Mark Scheme (Results)

Summer 2018

Pearson Edexcel GCE Further Mathematics
AS Further Pure FP2 Paper 8FM0_22

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General Marking Guidance

- All candidates must receive the same treatment. Examiners must mark the last candidate in exactly the same way as they mark the first.
- Mark schemes should be applied positively. Candidates must be rewarded for what they have shown they can do rather than penalised for omissions.
- Examiners should mark according to the mark scheme not according to their perception of where the grade boundaries may lie.
- All the marks on the mark scheme are designed to be awarded. Examiners should always award full marks if deserved, i.e. if the answer matches the mark scheme. Examiners should also be prepared to award zero marks if the candidate's response is not worthy of credit according to the mark scheme.
- Where some judgement is required, mark schemes will provide the principles by which marks will be awarded and exemplification/indicative content will not be exhaustive.

EDEXCEL GCE MATHEMATICS

General Instructions for Marking

1. The total number of marks for the paper is 40.

2. The Edexcel Mathematics mark schemes use the following types of marks:

- **M** marks: method marks are awarded for 'knowing a method and attempting to apply it', unless otherwise indicated.
- **A** marks: Accuracy marks can only be awarded if the relevant method (M) marks have been earned.
- **B** marks are unconditional accuracy marks (independent of M marks)
- Marks should not be subdivided.

3. Abbreviations

These are some of the traditional marking abbreviations that will appear in the mark schemes.

- bod – benefit of doubt
- ft – follow through
- the symbol $\sqrt{}$ will be used for correct ft
- cao – correct answer only
- cso - correct solution only. There must be no errors in this part of the question to obtain this mark
- isw – ignore subsequent working
- awrt – answers which round to
- SC: special case
- oe – or equivalent (and appropriate)
- dep – dependent
- indep – independent
- dp decimal places
- sf significant figures
- $\ast$ The answer is printed on the paper
- $|$ The second mark is dependent on gaining the first mark

4. For misreading which does not alter the character of a question or materially simplify it, deduct two from any A or B marks gained, in that part of the question affected.

5. Where a candidate has made multiple responses <u>and indicates which response they wish to submit</u>, examiners should mark this response.
If there are several attempts at a question <u>which have not been crossed out</u>, examiners should mark the final answer which is the answer that is the <u>most complete</u>.

6. Ignore wrong working or incorrect statements following a correct answer.

7. Mark schemes will firstly show the solution judged to be the most common response expected from candidates. Where appropriate, alternatives answers are provided in the notes. If examiners are not sure if an answer is acceptable, they will check the mark scheme to see if an alternative answer is given for the method used.

Question	Scheme	Marks	AOs
1(i)	$2 - 3 + 7 - 3 + 8 = \ldots$ or $2 + 7 + 8 - (3 + 3) = \ldots$	M1	1.1b
	$= 11$ so 23 738 is divisible by 11	A1	1.1b
		(2)	
(ii)	$2322 = 3\times 654 + 360, \quad 654 = 1\times 360 + 294$	M1	1.2
	$360 = 1\times 294 + 66, \quad 294 = 4\times 66 + 30$		
	$66 = 2\times 30 + 6, \quad 30 = 5\times 6 + 0$	A1	1.1b
	So HCF(2 322, 654) = 6	A1	1.1b
		(3)	
			(5 marks)

Notes

(i)
M1: Executes the correct process by adding and subtracting alternating digits or equivalent
A1: Completes correctly with a correct conclusion
(ii)
M1: Uses the Euclidean algorithm showing two stages (Must be Euclidean algorithm not e.g. using prime factors)
A1: Completes the algorithm correctly
A1: All correct and concludes HCF is 6

<table>
<tr><th>Question</th><th>Scheme</th><th>Marks</th><th>AOs</th></tr>
<tr><td>2(a)(i)</td><td>
<table>
<tr><td>*</td><td>I</td><td>X</td><td>Y</td><td>Z</td><td>R_1</td><td>R_2</td></tr>
<tr><td>I</td><td>$\boldsymbol{I}$</td><td>$\boldsymbol{X}$</td><td>$\boldsymbol{Y}$</td><td>$\boldsymbol{Z}$</td><td>$\boldsymbol{R_1}$</td><td>$\boldsymbol{R_2}$</td></tr>
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<tr><td>Y</td><td>$\boldsymbol{Y}$</td><td></td><td></td><td></td><td></td><td></td></tr>
<tr><td>Z</td><td>$\boldsymbol{Z}$</td><td></td><td></td><td></td><td></td><td></td></tr>
<tr><td>R_1</td><td>$\boldsymbol{R_1}$</td><td></td><td></td><td></td><td></td><td></td></tr>
<tr><td>R_2</td><td>$\boldsymbol{R_2}$</td><td></td><td></td><td></td><td></td><td></td></tr>
</table>
</td><td>B1</td><td>1.1b</td></tr>
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<tr><td>R_2</td><td>$\boldsymbol{R_2}$</td><td>$\boldsymbol{Z}$</td><td>$\boldsymbol{X}$</td><td>$\boldsymbol{Y}$</td><td>$\boldsymbol{I}$</td><td>$\boldsymbol{R_1}$</td></tr>
</table>
</td><td>B1
B1</td><td>1.1b
1.1b</td></tr>
<tr><td>(ii)</td><td>I is the identity and closure is shown (by the Cayley table)</td><td>M1</td><td>2.1</td></tr>
<tr><td></td><td>X, Y and Z are self-inverse, R_1 and R_2 are inverses, (I is the identity so is self-inverse)</td><td>M1</td><td>2.5</td></tr>
<tr><td></td><td>(Associative law may be assumed) so T forms a group</td><td>A1</td><td>1.1b</td></tr>
<tr><td></td><td></td><td>(6)</td><td></td></tr>
<tr><td>(b)</td><td>$R_2 * R_2 * R_2 = (R_2 * R_2) * R_2$ or $R_2 * (R_2 * R_2) = R_1 * R_2$ or $R_2 * R_1$</td><td>M1</td><td>2.1</td></tr>
<tr><td></td><td>$= I$ (the identity) so R_2 has order 3</td><td>A1</td><td>2.2a</td></tr>
<tr><td></td><td></td><td>(2)</td><td></td></tr>
<tr><td>(c)</td><td>R_1 (and R_2) have order 3, X, Y and Z have order 2 so:
There is no element of T that generates the group
or
There is no element of order 6</td><td>B1</td><td>2.4</td></tr>
<tr><td></td><td></td><td>(1)</td><td></td></tr>
<tr><td>(d)</td><td>$\{I, R_1, R_2\}$</td><td>B1</td><td>1.1b</td></tr>
<tr><td></td><td></td><td>(1)</td><td></td></tr>
<tr><td colspan="4" align="right">(10 marks)</td></tr>
<tr><td colspan="4" align="center">Notes</td></tr>
<tr><td colspan="4">(a)(i)
B1: Begins completing the table by having at least the first row and first column correct
B1: Mostly correct – three rows or three columns correct (so demonstrates an understanding of using *)
B1: Fully correct table
(a)(ii)
M1: States closure and identifies the identity as I
M1: States the inverse of each element (reference to the Identity not required here)
A1: Concludes that T is a group (must see a conclusion)
Special case: If the inverses are not stated explicitly but a statement such as “all elements have an inverse” is seen, score M1M1A0
(b)
M1: Clearly begins process to find $R_2 * R_2 * R_2$ reaching $R_1 * R_2$ or $R_2 * R_1$
A1: Gives answer as I states identity and deduces that the order is 3 or e.g. $(R_2)^3 = I$
(c)
B1: Demonstrates an understanding of the term cyclic by referring to the order of R_1, X, Y and Z and makes a suitable conclusion
(d)
B1: Indicates the set $\{I, R_1, R_2\}$ (Brackets not required)</td></tr>
</table>

Question	Scheme	Marks	AOs
3(a)	H_n is the measured height at the start of year n and this is decreased by 3% at the start of year n, so is multiplied by 97% = 0.97 to give 0.97 H_n as the new height due to trimming	B1	3.3
	0.15 is added to 0.97 H_n as 0.15 is 15 cm in m and this is how much the tree grows in a year.	B1	3.4
	And H_1 = 6 is the height of the tree at the start of year 1 before trimming	B1	1.1b
		(3)	
(b)	$n=1 \Rightarrow H_1 = (0.97)^{1-1} + 5 = 6$ So true for n = 1	B1	2.1
	Assume true for $n = k$ so $H_k = (0.97)^{k-1} + 5$ so $H_{k+1} = 0.97\left((0.97)^{k-1} + 5\right) + 0.15$	M1	2.4
	so $H_{k+1} = (0.97)^k + 4.85 + 0.15 = (0.97)^k + 5$	A1	1.1b
	<u>If true for $n = k$</u> then <u>true for $n = k + 1$</u>, <u>true for $n = 1$</u> so <u>true for all (positive integers) n</u> (Allow "for all values")	B1	2.2a
		(4)	
(c)	The height will approach 5m	B1	1.1b
		(1)	
(d)	Require 4 = 4x + 0.15	M1	3.1b
	x = 0.9625 so 3.75%	A1	1.1b
		(2)	
			(10 marks)

Notes

(a)
B1: Need to see 3% decrease linked to scale factor of 0.97
B1: Need to see that adding 0.15 corresponds to the yearly growth **in metres**. There must be some reference to the units for this mark.
B1: An explanation that H_1 is the first term (the starting height) and this is 6m
(b)
B1: Begins proof by induction by considering n = 1 and obtains H_1 = 6
M1: Assumes true for $n = k$ and uses iterative formula to consider $n = k + 1$
A1: Reaches $(0.97)^k + 5$ with no errors
B1: Correct conclusion. This mark is dependent on all previous marks apart from the first B mark. It is gained by conveying the ideas of **all** four underlined points **either** at the end of their solution **or** as a narrative in their solution.
(c)
B1: States the height will approach 5m
(d)
M1: Uses the model to adopt a correct strategy to find the required percentage
A1: Interprets their answer correctly in terms of the original context

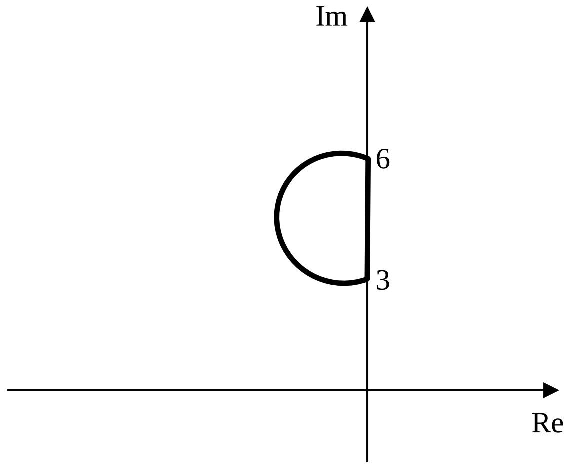
Question	Scheme	Marks	AOs
4	$\lvert\mathbf{A}-\lambda\mathbf{I}\rvert=0 \Rightarrow \begin{vmatrix}1-\lambda & 1 \\ -2 & 4-\lambda\end{vmatrix}=0 \Rightarrow (1-\lambda)(4-\lambda)+2=0$	M1	3.1a
	$\lambda_1=2,\quad \lambda_2=3$	A1	1.1b
	$\begin{pmatrix}1 & 1 \\ -2 & 4\end{pmatrix}\begin{pmatrix}x \\ y\end{pmatrix}=2\begin{pmatrix}x \\ y\end{pmatrix}$ or $\begin{pmatrix}1 & 1 \\ -2 & 4\end{pmatrix}\begin{pmatrix}x \\ y\end{pmatrix}=3\begin{pmatrix}x \\ y\end{pmatrix}$	M1	2.1
	$2,\begin{pmatrix}1 \\ 1\end{pmatrix}$ or $3,\begin{pmatrix}1 \\ 2\end{pmatrix}$	A1	1.1b
	$2,\begin{pmatrix}1 \\ 1\end{pmatrix}$ and $3,\begin{pmatrix}1 \\ 2\end{pmatrix}$	A1	1.1b
	$\mathbf{D}=\begin{pmatrix}2 & 0 \\ 0 & 3\end{pmatrix}$	B1ft	1.1b
	$\mathbf{P}=\begin{pmatrix}1 & 1 \\ 1 & 2\end{pmatrix}$	B1ft	2.2a
		(7)	
		(7 marks)	

Notes

M1: Correct strategy for finding eigenvalues
A1: Correct eigenvalues
M1: Uses at least one of their eigenvalues correctly to find a corresponding eigenvector
A1: One correct eigenvalue/eigenvector pair
A1: Both pairs correct
B1ft: Correct follow through **D** or **P** clearly identified as **D** or **P**
B1ft: **P** and **D** both correct and consistent and identified as **D** and **P**

Note that the correct matrices may be implied by e.g.

$$\begin{pmatrix}* & * \\ * & *\end{pmatrix}\begin{pmatrix}1 & 1 \\ -2 & 4\end{pmatrix}\begin{pmatrix}1 & 1 \\ 1 & 2\end{pmatrix}=\begin{pmatrix}2 & 0 \\ 0 & 3\end{pmatrix}$$

Question	Scheme	Marks	AOs
5(a)		M1 A1 A1	3.1a 1.1b 1.1b
		(3)	
(b)	y-coordinate of centre of circle is 4.5	B1	1.1b
	x-coordinate of centre of circle is $-\frac{1.5}{\tan\frac{\pi}{3}}\left(=-\frac{\sqrt{3}}{2}\right)$	M1	3.1a
	Radius of circle is $\frac{1.5}{\sin\frac{\pi}{3}}$ or $\sqrt{1.5^2+\left(\frac{1.5}{\tan\frac{\pi}{3}}\right)^2}$	M1	1.1b
	$d=\sqrt{4.5^2+0.75}+\sqrt{3}$	M1	3.1a
	$d=\sqrt{21}+\sqrt{3}$	A1	1.1b
		(5)	
			(8 marks)

Notes

(a)
M1: Interprets the locus correctly as a circle or as an arc of a circle
A1: A circle or an arc of a circle passing through or touching at 3 and 6 on the positive imaginary axis.
A1: Correct diagram – a major arc that is wholly to the left of the imaginary axis and wholly above the real axis with 3 and 6 marked on the imaginary axis
(b)
B1: Correct y-coordinate of the centre
M1: Correct strategy for finding the x-coordinate of the centre
M1: Correct strategy for finding the radius of the circle
M1: Fully correct method for the maximum using their values
A1: Correct value