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Surname MODEL SOLUTIONS

Other names

Pearson Edexcel Level 3 GCE

Centre Number

Candidate Number

Further Mathematics

Advanced Subsidiary
Further Mathematics options
Further Pure Mathematics 2

Sample Assessment Material for first teaching September 2017

Time: 50 minutes

Paper Reference

8FM0/2A

You must have:

Mathematical Formulae and Statistical Tables, calculator

Total Marks

Candidates may use any calculator permitted by Pearson regulations. Calculators must not have the facility for algebraic manipulation, differentiation and integration, or have retrievable mathematical formulae stored in them.

Instructions

- Use **black** ink or ball-point pen.
- If pencil is used for diagrams/sketches/graphs it must be dark (HB or B).
- **Fill in the boxes** at the top of this page with your name, centre number and candidate number.
- Answer the questions in the spaces provided – *there may be more space than you need.*
- You should show sufficient working to make your methods clear. Answers without working may not gain full credit.
- Answers should be given to three significant figures unless otherwise stated.

Information

- A booklet 'Mathematical Formulae and Statistical Tables' is provided.
- There are 5 questions in this question paper. The total mark for this paper is 40.
- The marks for **each** question are shown in brackets – *use this as a guide as to how much time to spend on each question.*

Advice

- Read each question carefully before you start to answer it.
- Try to answer every question.
- Check your answers if you have time at the end.

Turn over ▶

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Pearson

Answer ALL questions. Write your answers in the spaces provided.

1. Given that

$$\mathbf{A} = \begin{pmatrix} 3 & 1 \\ 6 & 4 \end{pmatrix}$$

(a) find the characteristic equation of the matrix **A**.

(2)

(b) Hence show that $\mathbf{A}^3 = 43\mathbf{A} - 42\mathbf{I}$.

(3)

a) $A - \lambda I = \begin{pmatrix} 3 & 1 \\ 6 & 4 \end{pmatrix} - \lambda \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$

$= \begin{pmatrix} 3 & 1 \\ 6 & 4 \end{pmatrix} - \begin{pmatrix} \lambda & 0 \\ 0 & \lambda \end{pmatrix}$

$= \begin{pmatrix} 3-\lambda & 1 \\ 6 & 4-\lambda \end{pmatrix}$

$\det(A - \lambda I) = 0$

$\Rightarrow (3-\lambda)(4-\lambda) - (6)(1) = 0$

$12 - 7\lambda + \lambda^2 - 6 = 0$

$\boxed{\lambda^2 - 7\lambda + 6 = 0}$

$\Rightarrow (\lambda - 6)(\lambda - 1) = 0$

$\Rightarrow \lambda = 6 \quad \lambda = 1$ are eigenvalues of A

not required

b) from (a) using the Cayley-Hamilton theorem...

$A^2 - 7A + 6I = 0$

$A^2 = 7A - 6I$

$\times 7$ $\quad 7A^2 = 49A - 42I$

$-6A$ $\quad 7A^2 - 6A = 43A - 42I$

consider $7A^2 - 6A$, $A^2 = \begin{pmatrix} 3 & 1 \\ 6 & 4 \end{pmatrix}\begin{pmatrix} 3 & 1 \\ 6 & 4 \end{pmatrix} = \begin{pmatrix} 9+6 & 3+4 \\ 18+24 & 6+16 \end{pmatrix}$

$A^2 = \begin{pmatrix} 15 & 7 \\ 42 & 22 \end{pmatrix}$

Question 1 continued

$$7A^2 = 7\begin{pmatrix} 15 & 7 \\ 42 & 22 \end{pmatrix}$$

$$= \begin{pmatrix} 105 & 49 \\ 294 & 154 \end{pmatrix}$$

$$6A = \begin{pmatrix} 18 & 6 \\ 36 & 24 \end{pmatrix}$$

$$\text{so } 7A^2 - 6A = \begin{pmatrix} 105 & 49 \\ 294 & 154 \end{pmatrix} - \begin{pmatrix} 18 & 6 \\ 36 & 24 \end{pmatrix}$$

$$= \begin{pmatrix} 87 & 43 \\ 258 & 130 \end{pmatrix}$$

$$\text{and } A^3 = \begin{pmatrix} 15 & 7 \\ 42 & 22 \end{pmatrix}\begin{pmatrix} 3 & 1 \\ 6 & 4 \end{pmatrix}$$

$$= \begin{pmatrix} 45+42 & 15+28 \\ 126+132 & 42+88 \end{pmatrix}$$

$$= \begin{pmatrix} 87 & 43 \\ 258 & 130 \end{pmatrix}$$

$$\text{hence } A^3 = 43A - 42I$$

(Total for Question 1 is 5 marks)

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2. (i) Without performing any division, explain why 8184 is divisible by 6

(2)

(ii) Use the Euclidean algorithm to find integers a and b such that

$$27a + 31b = 1$$

(4)

i) A number is divisible by 6 if it is divisible by both 2 & 3 :
- 8184 ends with 4 so it is an even number ∴ divisible by 2
- sum of digits : $8+1+8+4 = 21$
  $2+1 = 3$ → divisible by 3 ∴ 8184 is divisible by 3

⇒ 8184 is divisible by both 2 & 3 so it is also divisible by 6.

ii) $31 = 27(1) + 4$

$27 = 4(6) + 3$

$4 = 3(1) + \boxed{1}$ hcf

$3 = 3(1) + 0$

so hcf $(31, 27) = 1$ hence $27a + 31b = 1$

back substitution : $1 = 4 - 3$

$= 4 - [27 - 4(6)]$

$= (31-27) - (27) + (31-27)(6)$

$= 31 - 2(27) + 6(31) - 6(27)$

$= 7(31) - 8(27)$

so $a = -8$ and $b = 7$

Question 2 continued

(Total for Question 2 is 6 marks)

3. A curve C is described by the equation

$$|z - 9 + 12\mathrm{i}| = 2|z|$$

(a) Show that C is a circle, and find its centre and radius.

(4)

(b) Sketch C on an Argand diagram.

(2)

Given that w lies on C,

(c) find the largest value of a and the smallest value of b that must satisfy

$$a \leqslant \mathrm{Re}(w) \leqslant b$$

(2)

a) $|z - 9 + 12i| = |2z|$

$|x + iy - 9 + 12i| = 2|x + iy|$

$|(x-9) + (y+12)i| = 2|x+iy|$

$(x-9)^2 + (y+12)^2 = 2^2(x^2+y^2)$

$x^2 - 18x + 81 + y^2 + 24y + 144 = 4x^2 + 4y^2$

$3x^2 + 3y^2 + 18x - 24y = 225$

$\div 3$

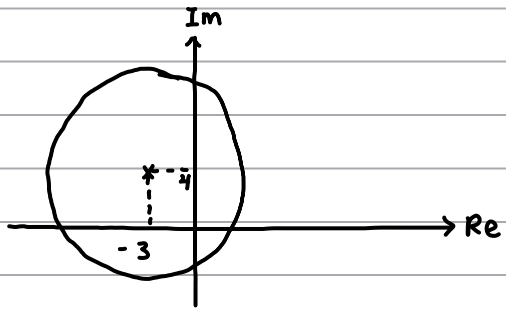
$x^2 + y^2 + 6x - 8y = 75$

$(x+3)^2 - 9 + (y-4)^2 - 16 = 75$

$(x+3)^2 + (y-4)^2 = 100$

centre : $(-3, 4)$
radius : $\sqrt{100} = 10$

b)



c) We want the points on the circle with the greatest / lowest real components

so ... $a = -3 - 10 = -13$

$b = -3 + 10 = 7$

real component of centre

radius

$\therefore a = -13$
$b = 7$

Question 3 continued

(Total for Question 3 is 8 marks)

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4. The operation * is defined on the set $S = \{0, 2, 3, 4, 5, 6\}$ by $x*y = x + y - xy \pmod 7$

*	0	2	3	4	5	6
0	0	2	3	4	5	6
2	2	0	6	5	4	3
3	3	6	4	2	0	5
4	4	5	2	6	3	0
5	5	4	0	3	6	2
6	6	3	5	0	2	4

(a) (i) Complete the Cayley table shown above

(ii) Show that S is a group under the operation *

(You may assume the associative law is satisfied.)

(6)

(b) Show that the element 4 has order 3

(2)

(c) Find an element which generates the group and express each of the elements in terms of this generator.

(3)

aii) Identity ✓: 0 is the identity

Closure ✓: all values in the table ∈ S so there is closure

Inverses ✓: 0 is self-inverse (identity)

2 is self-inverse

3 & 5 are inverses of each other

4 & 6 are inverses of each other

Associativity ✓: assumed

so... yes, S is a group under the operation *

Question 4 continued

b) $4 * 4 = 4+4 - (4)(4) \pmod 7$
$= 8 - 16 \pmod 7$
$= -8 \pmod 7$
$= 6 \pmod 7$

$4*4*4 = 4*(4*4) = (4)*(6)$
$= 4+6 - (4)(6) \pmod 7$
$= 10 - 24 \pmod 7$
$= -14 \pmod 7$
$= 0 \pmod 7$
identity $\therefore$ 4 has order 3

c) $2 * 2 = 0$ so 2 can't be the generator
try 3: $3*3 = 4$
$3*3*3 = 3*4 = 2$
$3*3*3*3 = 3*2 = 6$
$3*3*3*3*3 = 3*6 = 5$
$3*3*3*3*3*3 = 3*5 = 0$

so 3 is the generator.
$3^1 = 3$
$3^2 = 4$
$3^3 = 2$
$3^4 = 6$
$3^5 = 5$
$3^6 = 0$

(Total for Question 4 is 11 marks)

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5. A population of deer on a large estate is assumed to increase by 10% during each year due to natural causes.

The population is controlled by removing a constant number, Q, of the deer from the estate at the end of each year.

At the start of the first year there are 5000 deer on the estate.

Let P_n be the population of deer at the end of year n.

(a) Explain, in the context of the problem, the reason that the deer population is modelled by the recurrence relation

$$P_n = 1.1P_{n-1} - Q, \qquad P_0 = 5000, \qquad n \in \mathbb{Z}^+$$

(3)

(b) Prove by induction that $P_n = (1.1)^n (5000 - 10Q) + 10Q, \qquad n \geqslant 0$

(5)

(c) Explain how the long term behaviour of this population varies for different values of Q.

(2)

a) · $P_0 = 5000$ as at the start of the first year there are 5000 deers
· we have $1.1\,P_{n-1}$ as population increases by 10% each year. This is equivalent to multiplying by 1.1. P_{n-1} is the population at the end of year $n-1$.
· each year Q deers are removed so we subtract Q from $1.1\,P_{n-1}$ giving us $P_n = 1.1P_{n-1} - Q$

b) we want to prove $P_{k+1} = (1.1)^{k+1}(5000 - 10Q) + 10Q$

$n=0$: $P_0 = (1.1)^0(5000 - 10Q) + 10Q$
$= 5000 - 10Q + 10Q$
$= 5000$

$\therefore$ true when $n=0$

Now assume true for $n=k$:

$P_k = (1.1)^k(5000 - 10Q) + 10Q$

consider $n=k+1$: $P_{k+1} = 1.1\,P_k - Q$
$P_{k+1} = 1.1\left[(1.1)^k(5000 - 10Q) + 10Q\right] - Q$
$= (1.1)^{k+1}(5000 - 10Q) + 11Q - Q$
$= (1.1)^{k+1}(5000 - 10Q) + 10Q$

$\therefore$ true for $n = k+1$

Question 5 continued

· solution is true for $n=0$

· when assumed true for $n=k$, it is also true for $n=k+1$

So by mathematical induction it is true for all $n \in \mathbb{Z}^+$

c) when $Q = 500$, $5000 - 10Q = 0$

- so at $Q = 500$ the population will remain steady at 5000
- when $Q > 500$ the population will fall (since $5000 - 10Q < 0$)
- when $Q < 500$ the population will increase (since $5000 - 10Q > 0$)

Question 5 continued

Question 5 continued

(Total for Question 5 is 10 marks)

TOTAL FOR PAPER IS 40 MARKS