Please check the examination details below before entering your candidate information

Candidate surname	Other names

Centre Number | Candidate Number

Pearson Edexcel Level 3 GCE

Time 1 hour 30 minutes

Paper reference **9FM0/4A**

Further Mathematics

Advanced

PAPER 4A: Further Pure Mathematics 2

You must have:
Mathematical Formulae and Statistical Tables (Green), calculator

Total Marks

Candidates may use any calculator permitted by Pearson regulations. Calculators must not have the facility for symbolic algebraic manipulation, differentiation and integration, or have retrievable mathematical formulae stored in them.

Instructions

- Use **black** ink or ball-point pen.
- If pencil is used for diagrams/sketches/graphs it must be dark (HB or B).
- **Fill in the boxes** at the top of this page with your name, centre number and candidate number.
- Answer **all** questions and ensure that your answers to parts of questions are clearly labelled.
- Answer the questions in the spaces provided – *there may be more space than you need.*
- You should show sufficient working to make your methods clear. Answers without working may not gain full credit.
- Inexact answers should be given to three significant figures unless otherwise stated.

Information

- A booklet 'Mathematical Formulae and Statistical Tables' is provided.
- There are 10 questions in this question paper. The total mark for this paper is 75.
- The marks for **each** question are shown in brackets – *use this as a guide as to how much time to spend on each question.*

Advice

- Read each question carefully before you start to answer it.
- Try to answer every question.
- Check your answers if you have time at the end.

Turn over

1. The group S_4 is the set of all possible permutations that can be performed on the four numbers 1, 2, 3 and 4, under the operation of composition.

Apply one permutation, then another in sequence.

For the group S_4

(a) write down the identity element, **(1)**

(b) write down the inverse of the element a, where

$$a = \begin{pmatrix} 1 & 2 & 3 & 4 \\ 3 & 4 & 2 & 1 \end{pmatrix}$$

(1)

(c) demonstrate that the operation of composition is associative using the following elements

$$a = \begin{pmatrix} 1 & 2 & 3 & 4 \\ 3 & 4 & 2 & 1 \end{pmatrix} \quad b = \begin{pmatrix} 1 & 2 & 3 & 4 \\ 2 & 4 & 3 & 1 \end{pmatrix} \quad \text{and } c = \begin{pmatrix} 1 & 2 & 3 & 4 \\ 4 & 1 & 2 & 3 \end{pmatrix}$$

(2)

(d) Explain why it is possible for the group S_4 to have a subgroup of order 4
You do not need to find such a subgroup.

(2)

(a) $e = \begin{pmatrix} 1 & 2 & 3 & 4 \\ 1 & 2 & 3 & 4 \end{pmatrix}$ ① ← identity element leaves all other elements unchanged

(b) $\begin{pmatrix} 1 & 2 & 3 & 4 \\ 4 & 3 & 1 & 2 \end{pmatrix}$ ① ← reflect in leading diagonal

(c) $(a \circ b) \circ c = \left[\begin{pmatrix} 1 & 2 & 3 & 4 \\ 3 & 4 & 2 & 1 \end{pmatrix} \circ \begin{pmatrix} 1 & 2 & 3 & 4 \\ 2 & 4 & 3 & 1 \end{pmatrix}\right] \circ \begin{pmatrix} 1 & 2 & 3 & 4 \\ 4 & 1 & 2 & 3 \end{pmatrix}$

Apply right-hand bracket first

$1 \to 2 \to 4,\ 2 \to 4 \to 1,\ 3 \to 3 \to 2,\ 4 \to 1 \to 3$

$(a \circ b) \circ c = \begin{pmatrix} 1 & 2 & 3 & 4 \\ 4 & 1 & 2 & 3 \end{pmatrix} \circ \begin{pmatrix} 1 & 2 & 3 & 4 \\ 4 & 1 & 2 & 3 \end{pmatrix}$

$(a \circ b) \circ c = \begin{pmatrix} 1 & 2 & 3 & 4 \\ 3 & 4 & 1 & 2 \end{pmatrix}$

Question 1 continued

$$a \circ (b \circ c) = \begin{pmatrix} 1 & 2 & 3 & 4 \\ 3 & 4 & 2 & 1 \end{pmatrix} \circ \left[\begin{pmatrix} 1 & 2 & 3 & 4 \\ 2 & 4 & 3 & 1 \end{pmatrix} \circ \begin{pmatrix} 1 & 2 & 3 & 4 \\ 4 & 1 & 2 & 3 \end{pmatrix} \right]$$

$$a \circ (b \circ c) = \begin{pmatrix} 1 & 2 & 3 & 4 \\ 3 & 4 & 2 & 1 \end{pmatrix} \circ \begin{pmatrix} 1 & 2 & 3 & 4 \\ 1 & 2 & 4 & 3 \end{pmatrix}$$

$$a \circ (b \circ c) = \begin{pmatrix} 1 & 2 & 3 & 4 \\ 3 & 4 & 1 & 2 \end{pmatrix} \quad 1$$

$(a \circ b) \circ c = a \circ (b \circ c)$ ∴ it is associative. 1

(d) The order of the group is $4! = 24$ (using permutation formula). 1 4 is a factor of 24 therefore it is possible to have a subgroup of order 4 1 (by Lagrange's Theorem)

(Total for Question 1 is 6 marks)

2. Matrix **M** is given by

$$\mathbf{M} = \begin{pmatrix} 1 & 0 & a \\ -3 & b & 1 \\ 0 & 1 & a \end{pmatrix}$$

where a and b are integers, such that $a < b$

Given that the characteristic equation for **M** is

$$\lambda^3 - 7\lambda^2 + 13\lambda + c = 0$$

where c is a constant,

(a) determine the values of a, b and c.

(5)

(b) Hence, using the Cayley–Hamilton theorem, determine the matrix $\mathbf{M}^{-1}$

(3)

(a) $\det(M - \lambda I) = 0$

$$\det \begin{bmatrix} 1-\lambda & 0 & a \\ -3 & b-\lambda & 1 \\ 0 & 1 & a-\lambda \end{bmatrix} = 0$$

$$\det \begin{pmatrix} a & b & c \\ d & e & f \\ h & i & j \end{pmatrix} = a(ej-fi) - b(dj-fh) + c(di-eh)$$

$1-\lambda\left[(b-\lambda)(a-\lambda) - 1\right] + a(-3) = 0$ 1

$1-\lambda\left[ab - a\lambda - b\lambda + \lambda^2 - 1\right] - 3a = 0$

$1 - ab\lambda + a\lambda^2 + b\lambda^2 - \lambda^3 + \lambda + ab - a\lambda - b\lambda + \lambda^2 - 1 - 3a = 0$

$-\lambda^3 + (a+b+1)\lambda^2 + (-ab+1-a-b)\lambda + (ab-3a-1) = 0$

$\lambda^3 - (a+b+1)\lambda^2 - (-ab+1-a-b)\lambda - (ab-3a-1) = 0$ 1

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Question 2 continued

$a + b + 1 = 7$

$a + b = 6$

compare coefficients

$\underbrace{a + b}_{} + ab - 1 = 13$

$6 + ab - 1 = 13$

$ab = 8$

$b = \frac{8}{a}$

sub. into $a+b=6$

$a + \frac{8}{a} = 6$

$\times a$

$a^2 - 6a + 8 = 0$ 1

$(a-4)(a-2) = 0$

$a = 2$

since $a+b=6$ and $a<b$

$b = 4$ 1

$ab - 3a - 1 = 0$

$-(2 \times 4 - 3 \times 2 - 1) = -1$

$\therefore c = -1$ 1

Question 2 continued

(b) C-H Theorem states that a square matrix satisfies its own characteristic equation

$$M^3 - 7M^2 + 13M - I = 0 \quad (1)$$

$$I = M^3 - 7M^2 + 13M$$

$\div M$

$$M^{-1} = M^2 - 7M + 13I$$

$$M^{-1} = \begin{bmatrix} 1 & 0 & 2 \\ -3 & 4 & 1 \\ 0 & 1 & 2 \end{bmatrix}^2 - 7\begin{bmatrix} 1 & 0 & 2 \\ -3 & 4 & 1 \\ 0 & 1 & 2 \end{bmatrix} + \begin{bmatrix} 13 & 0 & 0 \\ 0 & 13 & 0 \\ 0 & 0 & 13 \end{bmatrix} \quad (1)$$

$$M^{-1} = \begin{bmatrix} 1 & 2 & 6 \\ -15 & 17 & 0 \\ -3 & 6 & 5 \end{bmatrix} - \begin{bmatrix} 7 & 0 & 14 \\ -21 & 28 & 7 \\ 0 & 7 & 14 \end{bmatrix} + \begin{bmatrix} 13 & 0 & 0 \\ 0 & 13 & 0 \\ 0 & 0 & 13 \end{bmatrix}$$

$$M^{-1} = \begin{bmatrix} 7 & 2 & -8 \\ 6 & 2 & -7 \\ -3 & -1 & 4 \end{bmatrix} \quad (1)$$

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Question 2 continued

(Total for Question 2 is 8 marks)

3.

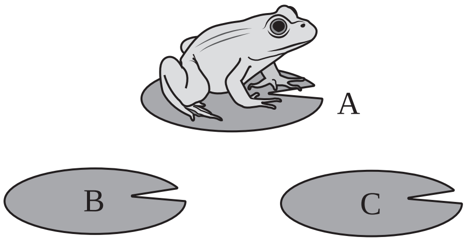

Figure 1

There are three lily pads on a pond. A frog hops repeatedly from one lily pad to another.

The frog starts on lily pad A, as shown in Figure 1.

In a model, the frog hops from its position on one lily pad to either of the other two lily pads with equal probability.

Let p_n be the probability that the frog is on lily pad A after n hops.

(a) Explain, with reference to the model, why $p_1 = 0$

(1)

The probability p_n satisfies the recurrence relation

$$p_{n+1} = \frac{1}{2}(1 - p_n) \qquad n \geqslant 1 \quad \text{where } p_1 = 0$$

(b) Prove by induction that, for $n \geqslant 1$

$$p_n = \frac{2}{3}\left(-\frac{1}{2}\right)^n + \frac{1}{3}$$

(6)

(c) Use the result in part (b) to explain why, in the long term, the probability that the frog is on lily pad A is $\frac{1}{3}$

(1)

(a) In the first hop, the frog must move from A onto a different lily pad 1

(b) When $n = 1$:

$$p_1 = \frac{2}{3}\left(-\frac{1}{2}\right)^1 + \left(\frac{1}{3}\right) = 0 \quad 1$$

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Question 3 continued

$\therefore$ True for $n = 1$, assume true for all $n = k$

such that:

$$p_k = \frac{2}{3}\left(\frac{-1}{2}\right)^k + \left(\frac{1}{3}\right) \quad ①$$

Consider $n = k+1$

$$p_{k+1} = \frac{1}{2}\left[1 - p_k\right]$$

$$p_{k+1} = \frac{1}{2}\left[1 - \left(\frac{2}{3}\left(\frac{-1}{2}\right)^k + \left(\frac{1}{3}\right)\right)\right] \quad ①$$

- goes inside brackets

$$p_{k+1} = \frac{1}{2} + \frac{2}{3}\left(\frac{-1}{2}\right)\left(\frac{-1}{2}\right)^k + \frac{1}{6}$$

$$p_{k+1} = \frac{1}{3} + \frac{2}{3}\left(\frac{-1}{2}\right)\left(\frac{-1}{2}\right)^k \quad ①$$

$$p_{k+1} = \frac{1}{3} + \frac{2}{3}\left(\frac{-1}{2}\right)^{k+1} \quad ①$$

$\therefore$ If true for $n = k$ then true for $n = k+1$, and as shown true for $n = 1$, the statement is true for all n. ①

(c) As $n \to \infty$, $\left(-\frac{1}{2}\right)^n \to 0$ $\quad \therefore p_n \to \frac{1}{3}$ ①

the only constant part

Question 3 continued

Question 3 continued

(Total for Question 3 is 8 marks)

PMT

4. (a) Use the Euclidean algorithm to show that 124 and 17 are relatively prime (coprime).
(2)

(b) Hence solve the equation

$$124x + 17y = 10$$
(3)

(c) Solve the congruence equation

$$124x \equiv 6 \bmod 17$$
(2)

prime factors

(a) $124 = 17 \times 7 + 5$ ← remainder

$17 = 3 \times 5 + 2$

$5 = 2 \times 2 + 1$ 1

Since the gcd is 1, 124 and 17 are coprime. 1

(b) $5 = 2 \times 2 + 1$

$1 = 5 - 2 \times 2$

$1 = 5 - 2(17 - 3 \times 5)$ 1

$1 = (1 - 2 \times -3) \times 5 - 2 \times 17$

$1 = 7 \times 5 - 2 \times 17$

$1 = 7 \times (124 - 17 \times 7) - 2 \times 17$

$1 = 7 \times 124 - 51 \times 17$ 1

$10 = 70 \times 124 - 510 \times 17$

$\therefore x = 70, \; y = -510$ 1

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P 6 5 5 0 6 A 0 1 2 3 2

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Question 4 continued

(c) $7 \times 124x \equiv 7 \times 6 \mod 17$ ← multiply by multiplicative inverse of 124 (deduce from (a)) to remove the coefficient.

$x \equiv 42 \mod 17$ 1

$x \equiv 8 \mod 17$ 1

$42 \div 17 = 2$ remainder 8

(Total for Question 4 is 7 marks)

5. The locus of points z satisfies

$$|z + a\mathrm{i}| = 3|z - a|$$

where a is an integer.

The locus is a circle with its centre in the third quadrant and radius $\frac{3}{2}\sqrt{2}$

Determine

(a) the value of a,

(4)

(b) the coordinates of the centre of the circle.

(2)

(a) $z = x + yi$

$|z + ai| = 3|z - a| \Rightarrow |x + (y + a)i| = 3|(x - a) + yi|$

$x^2 + (y+a)^2 = 3^2[(x-a)^2 + y^2]$ ① group real / imaginary parts

$x^2 + y^2 + 2ay + a^2 = 9x^2 - 18ax + 9a^2 + 9y^2$

$8x^2 - 18ax + 8y^2 - 2ay + 8a^2 = 0$ ①

$\div 8$

$x^2 - \frac{9}{4}ax + y^2 - \frac{1}{4}ay + a^2 = 0$

complete the square

$\left(x - \frac{9}{8}a\right)^2 + \left(y - \frac{1}{8}a\right)^2 - \left(\frac{9}{8}a\right)^2 - \left(\frac{1}{8}a\right)^2 + a^2 = 0$

Since $x^2 + y^2 = r^2$:

$r^2 = \left(\frac{9}{8}a\right)^2 + \left(\frac{1}{8}a\right)^2 - a^2$

$\left(\frac{3}{2}\sqrt{2}\right)^2 = \frac{9}{32}a^2$ ①

$a^2 = 16 \Rightarrow a = -4$ ① ← centre is in 3^{rd} quadrant

Question 5 continued

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(b) $\left(x - \frac{9}{8}(-4)\right)^2 + \left(y - \frac{1}{8}(-4)\right)^2 = \left(x + \frac{9}{2}\right)^2 + \left(y + \frac{1}{2}\right)^2$ 1

sub. $a = -4$

$\therefore$ centre is $\left(-\frac{9}{2}, -\frac{1}{2}\right)$ 1

(Total for Question 5 is 6 marks)

6. (a) Determine the general solution of the recurrence relation

$$u_n = 2u_{n-1} - u_{n-2} + 2^n \qquad n \geqslant 2$$

(4)

(b) Hence solve this recurrence relation given that $u_0 = 2u_1$ and $u_4 = 3u_2$

(2)

(a) Find the complementary solution by solving the homogeneous equation:

$U_n = 2U_{n-1} - U_{n-2}$

For the equation $U_n = aU_{n-1} + bU_{n-2}$, characteristic equation has the form $r^2 - ar - b = 0$.

$r^2 - 2r + 1 = 0$ 1

$(r-1)(r-1) = 0$

$\therefore r = 1$ ← root is α

Repeated real root, so general solution has

the form $U_n = (A + Bn)\alpha^n$ ← $\alpha = 1$ 1

The non-homogeneous term is 2^n, so try a particular solution of the form $U_n = \mu \times 2^n$

$\mu \times 2^n = 2\mu \times 2^{n-1} - \mu \times 2^{n-2} + 2^n$

$\mu \times 2^n = \mu \times 2^n - \mu \times 2^{n-2} + 2^n$

$0 = -\mu \times 2^{n-2} + 2^n$

$\mu \times 2^{n-2} = 2^n$

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Question 6 continued

$\mu = \dfrac{2^n}{2^{n-2}}$ $\quad x^a \div x^b = x^{a-b}$

$\mu = \dfrac{1}{2^{-2}}$

$\mu = 4$ ①

Combine general and particular equations.

$u_n = A + Bn + 4(2^n)$ ①

(b) $u_0 = 2u_1 \quad A + B(0) + 4(2^0) = 2[A + B(1) + 4 \times 2^1]$

$A + 4 = 2A + 2B + 16$

$-12 = A + 2B \quad (1)$

$u_4 = 3u_2. \quad A + B(4) + 4(2^4) = 3[A + B(2) + 4(2^2)]$

$A + 4B + 64 = 3A + 6B + 48$

$16 = 2A + 2B \quad (2)$

$(2) - (1): \quad A = 16 - (-12) = 28$ ①

$2B = -12 - 28 = -40$

$B = -20$

$\therefore u_n = 28 - 20n + 4(2^n)$ ①

(Total for Question 6 is 6 marks)

7. (i) The polynomial F(x) is a quartic such that

$$F(x) = px^4 + qx^3 + 2x^2 + rx + s$$

where p, q, r and s are distinct constants.

Determine the number of possible quartics given that

(a) the constants p, q, r and s belong to the set $\{-4, -2, 1, 3, 5\}$ **(1)**

(b) the constants p, q, r and s belong to the set $\{-4, -2, 0, 1, 3, 5\}$ **(1)**

(ii) A 3-digit positive integer $N = abc$ has the following properties

- N is divisible by 11
- the sum of the digits of N is even
- $N \equiv 8 \bmod 9$

(a) Use the first two properties to show that

$$a - b + c = 0$$

(2)

(b) Hence determine all possible integers N, showing all your working and reasoning. **(4)**

(i)(a) $\dfrac{5!}{(5-4)!} = 5! = 120$ ① ← choose 4 elements from 5 using permutation formula

(i)(b) $\dfrac{6!}{(6-4)!} - \dfrac{5!}{(5-3)!} = 300$ ①

all permutations

the number of permutations where $p=0$ (these are not quartics)

(ii)(a) Divisible by 11 implies $a-b+c = 11p$ where p is an integer ①

Since $0 \le a,b,c \le 9$, $p = 0$ or 1 ← max value would be $9-1+8=16$

$a+b+c$ is even $\Rightarrow$ $a+b+c-2b$ is even.

since $2b$ is even, and even − even = even.

$\therefore$ $p = 0$ and $a-b+c=0$ ①

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Question 7 continued

(ii)(b) $N = 100a + 10b + c$ ← split the digits

$100 \equiv 1 \bmod 9$ and $10 \equiv 1 \bmod 9$

So $100a + 10b + c \equiv a+b+c \bmod 9$ 1

$a+b+c \equiv 8 \bmod 9$ ← using the 3rd property

Using $a-b+c = 0 \Rightarrow a = b-c$

$b-c+b+c \equiv 8 \bmod 9$ ← substitute $a = b-c$

$2b \equiv 8 \bmod 9$

$b \equiv 4 \bmod 9$ 1

Since $b = a+c$ and $a+b+c$ is even, $a+c$ must also be even :

$0+4+4=8$ and $0-4+4=0$ ✗

$1+4+3=8$ and $1-4+3=0$ ✓

$3+4+1=8$ and $3-4+1=0$ ✓

$2+4+2=8$ and $2-4+2=0$ ✓

$4+4+0=8$ and $4-4+0=0$ ✓

$\therefore$ N could be 143, 242, 341, 440 2

Question 7 continued

Question 7 continued

(Total for Question 7 is 8 marks)

8. The locus of points $z = x + \mathrm{i}y$ that satisfy

$$\arg\left(\frac{z-8-5\mathrm{i}}{z-2-5\mathrm{i}}\right) = \frac{\pi}{3}$$

is an arc of a circle C.

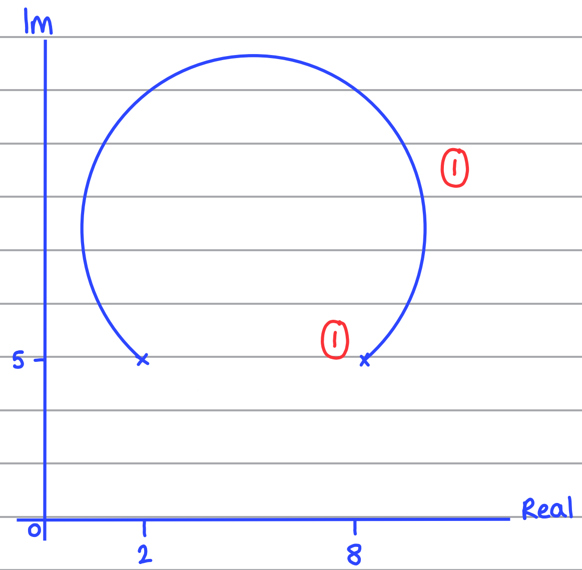
(a) On an Argand diagram sketch the locus of z. **(2)**

(b) Explain why the centre of C has x coordinate 5 **(1)**

(c) Determine the radius of C. **(2)**

(d) Determine the y coordinate of the centre of C. **(2)**

(a)



Locus of form $\arg\left(\frac{z_1}{z_2}\right) = \theta$ is an arc of a circle from

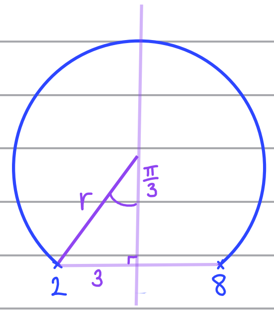
$z-8-5i \rightarrow$ point $(8,5)$

to

$z-2-5i \rightarrow$ point $(2,5)$

(b) The center is on the midpoint of 2 and 8. ① $\left(\frac{8+2}{2} = \frac{10}{2} = 5\right)$

(c)



From the diagram, $\sin = \frac{O}{H}$.

$\sin\frac{\pi}{3} = \frac{3}{r}$ ①

$\frac{\sqrt{3}}{2} = \frac{3}{r}$

$r = \frac{6}{\sqrt{3}} = 2\sqrt{3}$ ①

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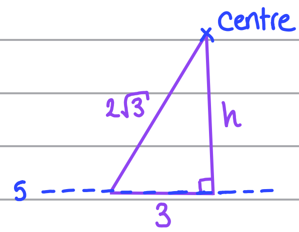
Question 8 continued

(d) $y = 5 + h$



$h^2 + 3^2 = (2\sqrt{3})^2$ 1

$h^2 = 12 - 3 = 9$

$h = \sqrt{3}$

$\therefore y = 5 + \sqrt{3}$ 1

(Total for Question 8 is 7 marks)

9.

$$I_n = \int_0^{\frac{\pi}{2}} \sin^n 2x \, dx$$

(a) Prove that for $n \geqslant 2$

$$I_n = \frac{n-1}{n} I_{n-2}$$

(4)

(b) Hence determine the exact value of

$$\int_0^{\frac{\pi}{2}} 64 \sin^5 x \cos^5 x \, dx$$

(3)

(a) $I_n = \int_0^{\pi/2} \sin^n 2x \, dx$

Apply integration by parts.

$I_n = \int_0^{\pi/2} \sin^{n-1} 2x \sin 2x \, dx$

$uv' = uv - \int vu'$

$u = \sin^{n-1} 2x$ $\qquad$ $v' = \sin 2x$

$u' = (n-1)\sin^{n-2} 2x \times 2\cos 2x$ $\qquad$ $v = -\frac{1}{2}\cos 2x$

$uv - \int vu' \, dx = \sin^{n-1} 2x \times -\frac{1}{2}\cos 2x - \int (n-1)\sin^{n-2} 2x \times 2\cos 2x \times -\frac{1}{2}\cos 2x \, dx$ 1

$I_n = \left[-\frac{1}{2}\sin^{n-1} 2x \cos 2x\right]_0^{\frac{\pi}{2}} + \int (n-1)\sin^{n-2} 2x \cos^2 2x \, dx$ 1

$\left[-\frac{1}{2}\sin^{\frac{\pi}{2}-1} 2\left(\frac{\pi}{2}\right)\cos 2\left(\frac{\pi}{2}\right)\right] - \left[-\frac{1}{2}\sin^{0-1} 2(0)\cos 2(0)\right] = 0$

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Question 9 continued

$$I_n = \int_0^{\frac{\pi}{2}} (n-1)\sin^{n-2}2x\cos^2 2x \, dx$$

$$I_n = (n-1)\int_0^{\frac{\pi}{2}} \sin^{n-2}2x \, dx - (n-1)\int_0^{\frac{\pi}{2}} \sin^n 2x \, dx \quad (1)$$

$$nI_n = (n-1)I_{n-2}$$

$$I_n = \frac{(n-1)}{n} I_{n-2} \quad (1)$$

(b) $$\int_0^{\frac{\pi}{2}} 64\sin^5 x\cos^5 x \, dx = \int_0^{\frac{\pi}{2}} 2\sin^5 2x \, dx \quad (1)$$

take out the constant, multiply in final step

$$I_5 = \frac{5-1}{5} I_3 = \frac{4}{5} I_3$$

$$I_3 = \frac{3-1}{3} I_1 = \frac{2}{3} I_1$$

$$I_1 = \int_0^{\frac{\pi}{2}} \sin 2x \, dx = \left[-\frac{1}{2}\cos 2x\right]_0^{\frac{\pi}{2}} = 1 \quad (1)$$

$$\therefore I_5 = 2 \times \frac{4}{5} \times \frac{2}{3} \times 1 = \frac{16}{15} \quad (1)$$

Question 9 continued

Question 9 continued

(Total for Question 9 is 7 marks)

10.

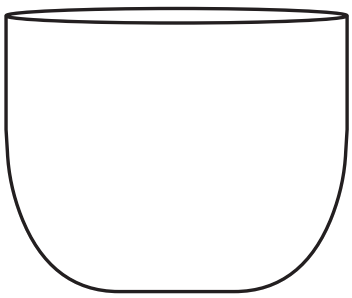

Figure 2

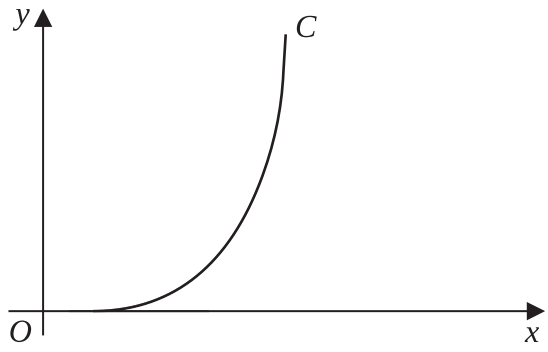

Figure 3

Figure 2 shows a picture of a plant pot.

The plant pot has

- a flat circular base of radius 10 cm
- a height of 15 cm

Figure 3 shows a sketch of the curve C with parametric equations

$$x = 10 + 15t - 5t^3 \qquad y = 15t^2 \qquad 0 \leqslant t \leqslant 1$$

The curved inner surface of the plant pot is modelled by the surface of revolution formed by rotating curve C through 2π radians about the y-axis.

(a) Show that, according to the model, the area of the curved inner surface of the plant pot is given by

$$150\pi \int_0^1 \left(2 + 3t + 2t^2 + 2t^3 - t^5\right) \mathrm{d}t$$

(5)

(b) Determine, according to the model, the total area of the inner surface of the plant pot.

(4)

Each plant pot will be painted with one coat of paint, both inside and outside.
The paint in one tin will cover an area of $12\,\text{m}^2$

(c) Use the answer to part (b) to estimate how many plant pots can be painted using one tin of paint.

(2)

(d) Give a reason why the model might not give an accurate answer to part (c).

(1)

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Question 10 continued

(a) $S_y = 2\pi \int_a^b x \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2}\, dt$

$x = 10 + 15t - 5t^3$ $\qquad y = 15t^2$

$\frac{dx}{dt} = 15 - 15t^2$ $\qquad \frac{dy}{dt} = 30t$

$S_y = 2\pi \int_0^1 (10 + 15t - 5t^3)\sqrt{(15 - 15t^2)^2 + (30t)^2}\, dt$ ① ①

$= 2\pi \int_0^1 (10 + 15t - 5t^3)\sqrt{225 - 450t^2 + 225t^4 + 900t^2}\, dt$ ①

$= 2\pi \int_0^1 (10 + 15t - 5t^3)\sqrt{225 + 450t^2 + 225t^4}\, dt$

$= 2\pi \int_0^1 (10 + 15t - 5t^3)\sqrt{(15 + 15t^2)^2}\, dt$

$= 2\pi \int_0^1 (10 + 15t - 5t^3)(15 + 15t^2)\, dt$ ①

$= 2\pi \int_0^1 (150 + 150t^2 + 225t + 225t^3 - 75t^3 - 75t^5)\, dt$

$= 2 \times 75\pi \int_0^1 (2 + 2t^2 + 3t + 2t^3 - t^5)\, dt$

$\therefore S_y = 150\pi \int_0^1 (2 + 2t^2 + 3t + 2t^3 - t^5)\, dt$ ①

Question 10 continued

(b) $S_y = 150\pi\left[2t + \frac{3}{2}t^2 + \frac{2}{3}t^3 + \frac{2}{4}t^4 - \frac{1}{6}t^6\right]_0^1$

$= 150\pi\left[\left(2(1) + \frac{3}{2}(1^2) + \frac{2}{3}(1^3) + \frac{2}{4}(1^4) - \frac{1}{6}(1^6)\right) - 0\right]$ ①

$= 150\pi \times \frac{9}{2}$

$= 675\pi$ ①

Area of circular base $= \pi \times 10^2 = 100\pi$ ①

$\therefore$ Total area $= 675\pi + 100\pi = 775\pi$ ①

(c) No plant pots $= \dfrac{120\,000}{2 \times 775\pi}$ ① $= 24.64\ldots$

convert to cm^2

inner + outer

round down

$\therefore$ 24 complete pots ①

(d) The outer surface area will be greater than the inner surface area ①

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Question 10 continued

Question 10 continued

(Total for Question 10 is 12 marks)

TOTAL FOR PAPER IS 75 MARKS