Mark Scheme (Result)
October 2020

Pearson Edexcel GCE in A Level Further Mathematics
Paper 9FM0/4A

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General Marking Guidance

- All candidates must receive the same treatment. Examiners must mark the first candidate in exactly the same way as they mark the last.
- Mark schemes should be applied positively. Candidates must be rewarded for what they have shown they can do rather than penalised for omissions.
- Examiners should mark according to the mark scheme not according to their perception of where the grade boundaries may lie.
- There is no ceiling on achievement. All marks on the mark scheme should be used appropriately.
- All the marks on the mark scheme are designed to be awarded. Examiners should always award full marks if deserved, i.e. if the answer matches the mark scheme. Examiners should also be prepared to award zero marks if the candidate's response is not worthy of credit according to the mark scheme.
- Where some judgement is required, mark schemes will provide the principles by which marks will be awarded and exemplification may be limited.
- When examiners are in doubt regarding the application of the mark scheme to a candidate's response, the team leader must be consulted.
- Crossed out work should be marked UNLESS the candidate has replaced it with an alternative response.

General Instructions for Marking

1. The total number of marks for the paper is 75.

2. The Edexcel Mathematics mark schemes use the following types of marks:

- M marks: method marks are awarded for 'knowing a method and attempting to apply it', unless otherwise indicated.
- A marks: Accuracy marks can only be awarded if the relevant method (M) marks have been earned.
- B marks are unconditional accuracy marks (independent of M marks)
- Marks should not be subdivided.

3. Abbreviations

These are some of the traditional marking abbreviations that will appear in the mark schemes.

- bod – benefit of doubt
- ft – follow through
- the symbol $\sqrt{}$ will be used for correct ft
- cao – correct answer only
- cso - correct solution only. There must be no errors in this part of the question to obtain this mark
- isw – ignore subsequent working
- awrt – answers which round to
- SC: special case
- oe – or equivalent (and appropriate)
- dep – dependent
- indep – independent
- dp decimal places
- sf significant figures
- ∗ The answer is printed on the paper
- ⌐ The second mark is dependent on gaining the first mark

4. All A marks are 'correct answer only' (cao.), unless shown, for example, as A1 ft to indicate that previous wrong working is to be followed through. After a misread however, the subsequent A marks affected are treated as A ft, but manifestly absurd answers should never be awarded A marks.

Question	Scheme	Marks	AOs
1(i)	${}^{26}C_8 = 1562275$	**B1**	1.1b
		(1)	
(ii)	Attempts ${}^{12}C_4 \times {}^{14}C_4$	**M1**	1.1b
	$= 495 \times 1001 = 495495$	**A1**	1.1b
		(2)	
(iii)	Attempts cases for 5, 6, 7 or 8 adults on team (only).	**M1**	3.1a
	${}^{12}C_5 \times {}^{14}C_3 + {}^{12}C_6 \times {}^{14}C_2 + {}^{12}C_7 \times {}^{14}C_1 + {}^{12}C_8 \times {}^{14}C_0 = \ldots$	**M1**	1.1b
	= 383955	**A1**	1.1b
		(3)	
		(6 marks)	

Notes:

(i)
B1: Correct answer 1562275 (no need to see calculation)
(ii)
M1: Attempts the product shown.
A1: 495495
(iii)
M1: Works out the different cases that give more than half adults – e.g. attempts to find combinations for 5, 6, 7 or 8 adults (or 0, 1, 2 or 3 juniors). Need not be the correct formula, look for evidence of the correct cases considered. Do not allow this mark if the case of 4 adults and 4 juniors is included.
M1: Sums the possibilities. Allow if the 4 adults/4 juniors case is included, but the binomial products must be correct but e.g. ${}^{14}C_0$ may not be seen (as it is 1). Must be an attempt at using the correct formulae.
A1: cao 383955

Question	Scheme	Marks	AOs
2	Auxiliary equation is $9r^2-12r+4=0$, so $r=\ldots$	M1	1.1b
	$(3r-2)^2=0 \Rightarrow r=\dfrac{2}{3}$ is repeated root.	A1	1.1b
	Complementary function is $x_n=(A+Bn)\left(\dfrac{2}{3}\right)^n$ or $A\left(\dfrac{2}{3}\right)^n+Bn\left(\dfrac{2}{3}\right)^n$	M1	2.2a
	Try particular solution $y_n=an+b \Rightarrow 9(a(n+2)+b)-12(a(n+1)+b)+4(an+b)=3n$	M1	2.1
	$\Rightarrow an+6a+b=3n \Rightarrow a=\ldots, b=\ldots$	dM1	1.1b
	$a=3, b=-18$	A1	1.1b
	General solution is $u_n=x_n+y_n=(A+Bn)\left(\dfrac{2}{3}\right)^n+3n-18$	B1ft	2.2a
	$\left.\begin{array}{l} u_1=1 \Rightarrow 1=\left(\dfrac{2}{3}\right)(A+B)-15 \\ u_2=4 \Rightarrow 4=\left(\dfrac{4}{9}\right)(A+2B)-12 \end{array}\right\} A=\ldots, B=\ldots$	M1	2.1
	$u_n=12(n+1)\left(\dfrac{2}{3}\right)^n+3n-18$ oe	A1	1.1b
		(9)	
		(9 marks)	

Notes:

M1: Forms and solves the auxiliary equation.
A1: Correct (repeated) root found.
M1: Forms the correct complementary function for their (real) root(s) to the equation, $(A+Bn)r^n$ if repeated root, or allow $Ar_1^n+Br_2^n$ if distinct real roots are found.
M1: Attempts to use a particular solution of the correct form (ie $an+b$ or a higher order polynomial in n containing this) in the recurrence relation.
dM1: Expands and solves for a and b
A1: Correct values for a and b
B1ft: Forms the general solution as the sum of their complementary function and a particular solution of correct form with their a and b
M1: Applies the initial values and solves for the constants
A1: Correct answer.

Question	Scheme	Marks	AOs
3(a)	Sight of $\det(\mathbf{M}-\lambda\mathbf{I})=0$	B1	1.1a
	$\begin{vmatrix} 1-\lambda & k & -2 \\ 2 & -4-\lambda & 1 \\ 1 & 2 & 3-\lambda \end{vmatrix}=0\Rightarrow$ $(1-\lambda)\left[(-4-\lambda)(3-\lambda)-2\right]-k\left[2(3-\lambda)-1\right]+(-2)\left[4-(-4-\lambda)\right]=0$	M1	1.1b
	$\Rightarrow(1-\lambda)(\lambda^2+\lambda-14)-k(5-2\lambda)-16-2\lambda=0$ $\Rightarrow\lambda^3-(2k+13)\lambda+5(k+6)=0*$	A1*	2.1
		(3)	
(b)	(i) $\pm5(k+6)=5\Rightarrow k=...$ or $(-12-2)-k(6-1)-2(4+4)=5\Rightarrow k=...$	M1	1.1b
	$k=-7$	A1	2.2a
	(ii) Hence by the C-H theorem $\mathbf{M}^3+\mathbf{M}-5\mathbf{I}=\mathbf{0}$	M1	2.1
	Multiplying by $\mathbf{M}^{-1}$ gives $\mathbf{M}^2+\mathbf{I}-5\mathbf{M}^{-1}=\mathbf{0}\Rightarrow\mathbf{M}^{-1}=...$	M1	3.1a
	So $\mathbf{M}^{-1}=\frac{1}{5}\left(\mathbf{M}^2+\mathbf{I}\right)$	A1	1.1b
	$=\frac{1}{5}\left(\begin{pmatrix} -15 & 17 & -15 \\ -5 & 4 & -5 \\ 8 & -9 & 9 \end{pmatrix}+\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}\right)=...$	M1	1.1b
	$=\frac{1}{5}\begin{pmatrix} -14 & 17 & -15 \\ -5 & 5 & -5 \\ 8 & -9 & 10 \end{pmatrix}$ or $\begin{pmatrix} -\frac{14}{5} & \frac{17}{5} & -3 \\ -1 & 1 & -1 \\ \frac{8}{5} & -\frac{9}{5} & 2 \end{pmatrix}$	A1	1.1b
		(7)	
			(10 marks)

Notes:
(a) **B1:** Recalls characteristic equation is found using $\det(\mathbf{M}-\lambda\mathbf{I})=0$ **M1:** Attempts to expand the determinant **A1*:** Achieves the correct equation with no errors and at least one intermediate step following the expansion. **(b)(i)** **M1:** Attempts to use determinant equals 5 to find *k*. May be attempted by finding determinant from original matrix, or attempt at using the $"-5(k+6)"$ from the expansion in (a) (allow $\pm$ for the method mark). **A1:** $k=-7$ **(ii)** **M1:** Attempts to use the Cayley-Hamilton theorem to set up a matrix equation. The equation should be correct for their *k*, including correct use of **I**. **M1:** Realises the need to multiply the equation through (either side) by $\mathbf{M}^{-1}$ and rearrange to make $\mathbf{M}^{-1}$ the subject. **A1:** $\mathbf{M}^{-1}=\frac{1}{5}\left(\mathbf{M}^2+\mathbf{I}\right)$ **M1:** Proceeds to find $\mathbf{M}^{-1}$ from their equation. **A1:** Correct answer.

Question	Scheme	Marks	AOs
4	Complete overall strategy evidenced – requires finding the area of the two sides, and the area of the curved surface via attempt at the arc length first.	**M1**	3.1a
	Area of each side is $\int \frac{1}{2}r^2\,\mathrm{d}\theta = 450\int_0^1 (1-\theta^2)^2\,\mathrm{d}\theta$	**B1**	1.1b
	$= 450\int_0^1 1-2\theta^2+\theta^4\,\mathrm{d}\theta = 450\left[\theta-\frac{2}{3}\theta^3+\frac{1}{5}\theta^5\right]_0^1$	**M1**	1.1b
	$= 450\left(1-\frac{2}{3}+\frac{1}{5}\right) = 240$ (cm^2)	**A1**	3.4
	$r^2+\left(\frac{\mathrm{d}r}{\mathrm{d}\theta}\right)^2 = 900\left(1-2\theta^2+\theta^4\right)+\left(30\times-2\theta\right)^2$	**M1**	1.1b
	$= 900\left(1+\theta^2\right)^2$	**A1**	2.2a
	Length of curve is $\int_0^1 \sqrt{r^2+\left(\frac{\mathrm{d}r}{\mathrm{d}\theta}\right)^2}\,\mathrm{d}\theta = 30\int_0^1 1+\theta^2\,\mathrm{d}\theta = 30\left[\theta+\frac{1}{3}\theta^3\right]_0^1$	**M1**	2.1
	$= 30\left(1+\frac{1}{3}-(0)\right) = 40$ (cm)	**A1**	3.4
	Surface area required is $2\times"240"+\underline{235\times"40"}=\ldots$	**M1**	1.1b
	$= 9880$ cm^2	**A1**	3.2a
		(10)	
			(10 marks)

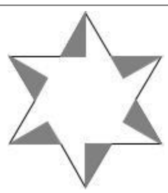
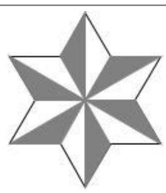
Notes:
M1: Shows a complete strategy for finding the required surface area – must include both sides, and attempt at area of curved surface using arc length using the correct formula. **B1:** Uses polar area formula for at least one of the two sides. May use $2\times\int \frac{1}{2}r^2\,\mathrm{d}\theta$, but should be clear they are finding area of both sides. (Limits not needed for this mark.) **M1:** Expands r^2 and integrates, powers to raise by 1. **A1:** Applies limits and finds the area of one (or both) sides. 240 cm^2 for one sides, or 480 cm^2 for both. Look to see if they double when combining to see if they have one or two sides. **M1:** Attempts $r^2+\left(\frac{\mathrm{d}r}{\mathrm{d}\theta}\right)^2$ with correct differentiation. May be errors in squaring. **A1:** Correct factorised expression, which may be implied by later work when they need to square root. **M1:** Applies the arc length formula to their expression, must be a valid attempt to square root. (Limits not needed.) **A1:** Applies the limits to the integral to obtain **correct** arc length. **M1:** Uses area of curved surface is arc length $\times$ width of bump, with correct units used (not 2.35 and 40 unless they recover before adding) and adds the areas of the sides. Allow even if the attempt at the arc length came from incorrect application of the formula. **A1:** cao 9880 cm^2

Question	Scheme	Marks	AOs
5(a)	$w=\dfrac{1-3z}{z+2\mathrm{i}} \Rightarrow w(z+2\mathrm{i})=1-3z \Rightarrow z=\ldots$ or $\ldots(z+\mathrm{i})=\ldots$	**M1**	2.1
	$z=\dfrac{1-2\mathrm{i}w}{w+3}$ or $(w+3)(z+\mathrm{i})=1-2\mathrm{i}w+(w+3)\mathrm{i}$ $(=1+(3-w)\mathrm{i})$	**A1**	1.1b
	$\left\lvert\dfrac{1-2\mathrm{i}w}{w+3}+\mathrm{i}\right\rvert=3 \Rightarrow \lvert 1-2\mathrm{i}w+\mathrm{i}(w+3)\rvert=3\lvert w+3\rvert$ or $3\lvert w+3\rvert=\lvert 1+(3-w)\mathrm{i}\rvert$ *	**M1** **A1***	3.1a 2.1a
		(4)	
(b)	(i) $w=u+\mathrm{i}v \Rightarrow \lvert(3-u)\mathrm{i}+(v+1)\rvert=3\lvert u+3+\mathrm{i}v\rvert$	**M1**	1.1b
	$\Rightarrow (3-u)^2+(v+1)^2=9\left[(u+3)^2+v^2\right]$	**M1** **A1**	2.1 1.1b
	(ii) $\Rightarrow 8u^2+60u+8v^2-2v+71=0 \Rightarrow (u+\ldots)^2+(v+\ldots)^2=\ldots$	**M1**	1.1b
	$\left[\left(u+\frac{15}{4}\right)^2+\left(v-\frac{1}{8}\right)^2=\frac{333}{64}\Rightarrow\right]$ Centre is $\left(-\dfrac{15}{4},\dfrac{1}{8}\right)$	**A1**	2.2a
	Radius is $\dfrac{3\sqrt{37}}{8}$	**A1**	2.2a
		(6)	
		(10 marks)	

Notes:

(a)
M1: Attempts to make *z* the subject or to extract $z+\mathrm{i}$ as a term.
A1: Correct expression for *z* or correct equation with $z+\mathrm{i}$ as only term(s) in *z*.
M1: Applies $\lvert z+\mathrm{i}\rvert=3$ to their equation and eliminates fractions.
A1*: Correctly completes to the given result with no errors.

(b)(i)
M1: Uses $w=u+\mathrm{i}v$ or $w=x+\mathrm{i}y$ (or other suitable notation) in the given equation
M1: Squares and applies Pythagoras to modulus to form an equation in just *u* and *v*. Allow if the 3 is not squared, or slips on sign inside the brackets (e.g. $(v-1)^2$ instead of $(v+1)^2$). However, there must be no i's involved in the equation, and must be sum of square terms on each side, for this mark to be awarded (cannot be recovered – this is for rigorous argument).
A1: Correct equation from correct work – see note on the previous M. (This is a Cartesian equation so satisfies the demand.)
(ii) Allow recovery for these three marks if the 2nd M was withheld dues to i's in the equation but which were later recovered.
M1: Expands their equation and gathers terms, then completes square or uses other valid method to find the centre and/or radius of the circle.
A1: Correct centre $\left(-\dfrac{15}{4},\dfrac{1}{8}\right)$ Accept as coordinates or complex number. Allow $\left(-\dfrac{15}{4},\dfrac{1}{8}\mathrm{i}\right)$
A1: Correct radius $\dfrac{3\sqrt{37}}{8}$

Question	Scheme	Marks	AOs
6(a)	A rotation …	**M1**	1.1b
	… about the centre of the shape, though an angle 120° anticlockwise.	**A1**	1.1b
		(2)	
(b)	$\begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 \\ 5 & 6 & 1 & 2 & 3 & 4 \end{pmatrix}$	**B1**	1.1b
	One of $\begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 \\ 1 & 6 & 5 & 4 & 3 & 2 \end{pmatrix}$, $\begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 \\ 3 & 2 & 1 & 6 & 5 & 4 \end{pmatrix}$ or $\begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 \\ 5 & 4 & 3 & 2 & 1 & 6 \end{pmatrix}$	**M1**	1.1b
	Two of $\begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 \\ 1 & 6 & 5 & 4 & 3 & 2 \end{pmatrix}$, $\begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 \\ 3 & 2 & 1 & 6 & 5 & 4 \end{pmatrix}$ & $\begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 \\ 5 & 4 & 3 & 2 & 1 & 6 \end{pmatrix}$	**A1**	1.1b
	All the above and $\begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 \\ 1 & 2 & 3 & 4 & 5 & 6 \end{pmatrix}$ and no extra symmetries given. Two line form required for this mark.	**A1**	2.5
		(4)	
(c)	*G* has order 6 so can have no subgroup of order 4 by Lagrange's Theorem.	**B1**	2.4
	There is no element of order 6 that generates the group.	**B1**	2.4
		(2)	
(d)	E.g.  or  | M1 – breaks the reflections	**M1**	3.1a
	A1 – keeps all rotations and no reflections	**A1**	1.1b
		(2)	
			(10 marks)

Notes:
(a) **M1:** For identifying the permutation as representing a rotation. **A1:** A complete description including rotation, centre, angle (degrees or radians) and direction. **(b)** *NB accept in cycle form for the first two marks in (b).* **B1:** For giving the other rotation of order 3 (rotation through 240º anticlockwise). **M1**: For one of the three reflections correctly given. **A1**: For one of the other two reflections. **A1:** For all reflections and the identity, and no extra symmetries given, and all in two-line notation. **(c)** **B1:** Correct reason given. Stating there are no elements of order 4 is **not** sufficient. A longer, valid method would be to state a subgroup of order 4 would need to comprise of the identity and the three elements of order 2 (as it cannot contain any of order 3), but this is not closed as the composite of any two reflection is a rotation. **B1:** Refers to there being no element (of order 6) to generate the group. May reason using orders of elements or via geometric restrictions (e.g. cannot generate a rotation from a single reflection and vice versa). **(d)** **M1**: Realises that the shape needs only the rotations preserved so shades in a way that breaks the reflection symmetries, but which preserves at least one non-trivial rotation. **A1:** Any correctly shaded shape, e.g. the ones shown above. There are many variations!

Question	Scheme	Marks	AOs
7(a)	$I_n = \int 1\times(4-x^2)^{-n}\,\mathrm{d}x = x(4-x^2)^{-n} \pm \int x.n(4-x^2)^{-n-1}.2x\mathrm{d}x$	M1	3.1a
	$I_n = x(4-x^2)^{-n} - \int x.-n(4-x^2)^{-n-1}.-2x\mathrm{d}x$	A1	1.1b
	$= x(4-x^2)^{-n} + 2n\int (4-x^2-4)(4-x^2)^{-n-1}\,\mathrm{d}x$ $= x(4-x^2)^{-n} + 2n\int (4-x^2)(4-x^2)^{-n-1} - 4(4-x^2)^{-n-1}\,\mathrm{d}x$	M1	2.1
	$I_n = x(4-x^2)^{-n} + 2n\int (4-x^2)^{-n}\,\mathrm{d}x - 8n\int (4-x^2)^{-(n+1)}\,\mathrm{d}x$ $= x(4-x^2)^{-n} + 2nI_n - 8nI_{n+1}$	M1	1.1b
	$\Rightarrow 8nI_{n+1} = x(4-x^2)^{-n} + (2n-1)I_n \Rightarrow I_{n+1} = \dfrac{x}{8n(4-x^2)^n} + \dfrac{2n-1}{8n}I_n$ *	A1*	2.1
		(5)	
(b)	$I_1 = \int \dfrac{1}{4-x^2}\mathrm{d}x = \dfrac{1}{2}\text{artanh}\left(\dfrac{x}{2}\right)$ or $\dfrac{1}{4}\ln\left\lvert\dfrac{2+x}{2-x}\right\rvert$ oe	B1	2.2a
	$I_2 = \dfrac{x}{8(4-x^2)} + \dfrac{1}{8}I_1$	M1	1.1b
	$= \dfrac{x}{8(4-x^2)} + \dfrac{1}{16}\text{artanh}\left(\dfrac{x}{2}\right)(+c)$ oe e.g. $\dfrac{x}{8(4-x^2)} + \dfrac{1}{32}\ln\left\lvert\dfrac{2+x}{2-x}\right\rvert(+c)$	A1	1.1b
		(3)	
		(8 marks)	

Notes:

(a)

M1: Splits integrand as $1\times(4-x^2)^{-n}$ and attempts parts (the right way round). There are lots of minus signs around, so accept with $\pm$ between as it is hard to tell if the formula is incorrect or it is sign error.

A1: Correct result obtained, need not be simplified.

M1: Gathers the terms and writes the x^2 as $-(4-x^2-4)$ (oe). Accept either line shown in the scheme (with their coefficients from the parts expansion).

M1: Sorts out the indices and replaces the appropriate integrals by I_n and I_{n+1} respectively.

A1*: Completes to the correct printed result, with no errors or ambiguities.

(b)

B1: Deduces the correct result for I_1

M1: Applies the reduction formula correct with $n=1$ only.

A1: Correct answer, accept any equivalent form as long as fractions are simplified. Constant of integration is not needed.

PMT

Question	Scheme	Marks	AOs
8(a)	The integer n can be written as $n = 1000a + 100b + 10c + d$	**M1**	1.1b
	As $1000 = 142\times 7 + 6$, $100 = 14\times 7 + 2$ and $10 = 7 + 3$, reducing coefficients modulo 7 gives $n \equiv 6a + 2b + 3c + d \pmod 7$ *	**A1***	2.4
		(2)	
(b)	n divisible by 9 means $a+b+c+d = 9k$ for some integer k	**B1**	1.1b
	$a+b = c+d \Rightarrow 2(a+b) = 9k$, hence k even	**M1**	3.1a
	But $a+b+c+d$ must be at least 3 and at most 35 (as b smaller than all other numbers).	**B1**	2.1
	So since k must be even, only possibility is $k = 2$ to keep $a+b+c+d$ in range 0-36, hence $a+b = 9$ *	**A1***	2.2a
		(4)	
(c)	Combining (a) and (b) gives $3 \equiv 2(a+b) + 4a + (c+d) + 2c \equiv 4a + 2c + 27 \pmod 7$	**M1**	3.1a
	$\Rightarrow 2c \equiv -4a - 24 \equiv -4(a+6) \equiv 3(a-1) \pmod 7$ $\Rightarrow 8c \equiv 12(a-1) \pmod 7 \Rightarrow c \equiv 5(a-1) \pmod 7$ *	**A1***	2.1
		(2)	
(d)	As $b < a$ and $a+b = 9$ we must have $a..5$	**M1**	3.1a
	If $a = 9$ then $c \equiv 5 \pmod 7 \Rightarrow c = 5$, contradicting c is even. If $a = 8$ then $c \equiv 0 \pmod 7 \Rightarrow c = 0$ or 7 but 7 not even, and $c > b$ so can't be zero. If $a = 7$ then $c \equiv 2 \pmod 7 \Rightarrow c = 2$, but also $b = 2$ for $a = 7$, and $b < c$ so not possible. If $a = 6$ then $c \equiv 4 \pmod 7 \Rightarrow c = 4$, $d = 5$ and $b = 3$ which works. If $a = 5$ then $c \equiv 6 \pmod 7 \Rightarrow c = 6$, $d = 3$ and $b = 4$ but then $d < b$, not allowed.	**dM1** **A1**	1.1b 1.1b
	Hence $n = 6345$ (only)	**B1**	2.2a
		(4)	
			(12 marks)

Notes: Allow credit for any part for relevant work seen throughout.
(a) **M1:** Writes $n = abcd$ as sum of multiples of the digits, as shown in scheme. **A1*:** Explains how each coefficient reduces modulo 7 to give the stated answer. **(b)** **B1:** Applies the divisibility test for 9, either as in scheme or stating $a+b+c+d$ is a multiple of 9 (or similar). $a+b+c+d \equiv 0 \pmod 9$ is fine. **M1:** Uses the third fact in combination with the second to eliminate c and d and deduce k even. Alternatively, $a+b+c+d \equiv 2(a+b) \equiv 0 \pmod 9 \Rightarrow 10(a+b) \equiv 5 \times 0 \pmod 9 \Rightarrow a+b \equiv 0 \pmod 9$ **B1:** Eliminates the possibilities that $k = 0$ or 4 using the fourth fact. Alt: reasons as $b < a$ then $a + b$ cannot be 0 or 18. **A1*:** Completes the proof, with all steps explained. Allow if the previous B has not been earned. **(c)** **M1:** Combines the results of (a) and (b) to eliminate b and d from the result in (a). **A1*:** Correct completion to the given statement. There will be different ways to achieve this, one example is shown. Check work carefully! **(d)** **M1:** There may be many approaches here. Scored for beginning a process of eliminating possibilities for one of the digits, e.g. deducing a is at least 5 as shown. (May be done by individually eliminating the lower values.) **dM1:** Continues the process eliminating at least two more cases. **A1:** Complete argument, leading to one solution only (and no possibility of others). Need not be expressed as eloquently as above, but should be clear all cases have been considered. **B1:** For $n = 6345$ obtained in any way.