Further Pure Mathematics 2 Mark Scheme

Question	Scheme	Marks	AOs
1(a)	Consider $\det\begin{pmatrix}3-\lambda & 1\\ 6 & 4-\lambda\end{pmatrix}=(3-\lambda)(4-\lambda)-6$	M1	1.1b
	So $\lambda^2-7\lambda+6=0$ is characteristic equation	A1	1.1b
		(2)	
	So $\mathbf{A}^2=7\mathbf{A}-6\mathbf{I}$	B1ft	1.1b
(b)	Multiplies both sides of their equation by **A** so $\mathbf{A}^3=7\mathbf{A}^2-6\mathbf{A}$	M1	3.1a
	Uses $\mathbf{A}^3=7(7\mathbf{A}-6\mathbf{I})-6\mathbf{A}$ So $\mathbf{A}^3=43\mathbf{A}-42\mathbf{I}$ *	A1*cso	1.1b
		(3)	
	(5 marks)		

Notes:

(a)

M1: Complete method to find characteristic equation

A1: Obtains a correct three term quadratic equation – may use variable other than λ

(b)

B1ft: Uses Cayley Hamilton Theorem to produce equation replacing λ with **A** and constant term with constant multiple of identity matrix, **I**

M1: Multiplies equation by **A**

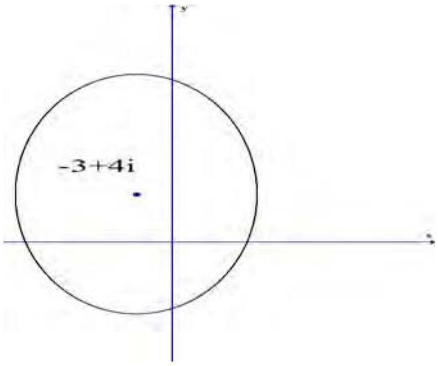
A1*: Replaces $\mathbf{A}^2$ by linear expression in **A** and achieves printed answer with no errors

Question	Scheme	Marks	AOs
2(i)	Adding digits 8 + 1 + 8 + 4 = 21 which is divisible by 3 (or continues to add digits giving 2+1=3 which is divisible by 3) so concludes that 8184 is divisible by 3	M1	1.1b
	8184 is even, so is divisible by 2 and as divisible by both 3 and 2, so it is divisible by 6	A1	1.1b
		(2)	
(ii)	Starts Euclidean algorithm $31 = 27 \times 1 + 4$ and $27 = 4 \times 6 + 3$	M1	1.2
	$4 = 3 \times 1 + 1$ (so hcf = 1)	A1	1.1b
	So $1 = 4 - 3 \times 1 = 4 - (27 - 4 \times 6) \times 1 = 4 \times 7 - 27 \times 1$	M1	1.1b
	$(31 - 27 \times 1) \times 7 - 27 \times 1 = 31 \times 7 - 27 \times 8$ $a = -8$ and $b = 7$	A1cso	1.1b
		(4)	
			(6 marks)

Notes:

(i)
M1: Explains divisibility by 3 rule in context of this number by adding digits
A1: Explains divisibility by 2, giving last digit even as reason and makes conclusion that number is divisible by 6

(ii)
M1: Uses Euclidean algorithm showing two stages
A1: Completes the algorithm. Does not need to state that hcf = 1
M1: Starts reversal process, doing two stages and simplifying
A1cso: Correct completion, giving clear answer following complete solution

Question	Scheme	Marks	AOs
3(a)	$(x-9)^2+(y+12)^2=4[x^2+y^2]$	M1	2.1
	$3x^2+3y^2+18x-24y-225=0$ which is the equation of a circle	A1*	2.2a
	As $x^2+y^2+6x-8y-75=0$ so $(x+3)^2+(y-4)^2=10^2$	M1	1.1b
	Giving centre at (-3, 4) and radius = 10	A1ft	1.1b
		(4)	
(b)		M1	1.1b
		A1	1.1b
		(2)	
(c)	Values range from **their** – 3 –10 to their –3 + 10	M1	3.1a
	So $-13 \le \text{Re}(w) \le 7$	A1ft	1.1b
		(2)	
		(8 marks)	

Notes:

(a)

M1: Obtains an equation in terms of *x* and *y* using the given information

A1: Expands and simplifies the algebra, collecting terms and obtains a circle equation correctly, deducing that this is a circle

M1: Completes the square for their equation to find centre and radius

A1ft: Both correct

(b)

M1: Draws a circle with centre and radius as given from **their** equation

A1: Correct circle drawn, as above, with centre at – 3 + 4i and passing through all four quadrants

(c)

M1: Attempts to find where a line parallel to the real axis, passing through the centre of the circle, meets the circle so using " their – 3 –10" to "their –3 + 10"

A1ft: Correctly obtains the correct answer for their centre and radius

Question	Scheme	Marks	AOs
4(a)(i)	<table><tr><td>*</td><td>0</td><td>2</td><td>3</td><td>4</td><td>5</td><td>6</td></tr><tr><td>0</td><td>0</td><td>2</td><td>3</td><td>4</td><td>5</td><td>6</td></tr><tr><td>2</td><td>2</td><td>0</td><td></td><td></td><td>4</td><td></td></tr><tr><td>3</td><td>3</td><td></td><td></td><td></td><td></td><td>5</td></tr><tr><td>4</td><td>4</td><td></td><td></td><td></td><td></td><td></td></tr><tr><td>5</td><td>5</td><td>4</td><td></td><td></td><td></td><td></td></tr><tr><td>6</td><td>6</td><td></td><td>5</td><td></td><td></td><td></td></tr></table>	M1	1.1b
	<table><tr><td>*</td><td>0</td><td>2</td><td>3</td><td>4</td><td>5</td><td>6</td></tr><tr><td>0</td><td>0</td><td>2</td><td>3</td><td>4</td><td>5</td><td>6</td></tr><tr><td>2</td><td>2</td><td>0</td><td>6</td><td>5</td><td>4</td><td>3</td></tr><tr><td>3</td><td>3</td><td>6</td><td>4</td><td>2</td><td>0</td><td>5</td></tr><tr><td>4</td><td>4</td><td>5</td><td>2</td><td>6</td><td>3</td><td>0</td></tr><tr><td>5</td><td>5</td><td>4</td><td>0</td><td>3</td><td>6</td><td>2</td></tr><tr><td>6</td><td>6</td><td>3</td><td>5</td><td>0</td><td>2</td><td>4</td></tr></table>	M1 A1	1.1b 1.1b
(ii)	Identity is zero and there is closure as shown above	M1	2.1
	3 and 5 are inverses, 4 and 6 are inverses, 2 is self-inverse, 0 is identity so is self-inverse	M1	2.5
	Associative law may be assumed so *S* forms a group	A1	1.1b
		(6)	
(b)	4*4*4 = 4* (4 * 4) = 4 * 6 or 4*4*4 = (4* 4) * 4 = 6* 4	M1	2.1
	= 0 (the identity) so 4 has order 3	A1	2.2a
		(2)	
(c)	3 and 5 each have order 6 so either generates the group	M1	3.1a
	Either $3^1 = 3,\ 3^2 = 4,\ 3^3 = 2,\ 3^4 = 6,\ 3^5 = 5,\ 3^6 = 0$ **Or** $5^1 = 5,\ 5^2 = 6,\ 5^3 = 2,\ 5^4 = 4,\ 5^5 = 3,\ 5^6 = 0$	A1, A1	1.1b 1.1b
		(3)	
		(11 marks)	

Question **4** notes:
(a)(i) **M1:** Begins completing the table – obtaining correct first row and first column and using symmetry **M1:** Mostly correct – three rows or three columns correct (so demonstrates understanding of using * **A1:** Completely correct
(a)(ii) **M1:** States closure and identifies the identity as zero **M1:** Finds inverses for each element **A1:** States that associative law is satisfied and so all axioms satisfied and *S* is a group
(b) **M1:** Clearly begins process to find 4*4*4 reaching 6*4 or 4*6 with clear explanation **A1:** Gives answer as zero, states identity and deduces that order is 3
(c) **M1:** Finds either 3 or 5 or both **A1:** Expresses four of the six terms as powers of either generator correctly (may omit identity and generator itself) **A1:** Expresses all six terms correctly in terms of either 3 or 5 (Do not need to give both)

Question	Scheme	Marks	AOs
5(a)	P_{n-1} is the population at the end of year n – 1 and this is increased by 10% by the end of year n, so is multiplied by 110% = 1.1 to give $1.1 \times P_{n-1}$ as new population by natural causes	B1	3.3
	Q is subtracted from $1.1 \times P_{n-1}$ as Q is the number of deer removed from the estate	B1	3.4
	So $P_n = 1.1P_{n-1} - Q, \quad P_0 = 5000$ as population at start is 5000 and $n \in \mathbb{Z}^+$	B1	1.1b
		(3)	
(b)	Let n = 0, then $P_0 = (5000 - 10Q)(1.1)^0 + 10Q = 5000$ so result is true when n = 0	B1	2.1
	Assume result is true for $n = k$, $P_k = (1.1)^k(5000 - 10Q) + 10Q$, then **as** $P_{k+1} = 1.1P_k - Q$, so $P_{k+1} = ...$	M1	2.4
	$P_{k+1} = 1.1 \times 1.1^k(5000 - 10Q) + 1.1 \times 10Q - Q$	A1	1.1b
	So $P_{k+1} = (5000 - 10Q)(1.1)^{k+1} + 10Q,$	A1	1.1b
	Implies result holds for $n = k + 1$ and so by induction $P_n = (5000 - 10Q)(1.1)^n + 10Q,$ is true for all integer n	B1	2.2a
		(5)	
(c)	For Q < 500 the population of deer will grow, for Q > 500 the population of deer will fall	B1	3.4
	For Q = 500 the population of deer remains steady at 5000,	B1	3.4
		(2)	
		(10 marks)	

Notes:

(a)
B1: Need to see 10% increase linked to multiplication by scale factor 1.1
B1: Needs to explain that subtraction of Q indicates the removal of Q deer from population
B1: Needs complete explanation with mention of $P_n = 1.1P_{n-1} - Q, \quad P_0 = 5000$ being the initial number of deer

(b)
B1: Begins proof by induction by considering n = 0
M1: Assumes result is true for $n = k$ and uses iterative formula to consider $n = k$ + 1
A1: Correct algebraic statement
A1: Correct statement for k + 1 in required form
B1: Completes the inductive argument

(c)
B1: Consideration of both possible ranges of values for Q as listed in the scheme
B1: Gives the condition for the steady state