Please check the examination details below before entering your candidate information

Candidate surname	Other names

Centre Number | Candidate Number

Pearson Edexcel Level 3 GCE

Time 1 hour 30 minutes

Paper reference **9FM0/3A**

Further Mathematics

Advanced
PAPER 3A: Further Pure Mathematics 1

You must have:
Mathematical Formulae and Statistical Tables (Green), calculator

Total Marks

Candidates may use any calculator permitted by Pearson regulations. Calculators must not have the facility for symbolic algebraic manipulation, differentiation and integration, or have retrievable mathematical formulae stored in them.

Instructions

- Use **black** ink or ball-point pen.
- If pencil is used for diagrams/sketches/graphs it must be dark (HB or B).
- **Fill in the boxes** at the top of this page with your name, centre number and candidate number.
- Answer **all** questions and ensure that your answers to parts of questions are clearly labelled.
- Answer the questions in the spaces provided – *there may be more space than you need.*
- You should show sufficient working to make your methods clear. Answers without working may not gain full credit.
- Inexact answers should be given to three significant figures unless otherwise stated.

Information

- A booklet 'Mathematical Formulae and Statistical Tables' is provided.
- There are 9 questions in this question paper. The total mark for this paper is 75.
- The marks for **each** question are shown in brackets – *use this as a guide as to how much time to spend on each question.*

Advice

- Read each question carefully before you start to answer it.
- Try to answer every question.
- Check your answers if you have time at the end.

Turn over ▶

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Q:1/1/1/1/
P 6 5 4 9 7 A 0 1 3 2

Pearson

1. An ellipse has equation $\frac{x^2}{16} + \frac{y^2}{4} = 1$ and eccentricity e_1

A hyperbola has equation $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ and eccentricity e_2

Given that $e_1 \times e_2 = 1$

(a) show that $a^2 = 3b^2$

(4)

Given also that the coordinates of the foci of the ellipse are the same as the coordinates of the foci of the hyperbola,

(b) determine the equation of the hyperbola.

(3)

(a) For ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$, $b^2 = a^2(1-e^2)$ where $e < 1$

$\frac{x^2}{16} + \frac{y^2}{4} = 1 \Rightarrow a^2 = 16$, $b^2 = 4$

$4 = 16(1-e^2)$ (1)

$\frac{1}{4} = 1 - e_1^2$

$e_1^2 = \frac{3}{4} \Rightarrow e_1 = \frac{\sqrt{3}}{2}$ (1)

For hyperbola $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$, $b^2 = a^2(e^2-1)$ where $e > 1$

$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1 \Rightarrow b^2 = a^2(e_2^2 - 1)$

$e_1 \times e_2 = 1 \Rightarrow e_2 = 1 \div \frac{\sqrt{3}}{2} = \frac{2}{\sqrt{3}} \Rightarrow e^2 = \frac{4}{3}$

$b^2 = a^2\left(\frac{4}{3} - 1\right)$ (1)

$b^2 = \frac{a^2}{3}$

$3b^2 = a^2$ (1)

P 6 5 4 9 7 A 0 2 3 2

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Question 1 continued

(b) $F_e = (\pm ae, 0)$ and $F_h = (\pm ae, 0)$

F_e : $x = \pm\sqrt{16} \times \frac{\sqrt{3}}{2}$ $\qquad$ F_h : $x = \pm\sqrt{a^2} \times \frac{2}{\sqrt{3}}$

$x = \pm 2\sqrt{3}$ ① $\qquad$ $x = \frac{2a}{\sqrt{3}}$

$F_e = F_h \implies 2\sqrt{3} = \frac{2a}{\sqrt{3}}$ $\quad \times\sqrt{3}$

$2\times 3 = 2\times a$ $\quad \div 2$

$a = 3$

$3b^2 = a^2 \leftarrow$ from part (a) ①

$b^2 = \frac{1}{3}(3^2)$

$b^2 = 3$

$\therefore$ Equation of hyperbola : $\frac{x^2}{9} - \frac{y^2}{3} = 1$ ①

(Total for Question 1 is 7 marks)

2. During 2029, the number of hours of daylight per day in London, H, is modelled by the equation

$$H = 0.3\sin\left(\frac{x}{60}\right) - 4\cos\left(\frac{x}{60}\right) + 11.5 \qquad 0 \leqslant x < 365$$

where x is the number of days after 1st January 2029 and the angle is in radians.

(a) Show that, according to the model, the number of hours of daylight in London on the 31st January 2029 will be 8.13 to 3 significant figures.

(1)

(b) Use the substitution $t = \tan\left(\frac{x}{120}\right)$ to show that H can be written as

$$H = \frac{at^2 + bt + c}{1 + t^2}$$

where a, b and c are constants to be determined.

(2)

(c) Hence determine, according to the model, the date of the first day of 2029 when there will be at least 12 hours of daylight in London.

(4)

(a) $31 - 1 = 30$ days $\Rightarrow$ $x = 30$ — sub x into H

$H = 0.3\sin\left(\frac{30}{60}\right) - 4\cos\left(\frac{30}{60}\right) + 11.5$

$H = 8.1334\ldots = 8.13$ hrs 1 (3.s.f)

(b) Using standard t-formulae, let $\tan\left(\frac{x}{2}\right) = t$.

$\sin x = \frac{2t}{1+t^2}$ $\qquad$ $\cos x = \frac{1-t^2}{1+t^2}$

When $t = \tan\left(\frac{x}{120}\right)$ then $\sin\left(\frac{x}{60}\right)$ and $\cos\left(\frac{x}{60}\right)$ can be substituted.

$H = 0.3\left(\frac{2t}{1+t^2}\right) - 4\left(\frac{1-t^2}{1+t^2}\right) + 11.5$ 1

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Question 2 continued

$$H = \frac{0.6t - 4 + 4t^2 + 11.5(1+t^2)}{(1+t^2)}$$

$$H = \frac{15.5t^2 + 0.6t + 7.5}{1 + t^2} \quad 1$$

(c) $$\frac{15.5t^2 + 0.6t + 7.5}{1 + t^2} = 12$$

$$15.5t^2 + 0.6t + 7.5 = 12(1+t^2)$$

$$3.5t^2 + 0.6t - 4.5 = 0 \quad 1$$

$$t = \frac{-0.6 \pm \sqrt{0.6^2 - 4(3.5)(-4.5)}}{2 \times 3.5} \leftarrow$$

use calculator or $\dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$$t = \frac{-3 \pm 12\sqrt{11}}{7}$$

$$t = \tan\left(\frac{x}{120}\right) \Rightarrow x = 120\tan^{-1}(t)$$

$$x_1 = \tan^{-1}\left(\frac{-3 + 12\sqrt{11}}{7}\right) \times 120 = 97.25$$

$$x_2 = \tan^{-1}\left(\frac{-3 - 12\sqrt{11}}{7}\right) \times 120 = -169.04$$

$x \geq 0$ so ignore

$\therefore x = 97$ 1 so 8^{th} April 1

Jan = 31, Feb = 28, March = 31, April = 30 and started 1^{st} Jan

(Total for Question 2 is 7 marks)

3. With respect to a fixed origin O, the points A and B have coordinates $(2, 2, -1)$ and $(4, 2p, 1)$ respectively, where p is a constant.

For each of the following, determine the possible values of p for which,

(a) OB makes an angle of 45° with the positive x-axis

(3)

(b) $\overrightarrow{OA} \times \overrightarrow{OB}$ is parallel to $\begin{pmatrix} 4 \\ -p \\ 2 \end{pmatrix}$

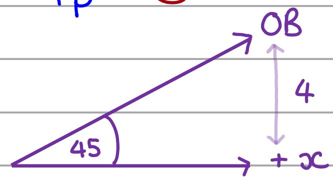
(3)

(c) the area of triangle OAB is $3\sqrt{2}$

(3)

(a) $|OB| = \sqrt{4^2 + (2p)^2 + 1^2} = \sqrt{17 + 4p^2}$ 1

$\cos 45 = \dfrac{4}{\sqrt{17 + 4p^2}}$ 1



$\dfrac{\sqrt{2}}{2} = \dfrac{4}{\sqrt{17 + 4p^2}}$

cross-multiply

$\sqrt{2} \times \sqrt{17 + 4p^2} = 8$

square both sides

$2 \times (17 + 4p^2) = 64$

$17 + 4p^2 = 32$

$4p^2 = 15$

$p^2 = \dfrac{15}{4}$

$p = \pm\dfrac{\sqrt{15}}{2}$ 1

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Question 3 continued

(b) $\overrightarrow{OA} \times \overrightarrow{OB} = \begin{pmatrix} 2 \\ 2 \\ -1 \end{pmatrix} \times \begin{pmatrix} 4 \\ 2p \\ 1 \end{pmatrix}$

$$= \begin{pmatrix} 2\times1 - 2p\times-1 \\ -1\times4 - 2\times1 \\ 2\times2p - 4\times2 \end{pmatrix} \qquad a\times b = \begin{pmatrix} a_2b_3 - a_3b_2 \\ a_3b_1 - a_1b_3 \\ a_1b_2 - a_2b_1 \end{pmatrix}$$

$$= \begin{pmatrix} 2+2p \\ -6 \\ 4p-8 \end{pmatrix} \quad 1$$

$$\begin{pmatrix} 2+2p \\ -6 \\ 4p-8 \end{pmatrix} = \begin{pmatrix} 4\lambda \\ -p\lambda \\ 2\lambda \end{pmatrix} \quad \leftarrow \text{parallel so same direction}$$

$2+2p = 4\lambda$ ①

$-6 = -p\lambda \quad \Rightarrow \lambda = \frac{6}{p}$ ②

$4p-8 = 2\lambda$ ③

$2+2p = 4\frac{6}{p}$

$2p + 2p^2 - 24 = 0 \quad \Rightarrow \quad (2p+8)(p-3) = 0$ 1

$p = -4, 3$

Try $p=-4$ in ③ $\quad 4(-4) - 8 = 2\left(\frac{6}{-4}\right)$

$-24 = -3$ ✗

Try $p=3$ in ③ $\quad 4(3) - 8 = 2\left(\frac{6}{3}\right)$

$4 = 4$ ✓

$\therefore p = 3$ 1

Question 3 continued

(c) Area OAB $= \frac{1}{2}|\vec{OA} \times \vec{OB}|$

$3\sqrt{2} = \frac{1}{2}\sqrt{(2+2p)^2 + (-6)^2 + (4p-8)^2}$ ①

$3\sqrt{2} = \frac{1}{2}\sqrt{4p^2 + 8p + 4 + 36 + 16p^2 - 64p + 64}$

$3\sqrt{2} = \frac{1}{2}\sqrt{20p^2 - 52p + 104}$

$(6\sqrt{2})^2 = 20p^2 - 52p + 104$

$72 = 20p^2 - 52p + 104$

$0 = 20p^2 - 52p + 32$ ①

$p = \dfrac{52 \pm \sqrt{(-52)^2 - 4 \times 20 \times 32}}{2 \times 20}$

$p = 2, \frac{4}{5}$ ①

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Question 3 continued

(Total for Question 3 is 9 marks)

4. The velocity v ms^{-1}, of a raindrop, t seconds after it falls from a cloud, is modelled by the differential equation

$$\frac{dv}{dt} = -0.1v^2 + 10 \qquad t \geqslant 0$$

Initially the raindrop is at rest.

(a) Use two iterations of the approximation formula $\left(\frac{dy}{dx}\right)_n \approx \frac{y_{n+1} - y_n}{h}$ to estimate the velocity of the raindrop 1 second after it falls from the cloud.

(5)

Given that the initial acceleration of the raindrop is found to be smaller than is suggested by the current model,

(b) refine the model by changing the value of one constant.

(1)

(a) When $t = 0$, $v = 0$: ← 'initially at rest'

2 iterations over 1 second so $h = \frac{1}{2}$

$\left(\frac{dv}{dt}\right)_0 = -0.1(0)^2 + 10 = 10$ ①

$\left(\frac{dv}{dt}\right)_0 = \frac{v_1 - v_0}{h} \Rightarrow v_1 = v_0 + h\left(\frac{dv}{dt}\right)_0$ ①

$v_1 = 0 + \frac{1}{2}(10) = 5$ ①

$\left(\frac{dv}{dt}\right)_1 = -0.1(5)^2 + 10 = 7.5$ ①

$v_2 = v_1 + h\left(\frac{dv}{dt}\right)_1$

$v_2 = 5 + \frac{1}{2}(7.5) = 8.75$ ms^{-1} ①

(b) $\frac{dv}{dt} = -0.1v^2 + A$ where $0 < A < 10$ ①

↑ still positive, but lower value

P 6 5 4 9 7 A 0 1 0 3 2

Question 4 continued

(Total for Question 4 is 6 marks)

5. The rectangular hyperbola H has equation $xy = 36$

(a) Use calculus to show that the equation of the tangent to H at the point $P\left(6t, \dfrac{6}{t}\right)$ is

$$yt^2 + x = 12t$$

(3)

The point $Q\left(12t, \dfrac{3}{t}\right)$ also lies on H.

(b) Find the equation of the tangent to H at the point Q.

(2)

The tangent at P and the tangent at Q meet at the point R.

(c) Show that as t varies the locus of R is also a rectangular hyperbola.

(4)

(a) $xy = 36 \Rightarrow y = \dfrac{36}{x}$

$\dfrac{dy}{dx} = -\dfrac{36}{x^2}$ $\leftarrow$ $\dfrac{d}{dx}36x^{-1} = -36x^{-2}$

$\dfrac{dy}{dx} = -\dfrac{36}{(6t)^2} = -\dfrac{36}{36t^2} = -\dfrac{1}{t^2}$ 1

$y - \dfrac{6}{t} = -\dfrac{1}{t^2}(x - 6t)$ 1 $\leftarrow$ $y - y_1 = m(x - x_1)$

$yt^2 - 6t = -x + 6t$

$yt^2 + x = 12t$ 1

(b) $\dfrac{dy}{dx} = -\dfrac{36}{x^2}$

$\dfrac{dy}{dx} = -\dfrac{36}{(12t)^2} = -\dfrac{1}{4t^2}$ 1

$y - \dfrac{3}{t} = -\dfrac{1}{4t^2}(x - 12t)$ 1

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Question 5 continued

(c) $y - \frac{3}{t} = -\frac{1}{4t^2}(x - 12t)$ ← from (b)

$4t^2y - 12t = -x + 12t$

$\left.\begin{aligned} 4t^2y + x &= 24t \\ yt^2 + x &= 12t \end{aligned}\right\}$ ① − ② . $3t^2y = 12t$ ①

↑ from (a)

$y = \frac{12t}{3t^2}$

$y = \frac{4}{t}$

$x = 12t - yt^2$

$x = 12t - \frac{4}{t}t^2$

$x = 12t - 4t$

$x = 8t, \quad y = \frac{4}{t}$ ①

$xy = 8t \times \frac{4}{t} = 32$ ① ∴ it is a rectangular hyperbola. ①

Question 5 continued

Question 5 continued

(Total for Question 5 is 9 marks)

6. The points P, Q and R have position vectors $\begin{pmatrix} 1 \\ -2 \\ 4 \end{pmatrix}$, $\begin{pmatrix} 3 \\ 1 \\ -5 \end{pmatrix}$ and $\begin{pmatrix} 2 \\ 0 \\ 3 \end{pmatrix}$ respectively.

(a) Determine a vector equation of the plane that passes through the points P, Q and R, giving your answer in the form $\mathbf{r} = \mathbf{a} + \lambda\mathbf{b} + \mu\mathbf{c}$, where λ and μ are scalar parameters.

(2)

(b) Determine the coordinates of the point of intersection of the plane with the x-axis.

(4)

(a) $r = a + \lambda b + \mu c$

point on the plane (a)

two non-parallel vectors on the plane (b, c)

$$a = \begin{bmatrix} 1 \\ -2 \\ 4 \end{bmatrix}$$

$$b = \overrightarrow{QR} = \begin{bmatrix} 2-3 \\ 0-1 \\ 3--5 \end{bmatrix} = \begin{bmatrix} -1 \\ -1 \\ 8 \end{bmatrix}$$

$$c = \overrightarrow{PR} = \begin{bmatrix} 2-1 \\ 0--2 \\ 3-4 \end{bmatrix} = \begin{bmatrix} 1 \\ 2 \\ -1 \end{bmatrix}$$

* or any other non-parallel vectors $\overrightarrow{PQ}$, $\overrightarrow{RQ}$, etc.

$$\therefore\ r = \begin{bmatrix} 1 \\ -2 \\ 4 \end{bmatrix} + \lambda \begin{bmatrix} -1 \\ -1 \\ 8 \end{bmatrix} + \mu \begin{bmatrix} 1 \\ 2 \\ -1 \end{bmatrix}$$

① ① ①

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Question 6 continued

(b) On the x-axis, $y=0$ and $z=0$.

$-2-1\lambda+2\mu=0$ ①
$4+8\lambda-1\mu=0$ ② set y and z components of r to zero. ①

①+2②: $6+15\lambda=0$

$15\lambda=-6$

$\lambda=-0.4$

$-2-1(-0.4)+2\mu=0$ ← sub λ into ①.

$2\mu=1.6$

$\mu=0.8$ ①

$x=1-\lambda+\mu$

$x=1-(-0.4)+(0.8)$ ① ← using x-components of r

$x=2.2$

∴ Point of intersection is $(2.2, 0, 0)$ ①

Question 6 continued

Question 6 continued

(Total for Question 6 is 6 marks)

7.

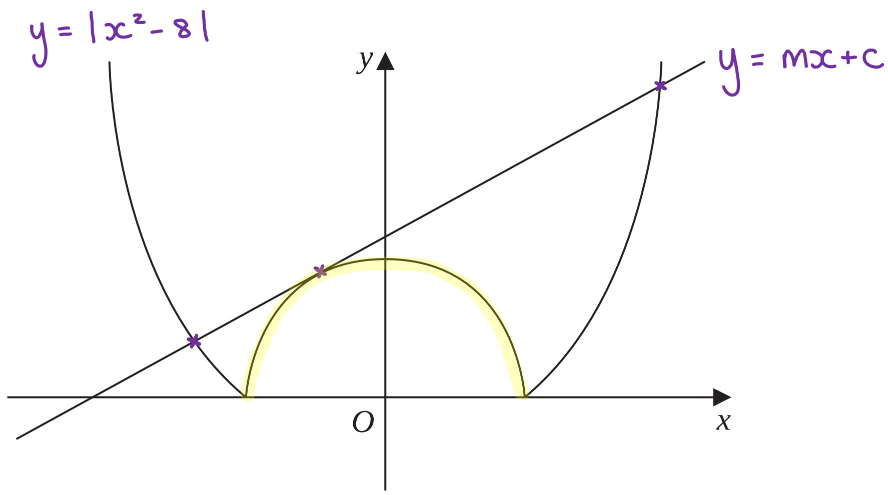


Figure 1

Figure 1 shows a sketch of the curve with equation $y = |x^2 - 8|$ and a sketch of the straight line with equation $y = mx + c$, where m and c are positive constants.

The equation

$$|x^2 - 8| = mx + c$$

has exactly 3 roots, as shown in Figure 1.

(a) Show that

$$m^2 - 4c + 32 = 0$$

(2)

Given that $c = 3m$

(b) determine the value of m and the value of c

(3)

(c) Hence solve

$$|x^2 - 8| \geqslant mx + c$$

(3)

(a) In the section $x^2 - 8 = -(mx + c)$ there is exactly one root ← highlighted on diagram

$x^2 + mx + (c-8) = 0$ ①

$m^2 - 4(c-8) = 0 \Rightarrow m^2 - 4c + 32 = 0$ ①

↑ set discriminant $b^2 - 4ac = 0$ for exactly one root.

Question 7 continued

(b) $m^2 - 4(3m) + 32 = 0$

$m^2 - 12m + 32 = 0$

$(m-8)(m-4) = 0 \quad \Rightarrow \quad m = 4, 8$ 1

$c = 3m \quad \Rightarrow \quad c = 12, 24$ 1

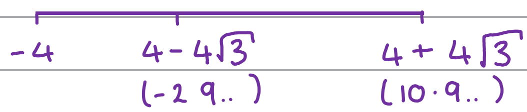
If $m = 8$, $c = 24$:

$x^2 - 8 = -8x - 24$ $\qquad$ $x^2 - 8 = 8x + 24$

$x^2 + 8x + 16 = 0$ $\qquad$ $x^2 - 8x - 32 = 0$

$(x+4)^2 = 0$ $\qquad$ $x = 4 \pm 4\sqrt{3}$

$x = -4$



This doesn't work because the repeated root $(x=-4)$ should lie between the two distinct roots according to the graph.

$\therefore \; m = 4, \; c = 12$ only 1

(c) $x^2 - 8 = mx + c$ $\qquad$ $-x^2 + 8 = mx + c$

$x^2 - 8 = 4x + 12$ $\qquad$ $-x^2 + 8 = 4x + 12$ 1

$x^2 - 4x - 20 = 0$ $\qquad$ $x^2 + 4x + 4 = 0$

$x = 2 \pm 2\sqrt{6}$ $\qquad$ $(x+2)^2 = 0 \quad \Rightarrow \quad x = -2$ 1

Question 7 continued

$|x^2 - 8|^2 \geqslant mx + c$ when the curve is above the line.

$x \leqslant 2 - 2\sqrt{6}$, $x \geqslant 2 + 2\sqrt{6}$, $x = -2$ 1

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Question 7 continued

(Total for Question 7 is 8 marks)

8. *The Taylor series expansion of* f(x) *about* $x = a$ *is given by*

$$\mathrm{f}(x) = \mathrm{f}(a) + (x-a)\mathrm{f}'(a) + \frac{(x-a)^2}{2!}\mathrm{f}''(a) + \ldots + \frac{(x-a)^r}{r!}\mathrm{f}^{(r)}(a) + \ldots$$

(i) (a) Use differentiation to determine the Taylor series expansion of $\ln x$, in ascending powers of $(x-1)$, up to and including the term in $(x-1)^2$

(4)

(b) Hence prove that

$$\lim_{x\to 1}\left(\frac{\ln x}{x-1}\right) = 1$$

(2)

(ii) Use L'Hospital's rule to determine

$$\lim_{x\to 0}\left(\frac{1}{(x+3)\tan(6x)\operatorname{cosec}(2x)}\right)$$

(*Solutions relying entirely on calculator technology are not acceptable.*)

(4)

(i)(a) $f(x) = \ln x \implies f(1) = 0$

$f'(x) = \frac{1}{x} \implies f'(1) = 1$

$f''(x) = \frac{1}{x^2} \implies f''(1) = 1$ ①

$\ln x = f(1) + (x-1)f'(1) - \frac{1}{2!}(x-1)^2 f''(1) + \ldots$

$\ln x = (x-1) - \frac{1}{2}(x-1)^2 + \ldots$ ①

(i)(b) $\lim_{x\to 1}\left(\frac{\ln x}{x-1}\right) = \lim_{x\to 1}\left(\frac{(x-1) - \frac{1}{2}(x-1)^2 + \ldots}{x-1}\right)$

$= \lim_{x\to 1}\left(1 - \frac{1}{2}(x-1) + \ldots\right)$ ①

As $x \mapsto 1$, $-\frac{1}{2}(x-1) \mapsto 0$ $\therefore \lim_{x\to 1}(1 - 0 + \ldots) = 1$ ①

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Question 8 continued

(ii) $\lim_{x \to 0} \left(\frac{1}{(x+3)\tan(6x)\operatorname{cosec}(2x)} \right)$ ← indeterminate because if $x=0$, $\operatorname{cosec}(0)$ is undefined.

Using $\lim_{x \to a} \frac{f(x)}{g(x)} = \lim_{x \to a} \frac{f'(x)}{g'(x)}$:

$$\frac{1}{(x+3)\tan(6x)\operatorname{cosec}(2x)} = \frac{\sin(2x)}{(x+3)\tan(6x)} \quad (1)$$

make top and bottom functions of x

let $f(x) = \sin(2x) \Rightarrow f'(x) = 2\cos 2x$ (1)

let $g(x) = (x+3)\tan 6x$

$u = x+3 \qquad v = \tan 6x$

$u' = 1 \qquad v' = 6\sec^2 6x$

$uv' + vu' = 6(x+3)\sec^2 6x + \tan 6x = g'(x)$ (1)

$$\lim_{x \to 0} \left(\frac{2\cos 2x}{6(x+3)\sec^2 6x + \tan 6x} \right) = \lim_{x \to 0} \left(\frac{2 \times 1}{18 \times \frac{1}{1} + 0} \right)$$

$$= \frac{2}{18}$$

$$= \frac{1}{9} \quad (1)$$

Question 8 continued

Question 8 continued

(Total for Question 8 is 10 marks)

9. A particle P moves along a straight line.

At time t minutes, the displacement, x metres, of P from a fixed point O on the line is modelled by the differential equation

$$t^2 \frac{d^2 x}{dt^2} - 2t\frac{dx}{dt} + 2x + 16t^2 x = 4t^3 \sin 2t \qquad \text{(I)}$$

(a) Show that the transformation $x = ty$ transforms equation (I) into the equation

$$\frac{d^2 y}{dt^2} + 16y = 4\sin 2t$$

(5)

(b) Hence find a general solution for the displacement of P from O at time t minutes.

(8)

(a) Need to find expressions for y and $\frac{dy}{dt}$.

$x = yt \Rightarrow y = x \times \frac{1}{t}$

let $u = x$, $v = \frac{1}{t}$

$u' = \frac{dx}{dt}$ $v' = -\frac{1}{t^2}$

differentiate terms w.r.t t

$uv' + vu' = -\frac{x}{t^2} + \frac{1}{t}\frac{dx}{dt} = \frac{dy}{dt}$ 1

differentiate again for $\frac{d^2y}{dx^2}$

$u = x$ $v = -\frac{1}{t^2}$ $\qquad$ $u = \frac{1}{t}$ $v = \frac{dx}{dt}$

$u' = \frac{dx}{dt}$ $v' = \frac{2}{t^3}$ $\qquad$ $u' = -\frac{1}{t^2}$ $v' = \frac{d^2x}{dt^2}$

$uv' + vu' = \frac{2x}{t^3} - \frac{1}{t^2}\frac{dx}{dt}$ $\qquad$ $uv' + vu' = \frac{1}{t}\frac{d^2x}{dt^2} - \frac{1}{t^2}\frac{dx}{dt}$

$\frac{d^2y}{dt^2} = \frac{2x}{t^3} - \frac{2}{t^2}\frac{dx}{dt} + \frac{1}{t}\frac{d^2x}{dt^2}$ 2

adding $16t^2x$ to this makes (I), and makes it equal to $4t^3\sin 2t$

$t^3\frac{d^2y}{dt^2} + 16t^2x = 4t^3\sin 2t$ 1

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Question 9 continued

$t^3 \frac{d^2y}{dt^2} + 16t^2(ty) = 4t^3 \sin 2t$ $\div t^3$

$\frac{d^2y}{dt^2} + 16y = 4\sin 2t$ 1

(b) $\frac{d^2y}{dt^2} + 16y = 4\sin 2t$ ← where $a = 1$, $b = 0$, $c = 16$, $f(x) = 4\sin 2t$

Auxiliary equation: $m^2 + 16 = 0$

$0^2 - 4 \times 1 \times 16 = -16$

$b^2 - 4ac < 0$ ∴ two complex conjugate roots $p \pm qi$

$m^2 = -16$

$m = 4i$ 1 ∴ $p \pm qi = 4i$, $p = 0$ and $q = 4$

Complementary function: $y = A\cos 4t + B\sin 2t$ 1

Particular Integral: $y = \lambda \sin 2t + \mu \cos 2t$ 1 ← using $f(x)$

$\frac{dy}{dt} = 2\lambda \cos 2t - 2\mu \sin 2t$

sub into original equation

$\frac{d^2y}{dx^2} = -4\lambda \sin 2t + 4\mu \cos 2t$ 1

$-4\lambda \sin 2t + 4\mu \cos 2t + 16(\lambda \sin 2t + \mu \cos 2t) = 4\sin 2t$

$(4\mu + 16\mu)\cos 2t + (16\lambda - 4\lambda)\sin 2t = 4\sin 2t$

$16\lambda - 4\lambda = 4$ $4\mu + 16\mu = 0$

$\lambda = \frac{1}{3}$ 1 $\mu = 0$

Question 9 continued

$GS = CF + PI$.

$y = A\cos 4t + B\sin 4t + \frac{1}{3}\sin 2t$ (1)

$x = ty$ (1)

$x = t\left[A\cos 4t + B\sin 4t + \frac{1}{3}\sin 2t\right]$ (1)

Question 9 continued

Question 9 continued

(Total for Question 9 is 13 marks)

TOTAL FOR PAPER IS 75 MARKS