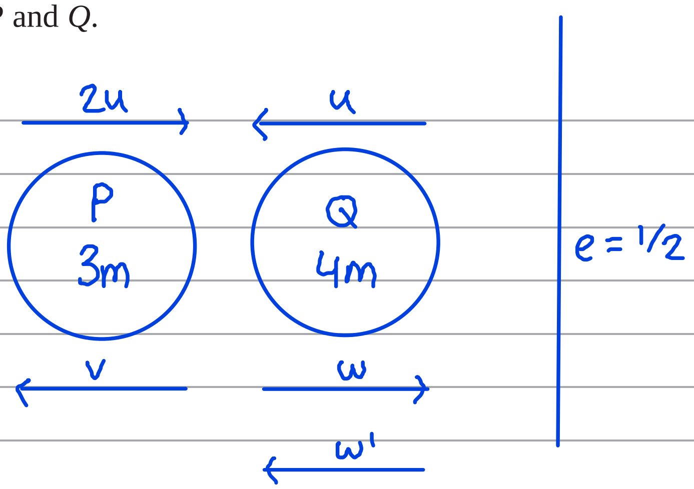
After the collision with P, particle Q collides directly with a fixed vertical wall and rebounds. The wall is perpendicular to the direction of motion of Q.

The coefficient of restitution between Q and the wall is $\frac{1}{2}$

(b) Find the complete range of possible values of e for which there is a second collision between P and Q.

(4)

a)



CLM: $(\rightarrow +)$ 1

$6mu - 4mu = -3mv + 4mw$ 1

$-3v + 4w = 2u$ (1)

NLR:

$e = \dfrac{v+w}{2u+u}$ 1

$3eu = v + w \Rightarrow 9eu = 3v + 3w$ (2) 1

(1) + (2): $-3v + 4w + 3v + 3w = 2u + 9eu$ 1

$7w = u(9e+2)$

$w = \dfrac{u}{7}(9e+2)$ 1

b) Method: find v, find w' and set $w' > v$

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Question 5 continued

$$V = 3eu - w$$
$$= 3eu - \frac{9eu}{7} - \frac{2u}{7}$$
$$= \frac{u}{7}(12e - 2) \quad ①$$

$$w' = ew$$
$$= \frac{1}{2}\left(\frac{u}{7}(9e+2)\right) = \frac{u}{14}(9e+2) \quad ①$$

$$w' > V \quad ①$$

$$\frac{u}{14}(9e+2) > \frac{u}{7}(12e-2)$$

$$(9e+2) > 2(12e-2)$$

$$6 > 15e$$

$$e < \frac{2}{5}$$

$$0 < e < \frac{2}{5} \quad ①$$

Question 5 continued

Question 5 continued

(Total for Question 5 is 10 marks)

6.

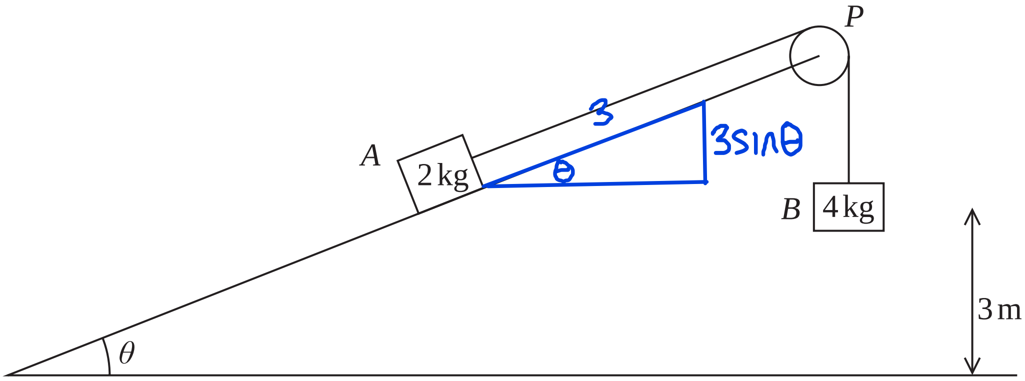


Figure 4

Two blocks, A and B, of masses 2 kg and 4 kg respectively are attached to the ends of a light inextensible string.

Initially A is held on a fixed rough plane. The plane is inclined to horizontal ground at an angle θ, where $\tan\theta = \frac{3}{4}$

The string passes over a small smooth light pulley P that is fixed at the top of the plane. The part of the string from A to P is parallel to a line of greatest slope of the plane.

Block A is held on the plane with the distance AP greater than 3 m.
Block B hangs freely below P at a distance of 3 m above the ground, as shown in Figure 4.

The coefficient of friction between A and the plane is μ

Block A is released from rest with the string taut.

By modelling the blocks as particles,

(a) find the potential energy lost by the whole system as a result of B falling 3 m.

(3)

Given that the speed of B at the instant it hits the ground is $4.5\,\text{m s}^{-1}$ and ignoring air resistance,

(b) use the work-energy principle to find the value of μ

(6)

After B hits the ground, A continues to move up the plane but does not reach the pulley in the subsequent motion.
Block A comes to instantaneous rest after moving a total distance of $(3 + d)$ m from its point of release.

Ignoring air resistance,

(c) use the work-energy principle to find the value of d

(4)

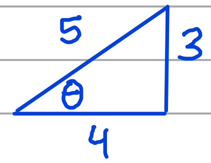
a) A and B have the same kinetic energy, so energy lost by system = GPE lost by B − GPE gained by A

Question 6 continued

$= 4 \times g \times 3 - 2 \times g \times 3\sin\theta$ (2)

$\tan\theta = 3/4$

$= 82.32$

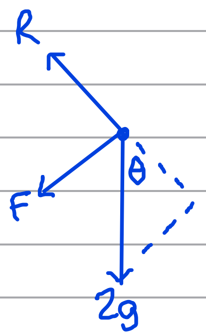


$= 82\text{J (2sf)}$ (1)

$\sin\theta = 3/5$
$\cos\theta = 4/5$

b) motion from A released to B hitting ground:

considering A:



Resolve $\nwarrow$: $R = 2g\cos\theta$

Work-energy equation:

A and B have the same speed

total KE gained $= \frac{1}{2} \times (2+4) \times 4.5^2$

$= 60.75\text{J}$ (1)

work done against friction: $\mu R d = \mu(2g\cos\theta)3 = 47.04\mu$ (1) (1)

GPE lost = KE gained + w.d. against friction (1)

$82.32 = 60.75 + 47.04\mu$ (1)

$\mu = 0.459...$

$= 0.46 \text{ (2dp)}$ (1)

Question 6 continued

c) motion from B hitting ground to A stopping:

KE before = GPE after + work done against friction

$\frac{1}{2} \times 2 \times 4.5^2 = 2g(d\sin\theta) + \mu(2g\cos\theta)d$ (1) (1) (1)

$20.25 = 11.76d + 7.19d$

$d = \frac{405}{379}$

$= 1.1\text{m}$ (2sf) (1)

Question 6 continued

(Total for Question 6 is 13 marks)

7. A spring of natural length a has one end attached to a fixed point A. The other end of the spring is attached to a package P of mass m.
The package P is held at rest at the point B, which is vertically below A such that $AB = 3a$.
After being released from rest at B, the package P first comes to instantaneous rest at A.
Air resistance is modelled as being negligible.

By modelling the spring as being light and modelling P as a particle,

(a) show that the modulus of elasticity of the spring is $2mg$

(5)

(b) (i) Show that P attains its maximum speed when the extension of the spring is $\frac{1}{2}a$

(ii) Use the principle of conservation of mechanical energy to find the maximum speed, giving your answer in terms of a and g.

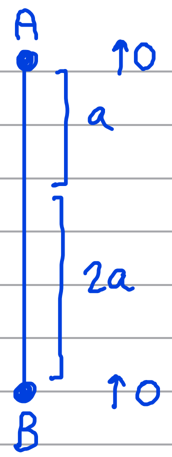
(6)

In reality, the spring is not light.

(c) State one way in which this would affect your energy equation in part (b).

(1)

a) spring:
$l = a$
$\lambda = ?$



EPE at B: $\dfrac{\lambda x^2}{2l} = \dfrac{\lambda(2a)^2}{2a} = 2\lambda a$ (1)

spring has been compressed by length a

EPE at A: $\dfrac{\lambda x^2}{2l} = \dfrac{\lambda(a)^2}{2a} = \dfrac{1}{2}\lambda a$

v=0 at both points so no need to consider KE

EPE at B + GPE at B = EPE at A + GPE at A

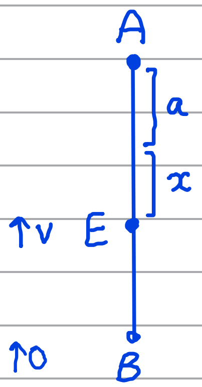
$$2\lambda a + 0 = \frac{1}{2}\lambda a + mg(3a)$$ (1) (1) (1)

$$\frac{3}{2}\lambda a = 3mga$$

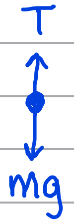
$$\lambda = 2mg$$ (1)

Question 7 continued

b) (i) max speed when $a=0$, i.e. at equilibrium point, E



Force diagram:



Resolve ↑, forces balanced

$T = mg$ (1)

using $T = \frac{\lambda x}{l} = \frac{2mgx}{a}$

$\frac{2mgx}{a} = mg$

$2x = a$

$x = \frac{a}{2}$ as required (1)

(ii)

KE before + GPE before + EPE before = KE after + GPE after + EPE after

$0 + 0 + \frac{2mg(2a)^2}{2a} = \frac{1}{2}mv^2 + mg(2a - x) + \frac{2mgx^2}{2a}$ (1) (1)

$4ag = \frac{1}{2}v^2 + \frac{3}{2}ag + \frac{1}{4}ag$ (1)

$\frac{1}{2}v^2 = \frac{9}{4}ag$

$v = \sqrt{\frac{9ag}{2}}$ (1)

c) The string's KE would need to be considered (1)

Question 7 continued

Question 7 continued

(Total for Question 7 is 12 marks)

8.

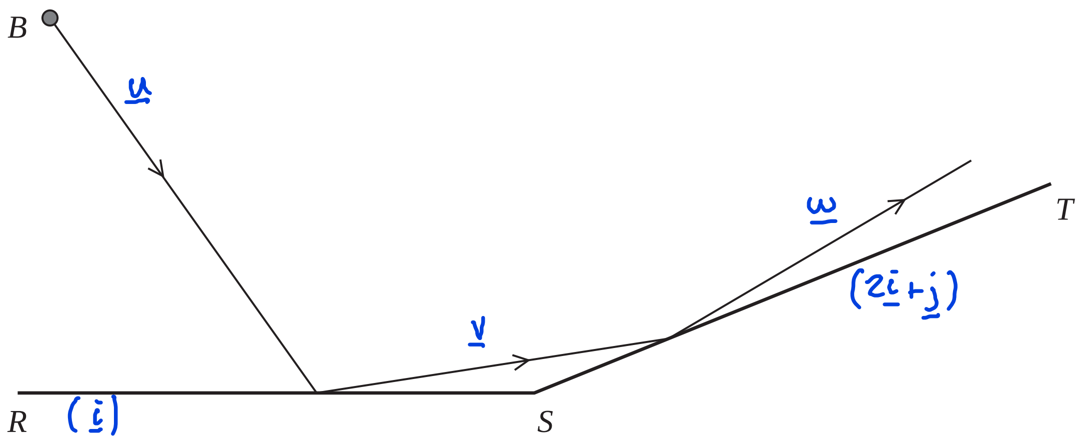


Figure 5

Figure 5 represents the plan view of part of a smooth horizontal floor, where RS and ST are smooth fixed vertical walls. The vector $\overrightarrow{RS}$ is in the direction of $\mathbf{i}$ and the vector $\overrightarrow{ST}$ is in the direction of $(2\mathbf{i} + \mathbf{j})$.

A small ball B is projected across the floor towards RS. Immediately before the impact with RS, the velocity of B is $(6\mathbf{i} - 8\mathbf{j})\,\text{m s}^{-1}$. The ball bounces off RS and then hits ST.

The ball is modelled as a particle.

Given that the coefficient of restitution between B and RS is e,

(a) find the full range of possible values of e.

(3)

It is now given that $e = \frac{1}{4}$ and that the coefficient of restitution between B and ST is $\frac{1}{2}$

(b) Find, in terms of $\mathbf{i}$ and $\mathbf{j}$, the velocity of B immediately after its impact with ST.

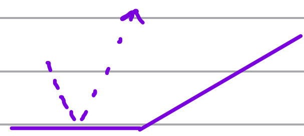
(7)

a) $\underline{u} = 6\underline{i} - 8\underline{j}$ so $\underline{v} = 6\underline{i} + 8e\underline{j}$ 1 1

if $\underline{v}$ is steeper than $(2\underline{i}+\underline{j})$, there will be no second collision

$\therefore \quad \frac{8e}{6} < \frac{1}{2} \Rightarrow e < \frac{3}{8}$

e.g.



$0 < e < \frac{3}{8}$ 1

b) $e = \frac{1}{4}$ so $\underline{v} = 6\underline{i} + 2\underline{j}$

Question 8 continued

using $\underline{u} \cdot \underline{W} = \underline{v} \cdot \underline{W}$, where $\underline{W}$ is the direction of the wall

$$\begin{pmatrix} 6 \\ 2 \end{pmatrix} \cdot \begin{pmatrix} 2 \\ 1 \end{pmatrix} = \begin{pmatrix} w_x \\ w_y \end{pmatrix} \cdot \begin{pmatrix} 2 \\ 1 \end{pmatrix} \quad 1$$

$$2w_x + w_y = 12 + 2 \quad (1) \quad 1$$

using $-e\underline{u} \cdot \underline{I} = \underline{v} \cdot \underline{I}$, where $\underline{I}$ is the direction of the impulse

$\underline{I}$ is perpendicular to $\underline{W}$: $\begin{pmatrix} 2 \\ 1 \end{pmatrix} \cdot \begin{pmatrix} I_x \\ I_y \end{pmatrix} = 0$

so let $I_x = -1$ and $I_y = 2$ 1

$$-\frac{1}{2}\begin{pmatrix} 6 \\ 2 \end{pmatrix} \cdot \begin{pmatrix} -1 \\ 2 \end{pmatrix} = \begin{pmatrix} w_x \\ w_y \end{pmatrix} \cdot \begin{pmatrix} -1 \\ 2 \end{pmatrix} \quad 1$$

$$-0.5(-6+4) = -w_x + 2w_y$$

$$-w_x + 2w_y = 1 \quad (2) \quad 1$$

solve (1) and (2) simultaneously: $w_x = 5.4 \quad w_y = 3.2$ 1

$$\underline{w} = 5.4\underline{i} + 3.2\underline{j} \quad 1$$

for vectors collision questions, $\underline{u} \cdot \underline{W} = \underline{v} \cdot \underline{W}$ and $-e\underline{u} \cdot \underline{I} = \underline{v} \cdot \underline{I}$ are the most useful identities to learn.

The magnitudes of W and I are not important as the extra scaling cancels out in the dot product.

Question 8 continued

Question 8 continued

Question 8 continued

(Total for Question 8 is 10 marks)

TOTAL FOR PAPER IS 75 MARKS