MODEL ANSWERS

Please check the examination details below before entering your candidate information

Candidate surname	Other names

Pearson Edexcel Level 3 GCE

Centre Number | Candidate Number

Thursday 14 May 2020

Afternoon | Paper Reference **8FM0/21**

Further Mathematics
Advanced Subsidiary
Further Mathematics options
21: Further Pure Mathematics 1
(Part of options A, B, C and D)

You must have:
Mathematical Formulae and Statistical Tables (Green), calculator

Total Marks

Candidates may use any calculator allowed by Pearson regulations. Calculators must not have the facility for symbolic algebra manipulation, differentiation and integration, or have retrievable mathematical formulae stored in them.

Instructions

- Use **black** ink or ball-point pen.
- If pencil is used for diagrams/sketches/graphs it must be dark (HB or B).
- **Fill in the boxes** at the top of this page with your name, centre number and candidate number.
- Answer **all** questions and ensure that your answers to parts of questions are clearly labelled.
- Answer the questions in the spaces provided – *there may be more space than you need.*
- You should show sufficient working to make your methods clear. Answers without working may not gain full credit.
- Inexact answers should be given to three significant figures unless otherwise stated.

Information

- A booklet 'Mathematical Formulae and Statistical Tables' is provided.
- The total mark for this part of the examination is 40. There are 5 questions.
- The marks for **each** question are shown in brackets – *use this as a guide as to how much time to spend on each question.*

Advice

- Read each question carefully before you start to answer it.
- Try to answer every question.
- Check your answers if you have time at the end.

Turn over ▶

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Pearson

1. The variables x and y satisfy the differential equation

$$\frac{d^2y}{dx^2} = 2y^2 - x - 1$$

where $\frac{dy}{dx} = 3$ and $y = 0$ at $x = 0$

Use the approximations

$$\left(\frac{d^2y}{dx^2}\right)_n \approx \frac{(y_{n+1} - 2y_n + y_{n-1})}{h^2} \quad \text{and} \quad \left(\frac{dy}{dx}\right)_n \approx \frac{(y_{n+1} - y_{n-1})}{2h}$$

with $h = 0.1$ to find an estimate for the value of y at $x = 0.2$

(7)

$\frac{d^2y}{dx^2} = 2y^2 - x - 1 \Rightarrow$ @ $(0,0)$, $\frac{d^2y}{dx^2} = 0 - 0 - 1 = -1$

find y' & y'' @ $n = 0$

given $\left.\frac{dy}{dx}\right|_{(0,0)} = 3$

using approximations given: $\left(\frac{dy}{dx}\right)_0 = \frac{y_1 - y_{-1}}{0.2} \simeq 3$

'reverse' approx. to find y @ next step

$y_0 = 0$

$\left(\frac{d^2y}{dx^2}\right)_0 \simeq \frac{(y_1 - 2y_0 + y_{-1})}{h^2} \simeq -1 \Rightarrow \frac{y_1 - 2(0) + y_{-1}}{(0.1)^2} \simeq -1$

so $y_1 - y_{-1} \simeq 0.6$

$+$

$y_1 + y_{-1} \simeq -0.01$

$2y_1 \simeq 0.6 - 0.01$

$y_1 \simeq \frac{0.59}{2} = 0.295$

@ $x = 0.1$

solve simultaneous equations for y_1

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Question 1 continued

find y' & y'' @ $n=1$

$$\frac{d^2y}{dx^2} = 2y^2 - x - 1 \Rightarrow \left.\frac{d^2y}{dx^2}\right|_{x=0.1} = 2(0.295)^2 - 0.1 - 1 = -0.92595$$

plug back in approx to find y_2

$$\left(\frac{d^2y}{dx^2}\right)_1 \simeq \frac{(y_2 - 2y_1 + y_0)}{h^2} \Rightarrow \frac{y_2 - 2(0.295) + 0}{0.01} \simeq -0.92595$$

$$\therefore y_2 \simeq -0.0092595 + 2(0.295) = 0.5807\ldots$$
$$= 0.581 \quad \text{(3sf)}$$

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Question 1 continued

Question 1 continued

(Total for Question 1 is 7 marks)

2. Use algebra to determine the values of x for which

$$\frac{x+1}{2x^2+5x-3} > \frac{x}{4x^2-1}$$

(5)

first, we factorise the denominators so we can identify common factors & subtract the fractions

$$\frac{x+1}{(2x-1)(x+3)} > \frac{x}{(2x+1)(2x-1)}$$

$$\frac{(x+1)(2x+1)-x(x+3)}{(2x+1)(2x-1)(x+3)} > 0$$

$$\frac{2x^2+3x+1-(x^2+3x)}{(2x+1)(2x-1)(x+3)} > 0$$

$$\frac{x^2+1}{(2x+1)(2x-1)(x+3)} > 0$$

for $x \in \mathbb{R}$, x^2+1 is always >0 $\Rightarrow$ we need denominator to be >0 for inequality to hold

$(2x+1)(2x-1)(x+3) > 0$

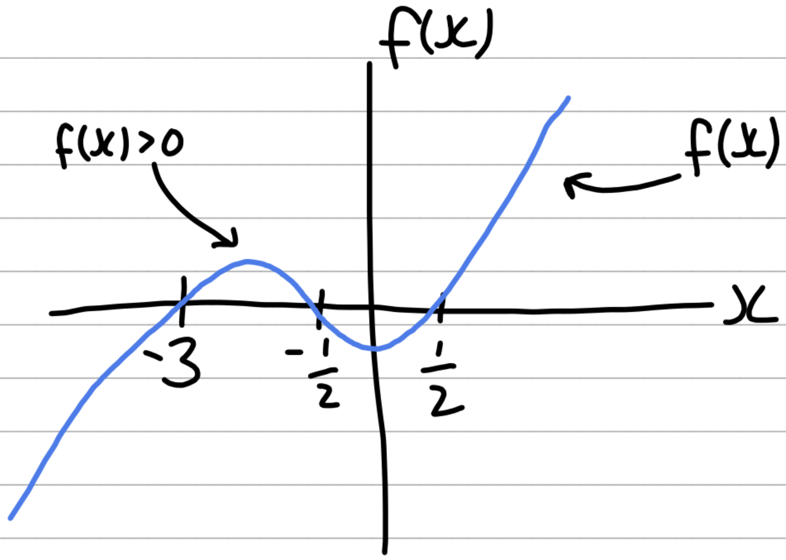
$\hookrightarrow$ roots: $\pm\frac{1}{2}, -3$

sketch polynomial:

when $x \to -\infty$, poly. is 3 -ve terms multiplied $\Rightarrow f(x) \to -\infty$

when $x \to +\infty$, poly. is 3 +ve terms multiplied $\Rightarrow f(x) \to +\infty$

Question 2 continued



solution : $\{x \in \mathbb{R} : -3 < x < -\frac{1}{2}\} \cup \{x \in \mathbb{R} : x > \frac{1}{2}\}$

(Total for Question 2 is 5 marks)

3. (i) Use the substitution $t = \tan\left(\frac{x}{2}\right)$ to prove that

$$\cot x + \tan\left(\frac{x}{2}\right) = \operatorname{cosec} x \qquad x \neq n\pi,\ n \in \mathbb{Z}$$

(2)

(ii)

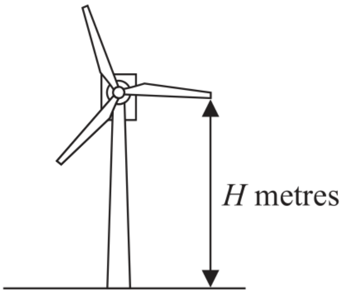


Figure 1

An engineer models the vertical height above the ground of the tip of one blade of a wind turbine, shown in Figure 1. The ground is assumed to be horizontal.

The vertical height of the tip of the blade above the ground, H metres, at time x seconds after the wind turbine has reached its constant operating speed, is modelled by the equation

$$H = 90 - 30\cos(120x)° - 40\sin(120x)° \qquad \text{(I)}$$

(a) Show that $H = 60$ when $x = 0$

(1)

Using the substitution $t = \tan(60x)°$

(b) show that equation (I) can be rewritten as

$$H = \frac{120t^2 - 80t + 60}{1 + t^2}$$

(3)

(c) Hence find, according to the model, the value of x when the tip of the blade is 100 m above the ground for the first time after the wind turbine has reached its constant operating speed.

(5)

i. $t = \tan\left(\frac{x}{2}\right)$

$\cot x = \frac{1}{\tan x}$

$\tan x = \frac{2\tan\left(\frac{x}{2}\right)}{1 - \tan^2\left(\frac{x}{2}\right)} \Rightarrow \cot x = \frac{1-t^2}{2t}$

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Question 3 continued

so LHS $= \frac{1-t^2}{2t} + t$ ← $\left(\frac{1-t^2}{2t} + \frac{2t^2}{2t}\right)$

$= \frac{1+t^2}{2t}$

standard result

$= \frac{1}{\sin x}$

$= \boxed{\operatorname{cosec} x}$

ii. a) $x = 0 \Rightarrow H = 90 - 30\cos(0) - 40\sin(0)$

$= 90 - 30$

$= \boxed{60}$

b) if $t = \tan\left(\frac{\theta}{2}\right)$, then $\cos\theta = \frac{1-t^2}{1+t^2}$ & $\sin\theta = \frac{2t}{1+t^2}$

$\theta = 120x \Rightarrow H = 90 - 30\left(\frac{1-t^2}{1+t^2}\right) - 40\left(\frac{2t}{1+t^2}\right)$

$= \frac{90(1+t^2) - 30 + 30t^2 - 80t}{(1+t^2)}$

$= \boxed{\frac{120t^2 - 80t + 60}{(1+t^2)}}$

c) $H = 100 = \frac{120t^2 - 80t + 60}{1+t^2}$

$\Rightarrow 120t^2 - 80t + 60 = 100 + 100t^2$

$20t^2 - 80t - 40 = 0 \Rightarrow 2t^2 - 8t - 4 = 0 \Rightarrow t^2 - 4t - 2 = 0$

(dividing both sides by 10) (dividing both sides by 2)

Question 3 continued

$$t = \frac{4 \pm \sqrt{4^2 + 4(2)}}{2(1)} = 2 \pm \sqrt{6}$$

so $60x = \tan^{-1}(2+\sqrt{6})$ or $60x = \tan^{-1}(2-\sqrt{6})$

x is positive (time) $\Rightarrow$ need positive root

$\tan^{-1}(2+\sqrt{6}) = 77.334$ (remember to use degrees)

$\therefore x = \frac{77.334}{60} = 1.29$ (3 s.f.)

Question 3 continued

(Total for Question 3 is 11 marks)

4.

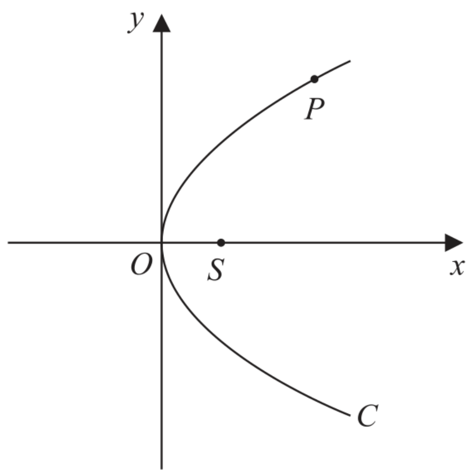

Figure 2

Figure 2 shows a sketch of the parabola C with equation $y^2 = 4ax$, where a is a positive constant. The point S is the focus of C and the point $P(ap^2, 2ap)$ lies on C where $p > 0$

(a) Write down the coordinates of S.

(1)

(b) Write down the length of SP in terms of a and p.

(1)

The point $Q(aq^2, 2aq)$, where $p \neq q$, also lies on C.
The point M is the midpoint of PQ.

Given that $pq = -1$

(c) prove that, as P varies, the locus of M has equation

$$y^2 = 2a(x - a)$$

(5)

a) $(a,0)$ for a curve of form $y^2=4ax$, its focus is @ $x=a$

b) focus-directrix relation: $SP=PX$, where $X(-a$, ycoord of $P)$

P has x-coord $ap^2 \Rightarrow PX = pa^2+a$

$\therefore SP = ap^2+a$

c) M is the midpoint of PQ so it has coordinates

$$\left(\frac{ap^2+aq^2}{2}, \frac{2ap+2aq}{2}\right)$$

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Question 4 continued

so $y_M = a(p+q) \Rightarrow y_M^2 = a^2(p^2+2pq+q^2)$

we are told $pq=-1 \Rightarrow y^2 = a^2(p^2-2+q^2)$

sub our x-value for M into the equation given:

$2a(x-a) = 2a(\frac{1}{2}ap^2+\frac{1}{2}aq^2-a) = a^2(p^2+q^2-2)$

equal

$\therefore y^2 = 2a(x-a)$ describes M

Question 4 continued

Question 4 continued

(Total for Question 4 is 7 marks)

5.

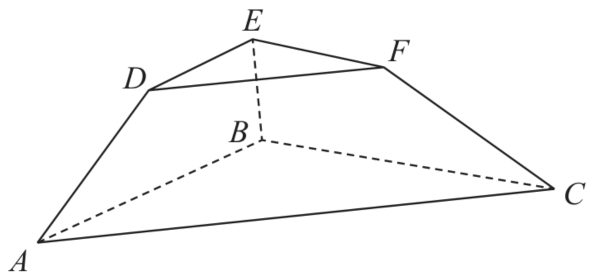

Figure 3

Figure 3 shows a solid display stand with parallel triangular faces *ABC* and *DEF*.
Triangle *DEF* is similar to triangle *ABC*.

With respect to a fixed origin *O*,
the points *A*, *B* and *C* have coordinates (3, –3, 1), (–5, 3, 3) and (1, 7, 5) respectively and the points *D*, *E* and *F* have coordinates (2, –1, 8), (–2, 2, 9) and (1, 4, 10) respectively.
The units are in centimetres.

(a) Show that the area of the triangular face *DEF* is $\frac{1}{2}\sqrt{339}$ cm^2 **(3)**

(b) Find, in cm^3, the exact volume of the display stand. **(7)**

a) area $= \frac{1}{2}|\underline{b} \times \underline{c}| \Rightarrow$ we need $\overrightarrow{DE}$ & $\overrightarrow{DF}$

$$\pm\overrightarrow{DE} = \pm\begin{pmatrix}-2\\2\\9\end{pmatrix} \mp \begin{pmatrix}2\\-1\\8\end{pmatrix} = \pm\begin{pmatrix}-4\\3\\1\end{pmatrix}$$

$$\pm\overrightarrow{DF} = \pm\begin{pmatrix}1\\4\\10\end{pmatrix} \mp \begin{pmatrix}2\\-1\\8\end{pmatrix} = \pm\begin{pmatrix}-1\\5\\2\end{pmatrix}$$

$$\text{area} = \frac{1}{2}|\overrightarrow{DE} \times \overrightarrow{DF}| = \begin{vmatrix}\underline{i} & \underline{j} & \underline{k}\\ -4 & 3 & 1\\ -1 & 5 & 2\end{vmatrix} \times \frac{1}{2} = \frac{1}{2}\left|\left[\underline{i}(3\times2 - 1\times5) - \underline{j}(-4\times2 - 1\times-1) + \underline{k}(-4\times5 - 3\times-1)\right]\right|$$

using the 'cross product'

$$= \frac{1}{2}\left|\begin{pmatrix}1\\7\\-17\end{pmatrix}\right|$$

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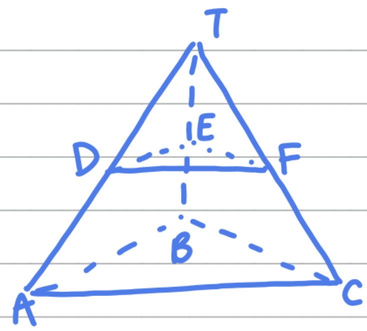
Question 5 continued

$= \frac{1}{2}\sqrt{1^2+7^2+17^2}$

$= \frac{1}{2}\sqrt{339}$ (cm²)

b) extend top of volume to give a tetrahedron

use similar triangles:



$DF = \sqrt{1^2+5^2+2^2}$

$= \sqrt{30}$

$\vec{AC} = \begin{pmatrix}1\\7\\5\end{pmatrix} - \begin{pmatrix}3\\-3\\1\end{pmatrix} = \begin{pmatrix}-2\\10\\4\end{pmatrix}$

$\Rightarrow AC = \sqrt{2^2+10^2+4^2}$

$= \sqrt{120}$

$\frac{AC}{DF} = 2$ = scale factor between triangles TDF & TAC

so $AT = 2AD$

$\vec{AD} = \begin{pmatrix}2\\-1\\8\end{pmatrix} - \begin{pmatrix}3\\-3\\1\end{pmatrix} = \begin{pmatrix}-1\\2\\7\end{pmatrix}$

$D + \vec{AD} = \begin{pmatrix}1\\1\\15\end{pmatrix} \Rightarrow T(1,1,15)$

Question 5 continued

$\overrightarrow{AT} = \begin{pmatrix}1\\1\\15\end{pmatrix} - \begin{pmatrix}3\\-3\\1\end{pmatrix} = \begin{pmatrix}-2\\4\\14\end{pmatrix}$ $\quad \overrightarrow{BT} = \begin{pmatrix}1\\1\\15\end{pmatrix} - \begin{pmatrix}-5\\3\\3\end{pmatrix} = \begin{pmatrix}6\\-2\\12\end{pmatrix}$

$\overrightarrow{CT} = \begin{pmatrix}1\\1\\15\end{pmatrix} - \begin{pmatrix}1\\7\\5\end{pmatrix} = \begin{pmatrix}0\\-6\\10\end{pmatrix}$

$V = \frac{1}{6}|\underline{a}\cdot(\underline{b}\times\underline{c})|$

$\overrightarrow{AT} \times \overrightarrow{BT}\cdot\overrightarrow{CT} = \begin{vmatrix}-2 & 4 & 14\\ 6 & -2 & 12\\ 0 & -6 & 10\end{vmatrix} = |-2(-2\times10-12\times-6)\underline{i} - 4(6\times10-0\times12)\underline{j} + 14(6\times-6-0\times-2)\underline{k}|$

V of tetrahedron $= \frac{1}{6}|-2(52)-4(60)+14(-36)|$

$= \frac{424}{3}$

linear scale factor is 2 $\Rightarrow$ volume s.f. is $8 = 2^3$

TEDF volume is $\frac{1}{8}$ volume of whole tetrahedron

$\therefore$ volume of stand $= \frac{7}{8} \times \frac{424}{3} = \frac{371}{3}\text{ cm}^3$

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Question 5 continued

Question 5 continued

(Total for Question 5 is 10 marks)

TOTAL FOR FURTHER PURE MATHEMATICS 1 IS 40 MARKS