Pearson Edexcel

Mark Scheme

Summer 2023

Pearson Edexcel GCE

A Level Further Mathematics (9FM0)

Paper 3A Further Pure Mathematics 1

Edexcel and BTEC Qualifications

Edexcel and BTEC qualifications are awarded by Pearson, the UK's largest awarding body. We provide a wide range of qualifications including academic, vocational, occupational and specific programmes for employers. For further information visit our qualifications websites at www.edexcel.com or www.btec.co.uk. Alternatively, you can get in touch with us using the details on our contact us page at www.edexcel.com/contactus.

Pearson: helping people progress, everywhere

Pearson aspires to be the world's leading learning company. Our aim is to help everyone progress in their lives through education. We believe in every kind of learning, for all kinds of people, wherever they are in the world. We've been involved in education for over 150 years, and by working across 70 countries, in 100 languages, we have built an international reputation for our commitment to high standards and raising achievement through innovation in education. Find out more about how we can help you and your students at: www.pearson.com/uk

Summer 2023
Publications Code 9FM0_3A_2306_MS*

General Marking Guidance

- All candidates must receive the same treatment. Examiners must mark the first candidate in exactly the same way as they mark the last.
- Mark schemes should be applied positively. Candidates must be rewarded for what they have shown they can do rather than penalised for omissions.
- Examiners should mark according to the mark scheme not according to their perception of where the grade boundaries may lie.
- There is no ceiling on achievement. All marks on the mark scheme should be used appropriately.
- All the marks on the mark scheme are designed to be awarded. Examiners should always award full marks if deserved, i.e. if the answer matches the mark scheme. Examiners should also be prepared to award zero marks if the candidate's response is not worthy of credit according to the mark scheme.
- Where some judgement is required, mark schemes will provide the principles by which marks will be awarded and exemplification may be limited.
- When examiners are in doubt regarding the application of the mark scheme to a candidate's response, the team leader must be consulted.
- Crossed out work should be marked UNLESS the candidate has replaced it with an alternative response.

EDEXCEL GCE MATHEMATICS

General Instructions for Marking

1. The total number of marks for the paper is 75.

2. The Edexcel Mathematics mark schemes use the following types of marks:

- **M** marks: method marks are awarded for 'knowing a method and attempting to apply it', unless otherwise indicated.
- **A** marks: Accuracy marks can only be awarded if the relevant method (M) marks have been earned.
- **B** marks are unconditional accuracy marks (independent of M marks)
- Marks should not be subdivided.

3. Abbreviations

These are some of the traditional marking abbreviations that will appear in the mark schemes.

- bod – benefit of doubt
- ft – follow through
- the symbol $\sqrt{}$ will be used for correct ft
- cao – correct answer only
- cso - correct solution only. There must be no errors in this part of the question to obtain this mark
- isw – ignore subsequent working
- awrt – answers which round to
- SC: special case
- oe – or equivalent (and appropriate)
- dep – dependent
- indep – independent
- dp decimal places
- sf significant figures
- ∗ The answer is printed on the paper
- □ The second mark is dependent on gaining the first mark

4. For misreading which does not alter the character of a question or materially simplify it, deduct two from any A or B marks gained, in that part of the question affected.

5. Where a candidate has made multiple responses <u>and indicates which response they wish to submit</u>, examiners should mark this response.

If there are several attempts at a question which have not been crossed out, examiners should mark the final answer which is the answer that is the most complete.

6. Ignore wrong working or incorrect statements following a correct answer.

7. Mark schemes will firstly show the solution judged to be the most common response expected from candidates. Where appropriate, alternatives answers are provided in the notes. If examiners are not sure if an answer is acceptable, they will check the mark scheme to see if an alternative answer is given for the method used.

Question	Scheme	Marks	AOs
1(a)	Step length = 0.5	B1	1.1b
	x: 0, 0.5, 1, 1.5, 2; y: y_0, y_1, y_2, y_3, y_4 = e^0, $e^{\sin^2\frac{1}{2}}$, $e^{\sin^2 1}$, $e^{\sin^2\frac{3}{2}}$, $e^{\sin^2 2}$ = 1, 1.258…, 2.030…, 2.704…, 2.286…	M1	1.1b
	$\int_0^2 e^{\sin^2 x}\,dx \approx \frac{0.5}{3}\{y_0+4y_1+2y_2+4y_3+y_4\} = \frac{0.5}{3}\times\{23.198...\}(=3.8664...)$	M1	1.1b
	= 3.87	A1cao	1.1b
		(4)	
(b)	eg It is accurate to 2 significant figures	B1	3.2b
		(1)	
		(5 marks)	

x	0	0.5	1	1.5	2
	y_0	y_1	y_2	y_3	y_4
y	e^0	$e^{\sin^2\frac{1}{2}}$	$e^{\sin^2 1}$	$e^{\sin^2\frac{3}{2}}$	$e^{\sin^2 2}$
	1	1.258…	2.030…	2.704…	2.286…

Notes

(a)

B1: Correct step length of 0.5 which may be implied e.g. by their 0, 0.5, etc.

M1: Attempts to find y values for **all** their x values – may be in terms of e or numerical values. Must be trying to find at least 3 values (e.g. if step length 1 is used). If substitution is not seen at least 2 of the values other than 1 must be correct to at least 2 s.f. rounded or truncated.

M1: Correct application of Simpson's rule: $\frac{h}{3}$(ends + 2evens + 4odds) (must have an odd number of ordinates). Must be y values **not** x values.

A1cao: 3.87. Must be to 3 s.f..

Note: must see evidence of Simpson's Rule so answer with no working scores no marks.

(b)

B1: Must have awrt 3.87 for part (a), makes a sensible comment regarding the accuracy. Must relate to the accuracy, not just the value.

Accept

- It is correct to 2 significant figures
- It is correct to 1 decimal place
- It is not correct to 3 significant figures
- It is awrt 99.6% accurate (using 3.87) or 99.7% accurate (using 3.866…)
- Percentage error is awrt 0.3%/0.4% so very accurate
- Very accurate as only out by 0.015 (requires reference to accuracy and evidence)

Do not accept e.g. it is out by 0.015 with no reference to what this means about the accuracy. Do not accept e.g. "very close to" as evidence, some quantification (absolute error, number of s.f. it agrees to) must be given.

Question	Scheme	Marks	AOs
2(a)(i)	$t=\mathrm{e}^x \Rightarrow \frac{\mathrm{d}t}{\mathrm{d}h}=\mathrm{e}^x\frac{\mathrm{d}x}{\mathrm{d}h} \Rightarrow \frac{\mathrm{d}h}{\mathrm{d}t}=\mathrm{e}^{-x}\frac{\mathrm{d}h}{\mathrm{d}x}$	M1	1.1b
	$\Rightarrow t\frac{\mathrm{d}h}{\mathrm{d}t}=\frac{\mathrm{d}h}{\mathrm{d}x}$ *	A1*	2.1
(ii)	$t\frac{\mathrm{d}h}{\mathrm{d}t}=\frac{\mathrm{d}h}{\mathrm{d}x} \Rightarrow t\frac{\mathrm{d}^2h}{\mathrm{d}t^2}+\frac{\mathrm{d}h}{\mathrm{d}t}=\frac{\mathrm{d}^2h}{\mathrm{d}x^2}\frac{\mathrm{d}x}{\mathrm{d}t}$	M1	1.1b
	$t\frac{\mathrm{d}^2h}{\mathrm{d}t^2}+\frac{\mathrm{d}h}{\mathrm{d}t}=\frac{\mathrm{d}^2h}{\mathrm{d}x^2}\frac{\mathrm{d}x}{\mathrm{d}t} \Rightarrow t\frac{\mathrm{d}^2h}{\mathrm{d}t^2}=\frac{1}{t}\frac{\mathrm{d}^2h}{\mathrm{d}x^2}-\frac{\mathrm{d}h}{\mathrm{d}t}$ $\Rightarrow t^2\frac{\mathrm{d}^2h}{\mathrm{d}t^2}=\frac{\mathrm{d}^2h}{\mathrm{d}x^2}-t\frac{\mathrm{d}h}{\mathrm{d}t}=\frac{\mathrm{d}^2h}{\mathrm{d}x^2}-\frac{\mathrm{d}h}{\mathrm{d}x}$ *	A1*	2.1
		(4)	
(b)	$t^2\frac{\mathrm{d}^2h}{\mathrm{d}t^2}-2t\frac{\mathrm{d}h}{\mathrm{d}t}+2h=t^3 \Rightarrow \frac{\mathrm{d}^2h}{\mathrm{d}x^2}-\frac{\mathrm{d}h}{\mathrm{d}x}-2\frac{\mathrm{d}h}{\mathrm{d}x}+2h=\mathrm{e}^{3x}$ $\Rightarrow \frac{\mathrm{d}^2h}{\mathrm{d}x^2}-3\frac{\mathrm{d}h}{\mathrm{d}x}+2h=\mathrm{e}^{3x}$ *	B1*	1.1b
		(1)	
(c)	$m^2-3m+2=0 \Rightarrow m=1,\ 2$	M1	1.1b
	$(h=)\,A\mathrm{e}^x+B\mathrm{e}^{2x}$	A1ft	1.1b
	PI is $C=k\mathrm{e}^{3x}$	B1	2.2a
	$\frac{\mathrm{d}h}{\mathrm{d}x}=3k\mathrm{e}^{3x}, \frac{\mathrm{d}^2h}{\mathrm{d}x^2}=9k\mathrm{e}^{3x} \Rightarrow 9k-9k+2k=1 \Rightarrow k=\frac{1}{2} \Rightarrow$ PI: $(h=)\frac{1}{2}\mathrm{e}^{3x}$	M1 A1	1.1b 1.1b
	$h=A\mathrm{e}^x+B\mathrm{e}^{2x}+\frac{1}{2}\mathrm{e}^{3x} \Rightarrow h=At+Bt^2+\frac{1}{2}t^3$ *	A1*	2.2a
		(6)	
(d)	$t=1, h=2.5 \Rightarrow 2.5=A+B+\frac{1}{2}$	M1	3.4
	$\frac{\mathrm{d}h}{\mathrm{d}t}=A+2Bt+\frac{3}{2}t^2 \Rightarrow -1=A+4B+6$	M1	3.4
	$A=5, B=-3 \Rightarrow h=5t-3t^2+\frac{1}{2}t^3$	A1	1.1b
	$t=5 \Rightarrow h=5\times5-3\times5^2+\frac{5^3}{2}$	M1	1.1b
	height is 12.5 m (including units)	A1	3.2a
		(5)	
		(16 marks)	

Notes

(a)(i)

M1: Uses the chain rule (must be a clear statement of this before substitution or clear implication of it use) with an attempt to differentiate $t=\mathrm{e}^x$ to form an equation linking $\frac{\mathrm{d}h}{\mathrm{d}t}$ and $\frac{\mathrm{d}h}{\mathrm{d}x}$ (or their reciprocals) in terms of either x or t. Note they may take $\ln t$ first, which is fine. Note that $\frac{\mathrm{d}h}{\mathrm{d}x}=\mathrm{e}^x\frac{\mathrm{d}h}{\mathrm{d}t}$ with no supporting working is M0.

A1*: Correct proof with no errors.

(ii)

M1: Differentiates again with the product and chain rule, fully and correctly on at least one product, in order to establish any second derivative equation linking $\frac{\mathrm{d}^2h}{\mathrm{d}t^2}$ and $\frac{\mathrm{d}^2h}{\mathrm{d}x^2}$

A1*: Correct proof with no errors.

(b)

B1*: Shows clearly the substitution of the given results into the differential equation and obtains the given transformed equation with no errors.

(c)

M1: Forms and solves a quadratic auxiliary equation.

A1ft: Correct form for the CF for their AE solutions which must be $\pm1, \pm2$ (ie accept sign errors solving the AE). The *h* = is not needed and condone e.g. $Ae^{x} + Be^{2x} = 0$

B1: Deduces the correct form for the PI. Must be in terms of *x* for the equation II.

M1: Differentiates their PI twice and substitutes their derivatives into the DE to find "*k*".

A1: Correct PI (for equation II) seen or implied by working. May or may not be combined with the CF for this mark.

A1*: Forms the correct GS for *h* in terms of *x* and deduces the correct GS for the height in terms of *t* with no errors. This is a given answer, so must have seen the GS in terms of *h* before proceeding to this answer. Accept with the constants *A* and *B* either way round (or with other constants).

(d)

M1: Uses the conditions of the model (*t* = 1, *h* = 2.5) to form an equation in *A* and *B*,

M1: Differentiates and shows of evidence of using the conditions of the model $\left(t = 2, \frac{\mathrm{d}h}{\mathrm{d}t} = -1\right)$ to form another equation in *A* and *B*.

A1: Solves simultaneously to obtain correct constants and hence a correct equation connecting *h* with *t*.

M1: Substitutes *t* = 5. If substitution is not seen you will need to check their answer matches their expression.

A1: Obtains 12.5 m using the model. Must include the units.

Question	Scheme	Marks	AOs
3.	$\frac{x^2-2x-24}{\lvert x+6\rvert}=5-4x \Rightarrow x^2-2x-24=\lvert x+6\rvert(5-4x)$ $x>-6 \Rightarrow x^2-2x-24=(x+6)(5-4x) \Rightarrow x=...$ **or** $x<-6 \Rightarrow x^2-2x-24=-(x+6)(5-4x) \Rightarrow x=...$ **or** $(x^2-2x-24)^2=(x+6)^2(5-4x)^2 \Rightarrow$ $15x^4+156x^3+165x^2-1236x+324=0 \Rightarrow x=...$	M1	1.1b
	$5x^2+17x-54=0 \Rightarrow x=2, -\frac{27}{5}$ **or** $3x^2+21x-6=0 \Rightarrow x=\frac{-7\pm\sqrt{57}}{2}\left(\text{or just } \frac{-7-\sqrt{57}}{2}\right)$ **or** any two correct roots	A1	1.1b
	$x^2-2x-24<(x+6)(5-4x) \Rightarrow x=...$ **and** $x^2-2x-24<-(x+6)(5-4x) \Rightarrow x=...$ (or at least three roots from quartic)	M1	3.1a
	$x=2, -\frac{27}{5}, \frac{-7\pm\sqrt{57}}{2}\left(\text{or just } \frac{-7-\sqrt{57}}{2}\right)$	A1	1.1b
	Forms $x<\alpha$ and $\beta<x<\gamma$ (see notes)	M1	3.1a
	Either $x<\frac{-7-\sqrt{57}}{2}$ or $-\frac{27}{5}<x<2$	A1ft	2.2a
	Both $x<\frac{-7-\sqrt{57}}{2}$ and $-\frac{27}{5}<x<2$	A1	2.2a
		(7)	
			(7 marks)

Notes

M1: Multiplies through by $\lvert x+6\rvert$ or (x + 6) and considers either (x + 6) or –(x + 6) and attempts to solve the resulting 3TQ (do not be concerned about the method of solving), or squares both sides and simplifies to a quartic and attempts to solve. May use "=" or any inequality for this and the next 3 marks. Allow if an extra factor (x + 6) is included (i.e. multiplies through by$(x+6)^2$) .

A1: Correct roots for either equation, or any two correct roots from the (correct) quartic. Allow if −6 is also included but do not count this as one of the roots.

M1: Recognises the requirement to consider both (x + 6) and –(x + 6) and attempts to solve the resulting 3TQ's (allow for any two values following the quadratic), or implied by an attempt to solve their quartic equation to find for 3 or 4 answers for their quartic, and allow for decimals, 0.2729…, −7.2749... for this mark. Again allow if extra factor (x + 6) included.

Allow if this is carried out but later rejected (e.g. crossed out) as they think the answers are inadmissible.

A1: All correct and exact critical values (and may include −6 and $\frac{-7+\sqrt{57}}{2}$ at this stage) and no other incorrect values.

M1: Produces an answer with the correct form of the solution set from the graph, allowing for non-strict inequalities for this mark, using three distinct answer from their critical values, α, β, γ, with $\alpha \leqslant -6 \leqslant \beta < 0 < \gamma$ and allow with either $\alpha = -6$ or $\beta = -6$ (but not both) for this mark. SC allow M1 if they list as 3 separate inequalities $x < \frac{-7-\sqrt{57}}{2}, x > -\frac{27}{5}, x < 2$ for this mark.

A1ft: Deduces one of the correct ranges, following through on appropriate critical values. If the two quadratics are solved independently then allow for either their $\frac{-27}{5} < x <$ their 2 from the "$x > -6$" equation (one positive, one negative) or for $x <$ their $\frac{-7-\sqrt{57}}{2}$ from the "$x < -6$" equation choosing a root less than −6. If a quartic is solved then allow with any three of their CVs satisfying the M used to get one interval of correct form. (Answer may be inexact here - follow through decimal roots.)

A1: Fully correct solution set, in any suitable form. May give the two inequality statements, in which case accept with “or” or “and” between. Accept in interval notation. In formal set notation accept with union ($\cup$) but intersection ($\cap$) is A0. Do not accept $x > -\frac{27}{5}, x < 2$ for the second interval for this mark. Values must be exact.

Question	Scheme	Marks	AOs
4(a)	$b^2 = a^2\left(1-e^2\right) \Rightarrow 9 = 16\left(1-e^2\right) \Rightarrow e^2 = ...$	M1	1.1b
	$e = \dfrac{\sqrt{7}}{4}$	A1	1.1b
		(2)	
(b)	$\dfrac{x^2}{16}+\dfrac{y^2}{9}=1 \Rightarrow \dfrac{x}{8}+\dfrac{2yy'}{9}=0 \Rightarrow y' = -\dfrac{9x}{16y} = -\dfrac{36\cos\theta}{48\sin\theta}$ or $x = 4\cos\theta, y = 3\sin\theta \Rightarrow \dfrac{\mathrm{d}y}{\mathrm{d}x} = -\dfrac{3\cos\theta}{4\sin\theta}$	M1 A1	2.1 1.1b
	$m_N = \dfrac{4\sin\theta}{3\cos\theta} \Rightarrow y - 3\sin\theta = \dfrac{4\sin\theta}{3\cos\theta}\left(x - 4\cos\theta\right)$	M1	1.1b
	$3y\cos\theta - 9\sin\theta\cos\theta = 4x\sin\theta - 16\sin\theta\cos\theta$ $\Rightarrow 4x\sin\theta - 3y\cos\theta = 7\sin\theta\cos\theta$ *	A1*	2.1
		(4)	
(c)	$y = -\dfrac{3\sin\theta}{4\cos\theta}x$ or $y + 3\sin\theta = -\dfrac{3\sin\theta}{4\cos\theta}\left(x - 4\cos\theta\right)$	B1	2.2a
	$4x\sin\theta + \dfrac{9x\sin\theta\cos\theta}{4\cos\theta} = 7\sin\theta\cos\theta \Rightarrow x = ...$ or $-\dfrac{16y\sin\theta\cos\theta}{3\sin\theta} - 3y\cos\theta = 7\sin\theta\cos\theta \Rightarrow y = ...$	M1	3.1a
	$x = \dfrac{28\cos\theta}{25}$ or $y = -\dfrac{21\sin\theta}{25}$	A1	1.1b
	$x = \dfrac{28\cos\theta}{25}$ and $y = -\dfrac{21\sin\theta}{25}$	A1	1.1b
		(4)	
(d)	$x = \dfrac{28\cos\theta}{25}, \quad y = -\dfrac{21\sin\theta}{25} \Rightarrow a = \dfrac{28}{25}, b = -\dfrac{21}{25}$ $\left(\dfrac{21}{25}\right)^2 = \left(\dfrac{28}{25}\right)^2\left(1-e^2\right) \Rightarrow \dfrac{441}{625} = \dfrac{784}{625}\left(1-e^2\right) \Rightarrow e = ...$	M1	3.1a
	Of form $\left(a\cos\phi, b\sin\phi\right)$ (where $\phi = -\theta$) so an ellipse, and $e = \dfrac{\sqrt{7}}{4}$ as required.	A1cso	1.1b
		(2)	
		(12 marks)	

Notes

(a)
M1: Uses the correct eccentricity formula and the given equation to find a value for e or e^2.

A1: Correct exact value $\dfrac{\sqrt{7}}{4}$, must be simplified. Must reject the negative value, so A0 if $-\dfrac{\sqrt{7}}{4}$ also included.

(b)

M1: Starts the process of establishing the gradient of the normal by adopting a suitable method to differentiate such as using implicit differentiation or parametric differentiation to obtain an expression for $\frac{\mathrm{d}y}{\mathrm{d}x}$ in terms of θ. The substitution for x and y may happen later, but must be in terms of θ when used in the equation for normal.

A1: Correct unsimplified $\frac{\mathrm{d}y}{\mathrm{d}x}$ in terms of θ. May be implied (if substitution happens later).

M1: Applies the negative reciprocal to their gradient and uses this and the coordinates of P to establish an equation of the normal. If using $y = mx + c$ they must proceed as far as finding c.

A1*: Fully correct proof with sufficient working shown. Must be an intermediate step between the initial equation and the final answer. Do not penalise minor notational slips such as missing some θ's if the working is clear, but award A0 if they are consistently missed.

(c)

B1: Deduces the correct equation of OQ. Need not be simplified - may use point Q.

M1: Proceeds to solve their OQ line simultaneously with the given l_1 to find either the x or the y coordinate of the intersection.

A1: One correct coordinate simplified or unsimplified.

A1: Both correct simplified coordinates. (Accept decimal equivalents.)

(d)

M1: Uses their coordinates of R (of an appropriate form) correctly to determine the values of "a" and "b" for the second ellipse and uses their values with the correct eccentricity formula in an attempt to show that the eccentricities are the same.

A1cso: Must have had correct coordinates in (c), a statement or deduction (or work shown) to verify the locus of R is an ellipse (conclusion not needed, e.g. accept if the equation is given to show it is an ellipse), and proceeds to show that the value of e (or e^2 - do not penalise inclusion of the negative value a second time) is the same as that obtained in part (a) or equivalent work. Note that the correct eccentricity can arise from coordinates of R where the denominator was incorrect, but these will score A0.

Question	Scheme	Marks	AOs
5(a)	$\frac{dt}{dx}=\frac{1+t^2}{2}$ or $\frac{dx}{dt}=\frac{2}{1+t^2}$ or $dx=\frac{2dt}{1+t^2}$ or $dt=\frac{1+t^2}{2}dx$ oe	B1	1.1b
	$2\sin x-\cos x+5=2\left(\frac{2t}{1+t^2}\right)-\frac{1-t^2}{1+t^2}+5$	M1	1.1a
	$\int\frac{1}{2\sin x-\cos x+5}dx=\int\frac{1}{2\left(\frac{2t}{1+t^2}\right)-\left(\frac{1-t^2}{1+t^2}\right)+5}\times\frac{2dt}{1+t^2}$	M1	2.1
	$=\int\frac{2}{4t-1+t^2+5+5t^2}\times dt=\int\frac{1}{3t^2+2t+2}dt$ *	A1*	2.1
		(4)	
(b)	$\int\frac{1}{3t^2+2t+2}dt=\frac{1}{3}\int\frac{1}{t^2+\frac{2}{3}t+\frac{2}{3}}dt=\frac{1}{3}\int\frac{1}{\left(t+\frac{1}{3}\right)^2+...}dt$ or $\int\frac{1}{\left(\sqrt{3}t+\frac{1}{\sqrt{3}}\right)^2+...}dt$	M1	3.1a
	$\frac{1}{3}\int\frac{1}{\left(t+\frac{1}{3}\right)^2+\frac{5}{9}}dt$ or $\int\frac{1}{\left(\sqrt{3}t+\frac{\sqrt{3}}{3}\right)^2+\frac{5}{3}}dt$ (oe)	A1	1.1b
	$=\frac{1}{3}\times\frac{1}{\frac{\sqrt{5}}{3}}\tan^{-1}\left(\frac{t+\frac{1}{3}}{\frac{\sqrt{5}}{3}}\right)\ (+c)=\frac{1}{\sqrt{5}}\tan^{-1}\left(f(x)\right)\ (+c)$ (oe)	M1	3.1a
	$=\frac{1}{\sqrt{5}}\tan^{-1}\left(\frac{3\tan\left(\frac{x}{2}\right)+1}{\sqrt{5}}\right)\ (+c)$ (oe)	A1	1.1b
		(4)	
		(8 marks)	

Notes

(a)

B1: Correct equation in terms of dx, dt and t – can be implied if seen as part of their substitution.

M1: Express $2\sin x - \cos x + 5$ in terms of t using half angle formulae with at least one correct. Allow if e.g. there are slips in sign when substituting or the +5 is missed.

M1: Makes a **complete** substitution to obtain an integral in terms of t only. Allow slips with the substitution of "dx" but must be dx = f(t)dt where f(t) ≠ 1. This mark is also available if the candidate makes errors when attempting to simplify $2\left(\frac{2t}{1+t^2}\right)-\frac{1-t^2}{1+t^2}$ before attempting the substitution. Condone omission of the integral and dt if the intent is clear.

A1*: Obtains the printed answer (including dt) with at least one intermediate line or aside working showing elimination of the fractions and no errors seen.

(b)

M1: Adopts the correct strategy of completing the square in order to attempt the integration. Must achieve one of the forms shown in scheme. May be seen separately but must be applied to the integral, not just as an attempt to solve the quadratic.

A1: Correct integral with completed square integrand.

M1: For recognising the arctan form for the integration (look for e.g $k\arctan\left(\frac{"t+\frac{1}{3}"}{b}\right)$)and makes further progress by undoing the substitution to obtain an answer in terms of x

A1: Correct answer. Accept equivalents with simplified (and no nested) fractions. No need for c.

Question	Scheme	Marks	AOs
6(a)	$y=\ln\left(e^{2x}\cos 3x\right)\Rightarrow\frac{dy}{dx}=\frac{2e^{2x}\cos 3x-3e^{2x}\sin 3x}{e^{2x}\cos 3x}$ or $y=\ln e^{2x}+\ln\cos 3x=2x+\ln\cos 3x\Rightarrow\frac{dy}{dx}=2+\frac{-3\sin 3x}{\cos 3x}$	M1	2.1
	$=2-3\tan 3x$*	A1*	1.1b
		(2)	
(b)	$\frac{d^2y}{dx^2}=-9\sec^2 3x$	B1	1.1b
	$\frac{d^3y}{dx^3}=-54\sec^2 3x\tan 3x$ $\Rightarrow\frac{d^4y}{dx^4}=-162\sec^4 3x-324\sec^2 3x\tan^2 3x$ (oe)	M1 A1	2.1 1.1b
		(3)	
(c)	$(y)_0=0,\left(\frac{dy}{dx}\right)_0=2,\left(\frac{d^2y}{dx^2}\right)_0=-9,\left(\frac{d^3y}{dx^3}\right)_0=0,\left(\frac{d^4y}{dx^4}\right)_0=-162$	M1	1.1b
	$y=f(0)+xf'(0)+\frac{x^2}{2!}f''(0)+\frac{x^3}{3!}f'''(0)+\dots$	M1	2.1
	$(y=)0+2x-\frac{9x^2}{2}-\frac{162x^4}{24}+\dots=2x-\frac{9x^2}{2}-\frac{27x^4}{4}$	A1cso	1.1b
		(3)	
(d)	$\ln(1+kx)=kx-\frac{k^2x^2}{2}+\frac{k^3x^3}{3}-\frac{k^4x^4}{4}+\dots$	B1	2.2a
		(1)	
(e)	$\ln\frac{e^{2x}\cos 3x}{1+kx}=\ln\left(e^{2x}\cos 3x\right)-\ln(1+kx)$ $=\ln\left(e^{2x}\cos 3x\right)-\ln(1+kx)$ $=2x-\frac{9x^2}{2}-\frac{27x^4}{4}+\dots-\left(kx-\frac{k^2x^2}{2}+\dots\right)$	M1	3.1a
	$=\frac{1}{x^2}\left((2-k)x-\frac{(9-k^2)x^2}{2}-\frac{k^3x^3}{3}-\frac{(27-k^4)x^4}{4}+\dots\right)$ For the limit to exist $2-k=0\Rightarrow k=\dots$	M1	3.1a
	$k=2$	A1	2.2a
		(3)	
			(12 marks)

Notes

(a)

M1: Attempts differentiation of the given function by applying both the chain rule and the product rule. Look for the correct form but they may make slips with signs or coefficients. Alternatively, applies the sum law for logs and differentiates using the chain rule.

A1*: Correct proof.

(b)
B1: Correct second derivative.
M1: Continues the differentiation using the chain rule and product rule to reach the 4th derivative. Look for correct forms, allowing for slips in signs or coefficients only when differentiating.
May be given in terms of $\frac{dy}{dx}, \frac{d^2y}{dx^2}$ etc for this mark.
A1: Correct 4th derivative in terms of x. Accept alternatives. Like terms should be gathered and coefficients simplified but isw after a correct suitable answer.
(c)
M1: Attempts the values of the derivatives at $x = 0$ up to at least the third derivative. If substitution not seen at least two values should be correct for their derivatives, with values for others.
M1: Substitutes the values of their derivatives into the correct Maclaurin expansion formula up to at least the third non-zero value for their derivatives.
A1cso: Must have obtained a correct 4th derivative in (b) (though need not have been in terms of x). Correct expansion with simplified coefficients. May be missing the "$y =$" and allow "f(x)=". Note the correct series will be obtained from various incorrect 4th derivatives due to some terms evaluating to 0, but use of a clearly incorrect 4th derivative score A0 here. However, an initially correct 4th derivative, with incorrect simplification, can score A1 bod if no incorrect work is shown (ie correct values stated with no substitution shown, as they may have used the initially correct answer).
(d)
B1: Deduces the correct expansion in any form.
(e)
M1: Applies the subtraction law of logarithms and then substitutes their expansions.
M1: Realises that the x terms will determine the required value of k and so collects the x terms and sets the coefficient to zero and solves for k (may be implied).
A1: For $k = 2$

Question	Scheme	Marks	AOs
7(a)	Vector A to l is $\pm\left(\begin{pmatrix}12\\30\\39\end{pmatrix}+\lambda\begin{pmatrix}7\\13\\24\end{pmatrix}-\begin{pmatrix}3\\6\\5\end{pmatrix}\right)$	B1	2.2a
	$\begin{vmatrix}9+7\lambda\\24+13\lambda\\34+24\lambda\end{vmatrix}=15\Rightarrow(9+7\lambda)^2+(24+13\lambda)^2+(34+24\lambda)^2=15^2\Rightarrow\lambda=...$	M1	3.1a
	$794\lambda^2+2382\lambda+1588=0\Rightarrow\lambda=-1,\ -2$	A1	1.1b
	$B(-2,\ 4,\ -9)$ and $C(5,\ 17,\ 15)$	A1	2.2a
		(4)	
(b)	E.g. $\overrightarrow{AB}\times\overrightarrow{AC}=\begin{pmatrix}-5\\-2\\-14\end{pmatrix}\times\begin{pmatrix}2\\11\\10\end{pmatrix}=$ or $\overrightarrow{AB}\times\overrightarrow{BC}=\begin{pmatrix}-5\\-2\\-14\end{pmatrix}\times\begin{pmatrix}7\\13\\24\end{pmatrix}\left(=\pm\begin{pmatrix}134\\22\\-51\end{pmatrix}\right)$	M1	3.1a
	$\begin{pmatrix}134\\22\\-51\end{pmatrix}\bullet\begin{pmatrix}3\\6\\5\end{pmatrix}=279$	M1	1.1b
	$134x+22y-51z=279$	A1	1.1b
		(3)	
(c)	E.g. $\overrightarrow{DA}=\begin{pmatrix}3\\6\\5\end{pmatrix}-\begin{pmatrix}-2\\1\\\alpha\end{pmatrix}=\begin{pmatrix}5\\5\\5-\alpha\end{pmatrix}$ or $\overrightarrow{BD}=\begin{pmatrix}-2\\1\\\alpha\end{pmatrix}-\begin{pmatrix}-2\\4\\-9\end{pmatrix}=\begin{pmatrix}0\\-3\\9+\alpha\end{pmatrix}$	M1	3.1a
	$\begin{pmatrix}134\\22\\-51\end{pmatrix}\bullet\begin{pmatrix}5\\5\\5-\alpha\end{pmatrix}=...$ or $\begin{pmatrix}7\\13\\24\end{pmatrix}\bullet\left(\begin{pmatrix}5\\2\\14\end{pmatrix}\times\begin{pmatrix}0\\-3\\9+\alpha\end{pmatrix}\right)=...$	dM1	1.1b
	$\frac{1}{6}\lvert 525+51\alpha\rvert=\pm147\Rightarrow\alpha=...$	M1	3.1a
	$\alpha=7,\ -\frac{469}{17}$	A1	1.1b
		(4)	
(d)	E.g. Vector connecting A and l is $\pm\begin{pmatrix}9\\24\\34\end{pmatrix}$ **or** D and l is $\pm\begin{pmatrix}0\\-3\\16\end{pmatrix}$	B1ft	2.2a
	$\overrightarrow{AD}\times\overrightarrow{BC}=\begin{pmatrix}-5\\-5\\2\end{pmatrix}\times\begin{pmatrix}7\\13\\24\end{pmatrix}=\begin{pmatrix}-146\\134\\-30\end{pmatrix}$	M1	1.1b

	$d=\dfrac{\left\lvert\begin{pmatrix}-146\\134\\-30\end{pmatrix}\bullet\begin{pmatrix}9\\24\\34\end{pmatrix}\right\rvert}{\sqrt{146^2+134^2+30^2}}=\ldots$	M1	3.1a
	$d=$ awrt 4.4	A1	1.1b
		(4)	
			(15 marks)

Notes

Notes: Accept alternative vector forms throughout (including use of row vectors), and accept coordinates as vectors.

For cross products with no working shown accept two out three correct entries to imply the method.

(a)

B1: Deduces the correct vector for A to l in parametric form (either direction).

M1: Makes the key step of attempting the general distance from A to l, sets equal to 15 and attempts to solve for λ

A1: Correct values for λ

A1: Both coordinates correct

(b)

M1: Realises that the normal to the plane is required and applies the vector product to 2 vectors in the plane. (Many possibilities are possible, the two most common are in the scheme.)

M1: Attempts scalar product using a point in the plane and their normal vector.

A1: Correct equation (allow any multiple).

(c)

M1: Finds an appropriate vector using the coordinates of D e.g. joining D to A or B.

dM1: Forms and evaluates an appropriate scalar triple product.

M1: Realises that ±147 is possible for the value of 1/6 of the triple product and attempts to solve to obtain 2 distinct values for α. Allow this mark if one value is later rejected (or crossed out).

A1: Correct values, but A0 if one is later rejected.

(d)

B1ft: Deduces a correct vector joining A or D and l (follow through their B or C if used). Accept in either direction (do not be concerned about the labelling). Many other vectors are possible here.

M1: Calculates the vector product between the directions.

M1: Fully correct strategy for the shortest distance.

A1: Awrt 4.4

(b) **Alt**	Plane is e.g. $\mathbf{r}=\begin{pmatrix}3\\6\\5\end{pmatrix}+\lambda\begin{pmatrix}7\\13\\24\end{pmatrix}+\mu\begin{pmatrix}5\\2\\14\end{pmatrix}\Rightarrow\begin{cases}x=3+7\lambda+5\mu\\y=6+13\lambda+2\mu\\x=5+24\lambda+14\mu\end{cases}$	M1	1.1b
	$\Rightarrow\begin{cases}5y-2x=24+51\lambda\\7y-z=37+67\lambda\end{cases}\Rightarrow 51(7y-z-37)-67(5y-2x-24)=0$	M1	3.1a
	$134x+22y-51z=279$	A1	1.1b
		(3)	

(d) **Alt**	$\overrightarrow{AD}\times\overrightarrow{BC}=\begin{pmatrix}-5\\-5\\2\end{pmatrix}\times\begin{pmatrix}7\\13\\24\end{pmatrix}=\begin{pmatrix}-146\\134\\-30\end{pmatrix}$ and need distance between planes $\mathbf{r}\bullet\begin{pmatrix}-146\\134\\30\end{pmatrix}=\begin{pmatrix}3\\6\\5\end{pmatrix}\bullet\begin{pmatrix}-146\\134\\30\end{pmatrix}=216$ and $\mathbf{r}\bullet\begin{pmatrix}-146\\134\\30\end{pmatrix}=\begin{pmatrix}-2\\4\\-9\end{pmatrix}\bullet\begin{pmatrix}-146\\134\\30\end{pmatrix}=1098$ (one correct B1ft on their normal vectors, M1 attempt both)	B1ft M1	2.2a 1.1b
	$d=\dfrac{1098-216}{\sqrt{146^2+134^2+30^2}}$	M1	3.1a
	$d=$ awrt 4.4	A1	1.1b
		(4)	
(d) **Alt 2**	E.g. $\mathbf{r}=\begin{pmatrix}9\\24\\34\end{pmatrix}+\lambda\begin{pmatrix}7\\13\\24\end{pmatrix}+\mu\begin{pmatrix}5\\5\\-2\end{pmatrix}$ Sets up plane perpendicular to both direction vectors, passing through point represented by vector that joins the two lines. (Translates e.g. point A to the origin and considers parallel planes.)	B1ft	2.2a
	$\mathbf{r}\bullet\begin{pmatrix}9+7\lambda+5\mu\\24+13\lambda+5\mu\\34+24\lambda=2\mu\end{pmatrix}\bullet\begin{pmatrix}7\\13\\24\end{pmatrix}=0\Rightarrow a+b\lambda+c\mu=0$ and $\mathbf{r}\bullet\begin{pmatrix}9+7\lambda+5\mu\\24+13\lambda+5\mu\\34+24\lambda=2\mu\end{pmatrix}\bullet\begin{pmatrix}5\\5\\-2\end{pmatrix}=0\Rightarrow d+e\lambda+f\mu=0$ Attempts scalar products of the plane with both direction vectors (finds point on second plane normal the origin).	M1	1.1b
	$\lambda=...,\mu=...\Rightarrow$ $d=\sqrt{\left(9+7"\lambda"+5"\mu"\right)^2+\left(24+13"\lambda"+5"\mu"\right)^2+\left(34+24"\lambda"-2"\mu"\right)^2}=...$ Solves for λ and μ and proceeds to find the distance using their values.	M1	3.1a
	$d=$ awrt 4.4	A1	1.1b
		(4)	

Pearson Education Limited. Registered company number 872828

with its registered office at 80 Strand, London, WC2R 0RL, United Kingdom