Pearson Edexcel

Mark Scheme (Results)

October 2020

Pearson Edexcel GCE Further Mathematics
Advanced Subsidiary Level
in Further Pure 2 Paper 8FM0_22

Edexcel and BTEC Qualifications

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General Marking Guidance

- All candidates must receive the same treatment. Examiners must mark the last candidate in exactly the same way as they mark the first.
- Mark schemes should be applied positively. Candidates must be rewarded for what they have shown they can do rather than penalised for omissions.
- Examiners should mark according to the mark scheme not according to their perception of where the grade boundaries may lie.
- All the marks on the mark scheme are designed to be awarded. Examiners should always award full marks if deserved, i.e. if the answer matches the mark scheme. Examiners should also be prepared to award zero marks if the candidate's response is not worthy of credit according to the mark scheme.
- Where some judgement is required, mark schemes will provide the principles by which marks will be awarded and exemplification/indicative content will not be exhaustive.

EDEXCEL GCE MATHEMATICS

General Instructions for Marking

1. The total number of marks for the paper is 40.

2. The Edexcel Mathematics mark schemes use the following types of marks:
   - **M** marks: method marks are awarded for 'knowing a method and attempting to apply it', unless otherwise indicated.
   - **A** marks: Accuracy marks can only be awarded if the relevant method (M) marks have been earned.
   - **B** marks are unconditional accuracy marks (independent of M marks)
   - Marks should not be subdivided.

3. Abbreviations

These are some of the traditional marking abbreviations that will appear in the mark schemes.

- bod – benefit of doubt
- ft – follow through
- the symbol $\sqrt{}$ will be used for correct ft
- cao – correct answer only
- cso - correct solution only. There must be no errors in this part of the question to obtain this mark
- isw – ignore subsequent working
- awrt – answers which round to
- SC: special case
- oe – or equivalent (and appropriate)
- dep – dependent
- indep – independent
- dp decimal places
- sf significant figures
- * The answer is printed on the paper
- ⌈ The second mark is dependent on gaining the first mark

4. For misreading which does not alter the character of a question or materially simplify it, deduct two from any A or B marks gained, in that part of the question affected.

5. Where a candidate has made multiple responses <u>and indicates which response they wish to submit</u>, examiners should mark this response.
   If there are several attempts at a question <u>which have not been crossed out</u>, examiners should mark the final answer which is the answer that is the <u>most complete</u>.

6. Ignore wrong working or incorrect statements following a correct answer.

7. Mark schemes will firstly show the solution judged to be the most common response expected from candidates. Where appropriate, alternatives answers are provided in the notes. If examiners are not sure if an answer is acceptable, they will check the mark scheme to see if an alternative answer is given for the method used.

General Principles for Further Pure Mathematics Marking

(*But note that specific mark schemes may sometimes override these general principles*)

Method mark for solving 3 term quadratic:

1. Factorisation

$(x^2 + bx + c) = (x + p)(x + q)$, where $|pq| = |c|$, leading to $x = \ldots$

$(ax^2 + bx + c) = (mx + p)(nx + q)$, where $|pq| = |c|$ and $|mn| = |a|$, leading to $x = \ldots$

2. Formula

Attempt to use the correct formula (with values for *a*, *b* and *c*)

3. Completing the square

Solving $x^2 + bx + c = 0$: $\left(x \pm \frac{b}{2}\right)^2 \pm q \pm c = 0, \quad q \neq 0$, leading to $x = \ldots$

Method marks for differentiation and integration:

1. Differentiation

Power of at least one term decreased by 1. $(x^n \to x^{n-1})$

2. Integration

Power of at least one term increased by 1. $(x^n \to x^{n+1})$

Use of a formula

Where a method involves using a formula that has been learnt, the advice given in recent examiners' reports is that the formula should be quoted first.

Normal marking procedure is as follows:

Method mark for quoting a correct formula and attempting to use it, even if there are small errors in the substitution of values.

Where the formula is not quoted, the method mark can be gained by implication from correct working with values but may be lost if there is any mistake in the working.

Exact answers

Examiners' reports have emphasised that where, for example, an exact answer is asked for, or working with surds is clearly required, marks will normally be lost if the candidate resorts to using rounded decimals.

Question	Scheme	Marks	AOs
1(a)	1, 9, 11 and 19 are self-inverse	M1 A1	1.1b 1.1b
	<table><tr><td>3</td><td>7</td><td>13</td><td>17</td></tr><tr><td>7</td><td>3</td><td>17</td><td>13</td></tr></table>	B1	1.1b
		(3)	
(b)	<table><tr><td>1</td><td>3</td><td>7</td><td>9</td><td>11</td><td>13</td><td>17</td><td>19</td></tr><tr><td>1</td><td>4</td><td>4</td><td>2</td><td>2</td><td>4</td><td>4</td><td>2</td></tr></table>	M1 A1 A1	1.1b 1.1b 1.1b
		(3)	
(c)	{1, 3, 7, 9} or {1, 9, 13, 17} or {1, 9, 11, 19}	B1	2.5
		(1)	
(d)	Because 4 is a factor of 8	B1	2.4
		(1)	
			(8 marks)

Notes

(a)
M1: For any 2 of the self-inverse elements
A1: All 4 self-inverse elements correctly identified
B1: Correct inverses for the other elements
(b)
M1: At least 3 correct orders
A1: 6 correct orders
A1: All correct
(c)
B1: Describes a correct subgroup of order 4
(d)
B1: Correct explanation

Question	Scheme	Marks	AOs
2(a)	$963 = 657\times1+306 \quad 657 = 306\times2+45$	M1	1.2
	$306 = 45\times6+36$ $45 = 36\times1+9$ $36 = 9\times4+0$	A1	1.1b
	$\text{HCF}(963, 657) = 9$ or $c = 9$	A1	1.1b
		(3)	
(b)	$9 = 45-36\times1 \quad 9 = 45-(306-45\times6)$	M1	1.1b
	$9 = 7\times45-306$ $= 7\times(657-2\times306)-306 = 7\times657-15\times306$ $= 7\times657-15\times(963-1\times657)$	A1	2.1
	$9 = -15\times963+22\times657$	A1	1.1b
		(3)	
			(6 marks)

Notes

(a)
M1: Starts the process by showing 2 correct stages
A1: Completes the algorithm correctly
A1: Correct HCF
(b)
M1: Starts the reversal process by completing at least 2 stages
A1: Completes the algorithm correctly
A1: Correct values for *a* and *b*

Question	Scheme	Marks	AOs
3(i)(a)	$\begin{vmatrix} 1-\lambda & -2 \\ 1 & 4-\lambda \end{vmatrix} = (1-\lambda)(4-\lambda)+2=0$	M1	1.1b
	$\Rightarrow 4-5\lambda+\lambda^2+2=0 \Rightarrow \lambda^2-5\lambda+6=0$*	A1*	1.1b
		(2)	
(b)	$\mathbf{A}^2 - 5\mathbf{A} + 6\mathbf{I} = 0$	M1	1.1b
	$\mathbf{A}^3 - 5\mathbf{A}^2 + 6\mathbf{A} = 0 \Rightarrow \mathbf{A}^3 = 5(5\mathbf{A} - 6\mathbf{I}) - 6\mathbf{A}$	M1	3.1a
	$\mathbf{A}^3 = 19\mathbf{A} - 30\mathbf{I}$	A1	1.1b
		(3)	
	Alternative to part (b)		
	$\lambda^2-5\lambda+6=0 \Rightarrow \lambda^3-5\lambda^2+6\lambda=0 \Rightarrow \lambda^3=5(5\lambda-6)-6\lambda$	M1	3.1a
	$\mathbf{A}^3 = 5(5\mathbf{A} - 6\mathbf{I}) - 6\mathbf{A}$	M1	1.1b
	$\mathbf{A}^3 = 19\mathbf{A} - 30\mathbf{I}$	A1	1.1b
		(3)	
(ii)	$\mathbf{M} = \begin{pmatrix} a & b \\ c & d \end{pmatrix} \Rightarrow \begin{pmatrix} a & b \\ c & d \end{pmatrix}\begin{pmatrix} 1 \\ 2-\mathrm{i} \end{pmatrix} = (-1+\mathrm{i})\begin{pmatrix} 1 \\ 2-\mathrm{i} \end{pmatrix}$ or $\mathbf{M} = \begin{pmatrix} a & b \\ c & d \end{pmatrix} \Rightarrow \begin{pmatrix} a & b \\ c & d \end{pmatrix}\begin{pmatrix} 1 \\ 2+\mathrm{i} \end{pmatrix} = (-1-\mathrm{i})\begin{pmatrix} 1 \\ 2+\mathrm{i} \end{pmatrix}$	M1	1.1b
	$a+b(2-\mathrm{i})=-1+\mathrm{i} \quad a+b(2+\mathrm{i})=-1-\mathrm{i}$ $c+d(2-\mathrm{i})=-1+3\mathrm{i} \quad c+d(2+\mathrm{i})=-1-3\mathrm{i}$	A1	1.1b
	$a+b(2-\mathrm{i})=-1+\mathrm{i},\ a+b(2+\mathrm{i})=-1-\mathrm{i} \Rightarrow a=1,\ b=-1$ or $c+d(2-\mathrm{i})=-1+3\mathrm{i},\ c+d(2+\mathrm{i})=-1-3\mathrm{i} \Rightarrow c=5,\ d=-3$	M1 A1	3.1a 1.1b
	$\mathbf{M} = \begin{pmatrix} 1 & -1 \\ 5 & -3 \end{pmatrix}$	A1	2.2a
		(5)	
	Alternative to part (ii)		
	$\mathbf{P} = \begin{pmatrix} 1 & 1 \\ 2-\mathrm{i} & 2+\mathrm{i} \end{pmatrix} \Rightarrow \mathbf{P}^{-1} = \frac{1}{2\mathrm{i}}\begin{pmatrix} 2+\mathrm{i} & -1 \\ \mathrm{i}-2 & 1 \end{pmatrix}$	M1 A1	1.1b 1.1b
	$\mathbf{D} = \mathbf{P}^{-1}\mathbf{MP} \Rightarrow \mathbf{M} = \mathbf{PDP}^{-1}$ $\mathbf{M} = \frac{1}{2\mathrm{i}}\begin{pmatrix} 1 & 1 \\ 2-\mathrm{i} & 2+\mathrm{i} \end{pmatrix}\begin{pmatrix} -1+\mathrm{i} & 0 \\ 0 & -1-\mathrm{i} \end{pmatrix}\begin{pmatrix} 2+\mathrm{i} & -1 \\ \mathrm{i}-2 & 1 \end{pmatrix} = \ldots$	M1	3.1a
	$\mathbf{M} = \begin{pmatrix} 1 & -1 \\ 5 & -3 \end{pmatrix}$	A1 A1	1.1b 2.2a
		(5)	
		(10 marks)	
	Notes		

(i)(a)
M1: Attempts the determinant of **A** – λ**I**
A1*: Fully correct proof
(i)(b)
M1: Applies the Cayley-Hamilton theorem to the equation given in (a)(i)
M1: A full method leading to $\mathbf{A}^3$ by multiplying by **A** and substituting for $\mathbf{A}^2$
A1: Deduces the correct expression or correct values for *p* and *q*
Alternative
M1: A full method leading to λ^3 in terms of λ
M1: Applies the Cayley-Hamilton theorem
A1: Deduces the correct expression or correct values for *p* and *q*
(ii)
M1: Uses a general matrix and sets up at least one matrix equation using the information given in the question
A1: Correct equations in terms of *a*, *b*, *c* and *d*
M1: Solves simultaneously to find values for all of *a*, *b*, *c* and *d*
A1: One correct pair of values
A1: Deduces the correct matrix **M**
Alternative:
M1: Attempts to find the inverse of the matrix of eigenvectors
A1: Correct matrix
M1: Attempts $\mathbf{PDP}^{-1}$ where **D** is the diagonal matrix of eigenvalues
A1: At least 2 elements correct
A1: Deduces the correct matrix **M**

Question	Scheme	Marks	AOs
4(a)	$A_{n+1} = 1.05A_n - F$	B1	3.3
		(1)	
(b)	$n = 1 \Rightarrow A_1 = (10\ 000 - 20F)1.05^{1-1} + 20F$ $n = 1 \Rightarrow A_1 = 10\ 000 - 20F + 20F = 10\,000$ So true for $n = 1$	B1	2.1
	Assume true for $n = k$ so that $A_k = (10\ 000 - 20F)1.05^{k-1} + 20F$ $A_{k+1} = ...$	M1	2.4
	$A_{k+1} = 1.05((10\ 000 - 20F)1.05^{k-1} + 20F\ ) - F$	A1ft	1.1b
	$A_{k+1} = (10\ 000 - 20F)1.05^k + 21F - F$ $= (10\ 000 - 20F)1.05^k + 20F$	A1	1.1b
	So the result holds for $n = k + 1$ and so $A_n = (10\ 000 - 20F)1.05^{n-1} + 20F$ is true for all $n \geqslant 1$	A1	2.2a
		(5)	
(c)	$(10\ 000 - 20F)1.05^{16-1} + 20F\ \leqslant\ 0$	M1	3.1b
	$10\ 000 \times 1.05^{15}\ \leqslant 20F(1.05^{15} - 1) \Rightarrow F \geqslant ...$	M1	2.1
	$F \geqslant \dfrac{10\ 000 \times 1.05^{15}}{20(1.05^{15} - 1)}$	A1	1.1b
	So the smallest value of F is £963.43 or So the smallest value of F is £964	A1	3.2a
		(4)	
			(10 marks)

Notes

(a)
B1: Correct equation
(b)
B1: Demonstrates that the result is true for $n = 1$
M1: Makes a statement that assumes the result is true for some value of n
A1ft: Correct expression for A_{k+1}
A1: Reaches the correct statement for $k + 1$ in the required form
A1: Completes the inductive argument
(c)
M1: Identifies a correct strategy using $n = 16$ to obtain an equation in F
M1: Proceeds to obtain a value for the minimum value of F
A1: Correct numerical expression for F
A1: Correct answer (allow to the nearest penny or nearest £)

Question	Scheme	Marks	AOs
5(a)	$p=\frac{\pi}{4}$ and $q=\pi$ or $\frac{\pi}{4}\le \arg z\le \pi$	B1	1.1b
	$r=2$ or $\lvert z\rvert \le 2$	B1	1.1b
		(2)	
(b)	Position for z is at intersection in quadrant 1	B1	3.1a
	Angle between $y = x$ and "OW" is $\frac{\pi}{3}+\frac{\pi}{4}$	B1	2.2a
	$d^2=4^2+2^2-2\times4\times2\cos\left(\frac{\pi}{3}+\frac{\pi}{4}\right)$	M1	1.1b
	$=20-4\sqrt{2}+4\sqrt{6}$	A1	2.2a
		(4)	
	Alternative for (b):		
	Position for z is at intersection in quadrant 1	B1	3.1a
	$y = x$ intersects $x^2 + y^2 = 4$ at $\left(\sqrt{2},\sqrt{2}\right)$	B1	2.2a
	$d^2=\left("\sqrt{2}"+2\sqrt{3}\right)^2+\left("\sqrt{2}"-2\right)^2$	M1	1.1b
	$=20-4\sqrt{2}+4\sqrt{6}$	A1	2.2a
		(4)	
			(6 marks)

Notes

(a)

B1: Correct values or correct loci

B1: Correct value or correct loci

(b)

B1: Realises that the position for z is at the intersection in quadrant 1 and makes progress in finding the distance

B1: Deduces from the information given that the angle between $y = x$ and "OW" is $\pi/4+\pi/3$

M1: Correct use of the cosine rule in order to find the required length^2

A1: Deduces the required value in a simplified and exact form

Alternative:

B1: Realises that the position for z is at the intersection in quadrant 1 and makes progress in finding the distance

B1: Deduces from the information given the correct coordinates for z

M1: Correct use of Pythagoras in order to find the required length^2

A1: Deduces the required value in a simplified and exact form