MODEL ANSWERS

Please check the examination details below before entering your candidate information

Candidate surname	Other names

Pearson Edexcel Level 3 GCE

Centre Number | Candidate Number

Thursday 14 May 2020

Afternoon | Paper Reference **8FM0/25**

Further Mathematics
Advanced Subsidiary
Further Mathematics options
25: Further Mechanics 1
(Part of options C, E, H and J)

You must have:
Mathematical Formulae and Statistical Tables (Green), calculator

Total Marks

Candidates may use any calculator allowed by Pearson regulations. Calculators must not have the facility for symbolic algebra manipulation, differentiation and integration, or have retrievable mathematical formulae stored in them.

Instructions

- Use **black** ink or ball-point pen.
- If pencil is used for diagrams/sketches/graphs it must be dark (HB or B).
- **Fill in the boxes** at the top of this page with your name, centre number and candidate number.
- Answer **all** questions and ensure that your answers to parts of questions are clearly labelled.
- Answer the questions in the spaces provided – *there may be more space than you need.*
- You should show sufficient working to make your methods clear. Answers without working may not gain full credit.
- Unless otherwise indicated, whenever a value of g is required, take $g = 9.8\,\text{m s}^{-2}$ and give your answer to either 2 significant figures or 3 significant figures.

Information

- A booklet 'Mathematical Formulae and Statistical Tables' is provided.
- The total mark for this part of the examination is 40. There are 4 questions.
- The marks for **each** question are shown in brackets – *use this as a guide as to how much time to spend on each question.*

Advice

- Read each question carefully before you start to answer it.
- Try to answer every question.
- Check your answers if you have time at the end.

Turn over

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Pearson

1. Two particles P and Q have masses m and $4m$ respectively. The particles are at rest on a smooth horizontal plane. Particle P is given a horizontal impulse, of magnitude I, in the direction PQ. Particle P then collides directly with Q. Immediately after this collision, P is at rest and Q has speed w. The coefficient of restitution between the particles is e.

(a) Find I in terms of m and w.

(2)

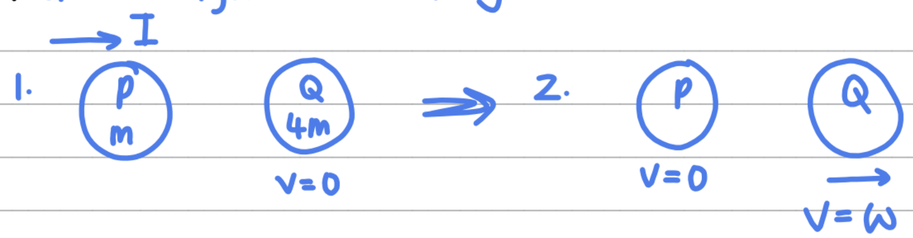
(b) Show that $e = \frac{1}{4}$

(1)

(c) Find, in terms of m and w, the total kinetic energy lost in the collision between P and Q.

(2)

a) draw diagram & identify directions



Impulse = Ft = Δp → $I = mv_1$ (∵ particles were at rest)

$p_1 = I = p_2 = 4mw$

conservation of linear momentum: $m \times \frac{I}{m} = 4mw$

$\Rightarrow I = 4mw$

b) $e = \frac{|V_{P2} - V_{Q2}|}{|V_{P1} - V_{Q1}|} = \frac{|-w|}{|(\frac{I}{m})|} = \frac{w}{4w} = \frac{1}{4}$

c) KE loss $= \Delta KE = \frac{1}{2}m(4w)^2 - \frac{1}{2}(4m)w^2$

$= \frac{16mw^2}{2} - \frac{4mw^2}{2}$

$= 6mw^2$

Question 1 continued

(Total for Question 1 is 5 marks)

P 6 1 8 6 8 A 0 3 1 6

2. A car of mass 1000 kg moves along a straight horizontal road.

In all circumstances, when the speed of the car is v m s^{-1}, the resistance to the motion of the car is modelled as a force of magnitude cv^2 N, where c is a constant.

The maximum power that can be developed by the engine of the car is 50 kW.

At the instant when the speed of the car is 72 km h^{-1} and the engine is working at its maximum power, the acceleration of the car is 2.25 m s^{-2}

(a) Convert 72 km h^{-1} into m s^{-1}

(1)

(b) Find the acceleration of the car at the instant when the speed of the car is 144 km h^{-1} and the engine is working at its maximum power.

(7)

The maximum speed of the car when the engine is working at its maximum power is V km h^{-1}.

(c) Find, to the nearest whole number, the value of V.

(4)

a) $72\text{kmh}^{-1} \times \frac{1000}{60\times60} = 20\text{ms}^{-1}$

b)

Power = force x velocity

$\frac{P}{v} - cv^2 = F_{net} = ma = 1000a$

we know $P = 50\,000$W, & that when $v = 72$ kmh^{-1}, $a = 2{\cdot}25$ms^{-2}

$\Rightarrow \frac{50000}{20} - c\times20^2 = 1000\times2{\cdot}25$

72 kmh^{-1} = 20 ms^{-1}

$\rightarrow 2500 - 2250 = 400c$

$\therefore c = \frac{250}{400} = \frac{5}{8}$

now $v = 144$ kmh^{-1}! this is 2×72kmh^{-1} so know 144 kmh^{-1} $= 2\times20$ms^{-1} $= 40$ms^{-1}

$\Rightarrow \frac{50000}{40} - \frac{5}{8}\times40^2 = 1000a$

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Question 2 continued

$\therefore 250 = 1000a$

$a = 0.25 ms^{-2}$

c) max. velocity $\Rightarrow a = 0$ $(\frac{dv}{dt} = a = 0)$

let W = max. v:

$\frac{50000}{W} - \frac{5}{8}W^2 = 0$

$\times W: 50000 - \frac{5}{8}W^3 = 0$

$\times \frac{8}{5}: 80000 = W^3$

$W = 43.089 ms^{-1}$

$43.089 \times \frac{3600}{1000} = 155 kmh^{-1}$ (3s.f.)

Question 2 continued

Question 2 continued

(Total for Question 2 is 12 marks)

3. Three particles A, B and C are at rest on a smooth horizontal plane. The particles lie along a straight line with B between A and C.

Particle B has mass $4m$ and particle C has mass km, where k is a positive constant. Particle B is projected with speed u along the plane towards C and they collide directly.

The coefficient of restitution between B and C is $\frac{1}{4}$

(a) Find the range of values of k for which there would be no further collisions.

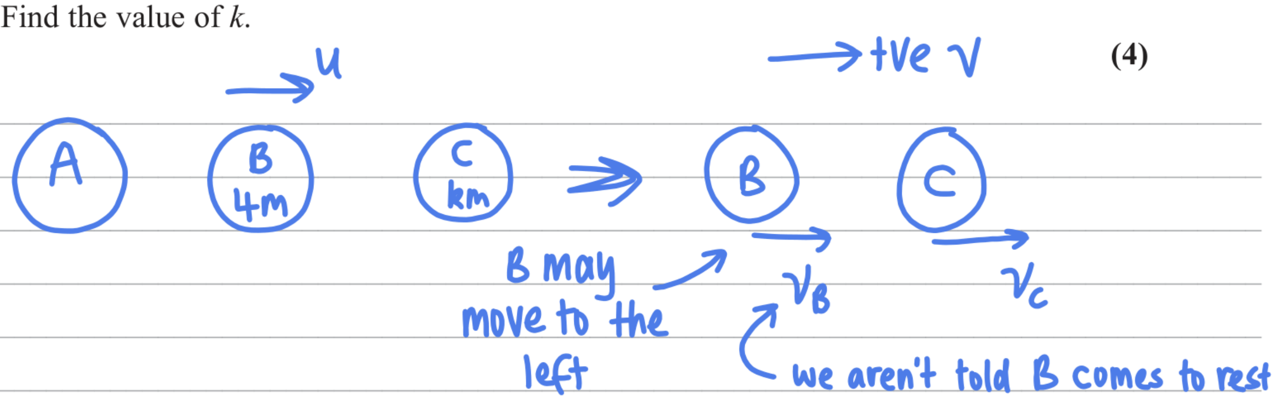
(8)

The magnitude of the impulse on B in the collision between B and C is $3mu$

(b) Find the value of k.

(4)

a)



conservation of linear momentum:

$4mu = 4mv_B + kmv_c$

Newton's Law of Restitution:

$e = \frac{|v_B - v_c|}{|u_B - u_c|} \Rightarrow \frac{1}{4} = \frac{v_c - v_B}{u} \rightarrow \frac{1}{4}u = -v_B + v_c$

solve for v_B: $4u = 4v_B + kv_c$

$4u = 4v_B + k(\frac{1}{4}u + v_B)$

$16u = 16v_B + ku + 4kv_B$

$u(16-k) = v_B(16+4k)$

$v_B = \frac{u(16-k)}{16+4k} = \frac{u(16-k)}{4(k+4)}$

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Question 3 continued

so that B doesn't collide with A, we need it to move to the right, i.e. $v_B \geq 0$

$$0 \leq \frac{u(16-k)}{4(k+4)}$$

$$0 \leq u(16-k)$$

$$ku \leq 16u$$

$$k \leq 16$$

& we are told $k > 0$ $\therefore$ $0 < k \leq 16$

b) impulse = Δ momentum = Δmv

impulse on B $\Rightarrow$ in -ve direction

$\therefore -3mu = 4m(v_B - u)$ ← Δp_B

$$-3mu = 4mv_B - 4mu$$

$$u = 4v_B \Rightarrow \frac{1}{4}u = -\frac{1}{4}u + v_c \Rightarrow \frac{1}{2}u = v_c$$

impulse on C: $3mu = kmv_c$

$$3(2v_c) = kv_c$$

$$\therefore k = 6$$

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Turn over

Question 3 continued

Question 3 continued

(Total for Question 3 is 12 marks)

4. A small ball, of mass m, is thrown vertically upwards with speed $\sqrt{8gH}$ from a point O on a smooth horizontal floor. The ball moves towards a smooth horizontal ceiling that is a vertical distance H above O. The coefficient of restitution between the ball and the ceiling is $\frac{1}{2}$

In a model of the motion of the ball, it is assumed that the ball, as it moves up or down, is subject to air resistance of constant magnitude $\frac{1}{2}mg$.

Using this model,

(a) use the work-energy principle to find, in terms of g and H, the speed of the ball immediately before it strikes the ceiling,

(5)

(b) find, in terms of g and H, the speed of the ball immediately before it strikes the floor at O for the first time.

(5)

In a simplified model of the motion of the ball, it is assumed that the ball, as it moves up or down, is subject to no air resistance.

Using this simplified model,

(c) explain, without any detailed calculation, why the speed of the ball, immediately before it strikes the floor at O for the first time, would still be less than $\sqrt{8gH}$

(1)

a)

work = Force × distance

So work done against air resistance = $\frac{1}{2}mgH$

speed @ $x=0$ is $\sqrt{8gH}$, let speed @ ceiling be v, so change in KE = $\frac{1}{2}m(8gH - v^2)$

work-energy principal: Work = ΔE

$$\frac{1}{2}mgH = \frac{1}{2}m(8gH - v^2) - mgH$$

$$\frac{1}{2}gH = 4gH - \frac{1}{2}v^2 - gH$$

$$\frac{1}{2}v^2 = \frac{5}{2}gH$$

$$\rightarrow v = \sqrt{5gH}$$

P 6 1 8 6 8 A 0 1 2 1 6

Question 4 continued

b) Newton's Law of Restitution: $e = \frac{|v_2 - v_1|}{|u_2 - u_1|} = \frac{V_{ball}}{\sqrt{5gH}} = \frac{1}{2}$

↑ speed before rebound

$\Rightarrow V_{ball} = \frac{1}{2}\sqrt{5gH}$

work-energy principle:

$\frac{1}{2}mgH = mgH - \frac{1}{2}m\left(v^2 - \left(\frac{\sqrt{5gH}}{2}\right)^2\right)$

force now in direction of v

$gH = v_1^2 - \frac{5}{4}gH$

$\frac{9}{4}gH = v_1^2$

$v_1 = \frac{3}{2}\sqrt{gH}$

SUVAT:

$a = g - \frac{1}{2}g = \frac{1}{2}g$

$v^2 = u^2 + 2as$

$= \frac{5gH}{4} + gH$

$v^2 = \frac{9}{4}gH$

$v = \frac{3}{2}\sqrt{gH}$

c) the coefficient of restitution, e, is < 1, so the ball loses energy in the collision & so cannot regain its former speed

P 6 1 8 6 8 A 0 1 3 1 6

Question 4 continued

Question 4 continued

Question 4 continued

(Total for Question 4 is 11 marks)

TOTAL FOR FURTHER MECHANICS 1 IS 40 MARKS