Pearson Edexcel Level 3 GCE

Further Mathematics

Advanced
Further Mathematics Option 2
Paper 4: Decision Mathematics 2

Sample Assessment Material for first teaching September 2017

Time: 1 hour 30 minutes

Paper Reference
9FM0/4G

You must have:
Decision Mathematics Answer Book (enclosed), calculator

Candidates may use any calculator permitted by Pearson regulations. Calculators must not have the facility for algebraic manipulation, differentiation and integration, or have retrievable mathematical formulae stored in them.

Instructions

- Use **black** ink or ball-point pen.
- If pencil is used for diagrams/sketches/graphs it must be dark (HB or B).
- Write your answers for this paper in the Decision Mathematics answer book provided.
- **Fill in the boxes** at the top of the answer book with your name, centre number and candidate number.
- Do not return the question paper with the answer book.
- Answer **all** questions and ensure that your answers to parts of questions are clearly labelled.
- Answer the questions in the spaces provided – *there may be more space than you need.*
- You should show sufficient working to make your methods clear. Answers without working may not gain full credit.
- Answers should be given to three significant figures unless otherwise stated.

Information

- There are 7 questions in this question paper. The total mark for this paper is 75.
- The marks for **each** question are shown in brackets – *use this as a guide as to how much time to spend on each question.*

Advice

- Read **each** question carefully before you start to answer it.
- Try to answer every question.
- Check your answers if you have time at the end.

Turn over

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Answer ALL questions. Write your answers in the answer book provided.

1. (a) Find the general solution of the recurrence relation

$$u_{n+2} = u_{n+1} + u_n, \quad n \geqslant 1$$

(3)

Given that $u_1 = 1$ and $u_2 = 1$

(b) find the particular solution of the recurrence relation.

(3)

(Total for Question 1 is 6 marks)

1. Auxiliary equation: $\lambda^2 - \lambda - 1 = 0$

solve using quadratic formula $x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$a = 1$

$b = -1$ $\quad x = \dfrac{1 \pm \sqrt{(-1)^2 - 4 \times 1 \times -1}}{2}$

$c = -1$

$= \dfrac{1 \pm \sqrt{5}}{2}$

$\Rightarrow u_n = A\left(\dfrac{1+\sqrt{5}}{2}\right)^n + B\left(\dfrac{1-\sqrt{5}}{2}\right)^n$ where A and B are arbitrary constants.

(1b) Use the given conditions $u_1 = 1$ and $u_2 = 1$ to obtain two equations in A and B:

$u_1 = A\left(\dfrac{1+\sqrt{5}}{2}\right)^1 + B\left(\dfrac{1-\sqrt{5}}{2}\right)^1 = 1$

$\Rightarrow A\left(\dfrac{1+\sqrt{5}}{2}\right) + B\left(\dfrac{1-\sqrt{5}}{2}\right) = 1$

$\Rightarrow A(1+\sqrt{5}) + B(1-\sqrt{5}) = 2 \quad (1)$

and

$u_2 = A\left(\dfrac{1+\sqrt{5}}{2}\right)^2 + B\left(\dfrac{1-\sqrt{5}}{2}\right)^2 = 1$

$\Rightarrow A\dfrac{(1+\sqrt{5})(1+\sqrt{5})}{4} + B\dfrac{(1-\sqrt{5})(1-\sqrt{5})}{4} = 1$

$\Rightarrow A\dfrac{(1+2\sqrt{5}+5)}{4} + B\dfrac{(1-2\sqrt{5}+5)}{4} = 1$

$\Rightarrow A\left(\dfrac{3+\sqrt{5}}{2}\right) + B\left(\dfrac{3-\sqrt{5}}{2}\right) = 1$

$\Rightarrow A(3+\sqrt{5}) + B(3-\sqrt{5}) = 2 \quad (2)$

Solve these equations simultaneously to obtain A and B.

$3A + \sqrt{5}A + 3B - \sqrt{5}B = 2$

$\underline{A + \sqrt{5}A + B - \sqrt{5}B = 2}$

$\Rightarrow 2A + 2B = 0 \quad \Rightarrow A = -B$

Question 1 continued

sub $A=-B$ into (1)

$$-B(1+\sqrt{5})+B(1-\sqrt{5})=2$$
$$\Rightarrow -B-\sqrt{5}B+B-\sqrt{5}B=2$$
$$\Rightarrow -2\sqrt{5}B=2$$
$$\Rightarrow \sqrt{5}B=-1$$
$$\Rightarrow B=\frac{-1}{\sqrt{5}}$$

so $A=-B=\frac{1}{\sqrt{5}}$

so we have

$$u_n=\frac{1}{\sqrt{5}}\left(\frac{1+\sqrt{5}}{2}\right)^n+B\left(\frac{1-\sqrt{5}}{2}\right)^n$$
$$=\frac{1}{\sqrt{5}}\left\{\left(\frac{1+\sqrt{5}}{2}\right)^n-\left(\frac{1-\sqrt{5}}{2}\right)^n\right\}$$

(Total for Question 1 is 6 marks)

2.

	D	E	F	Available
A	15	19	9	25
B	11	18	10	55
C	11	12	18	20
Required	38	24	38	

A company has three factories, A, B and C. It supplies mattresses to three shops, D, E and F. The table shows the transportation cost, in pounds, of moving one mattress from each factory to each shop. It also shows the number of mattresses available at each factory and the number of mattresses required at each shop. A minimum cost solution is required.

(a) Use the north-west corner method to obtain an initial solution.

(1)

(b) Show how the transportation algorithm is used to solve this problem.

You must state, at each appropriate step, the

- shadow costs,
- improvement indices,
- route,
- entering cell and exiting cell,

and explain clearly how you know that your final solution is optimal.

(11)

(Total for Question 2 is 12 marks)

2.

	D	E	F	Available
A	15	19	9	25
B	11	18	10	55
C	11	12	18	20
Required	38	24	38	

step ①
copy
↳ total amount allowed in each row

② total amount allowed in each column

③ need 38 in column but max for that row is 25.

④ 38 − 25 = 13

(a)

	D	E	F	Available
A	25			25
B	13	24	18	55
C			20	20
Required	38	24	38	

⑤ need a total of 55 in row but have to take into account max allowed per column

Check: All rows & columns add to required amount.

(b)

	D	E	F	
A				

b)

SCs

		15	22	14
		D	E	F
0	A	X	−3	−5
−4	B	X	X	X
4	C	−8	−14	X

Improvement indices
AE = 19 − 22 = −3
AF = 9 − 14 = −5
CD = 11 − 15 − 4 = −8
CE = 12 − 22 − 4 = −14 => most negative
so entering cell is CE

	D	E	F
A			
B		24−θ	18+θ
C		+θ	20−θ

θ = 20
CF is exiting cell

SCs

		15	22	14
		D	E	F
0	A	25	−3	−5
−4	B	13	4	38
−10	C	6	20	14

Improvement indices
AE = 19 − 22 = −3
AF = 9 − 14 = −5 => entering cell is AF
CD = 11 + 10 − 15 = 6
CF = 18 + 10 − 14 = 14

	D	E	F
A	25−θ		+θ
B	13+θ		38−θ
C			

θ = 25, exiting cell is AD

SCs

		10	17	9
		D	E	F
0	A	5	2	25
1	B	38	4	13
−5	C	6	20	14

Improvement indices
AD = 15 − 10 = 5
AE = 19 − 17 = 2
CD = 11 + 5 − 10 = 6
CF = 18 + 5 − 9 = 14

No negative improvement indices => solution is optimal at

25(9) + 13(10) + 4(18) + 38(11) + 20(12)
= £1085

Question 2 continued

	D	E	F	
A				
B				
C				

	D	E	F	
A				
B				
C				

	D	E	F	
A				
B				
C				

	D	E	F	
A				
B				
C				

	D	E	F	
A				
B				
C				

	D	E	F	
A				
B				
C				

	D	E	F	
A				
B				
C				

(Total for Question 2 is 12 marks)

3. Four workers, A, B, C and D, are to be assigned to four tasks, P, Q, R and S.

Each worker must be assigned to at most one task and each task must be done by just one worker.

The amount, in pounds, that each worker would earn while assigned to each task is shown in the table below.

	P	Q	R	S
A	32	32	33	35
B	28	35	31	37
C	35	29	33	36
D	36	30	36	33

The Hungarian algorithm is to be used to find the maximum total amount which may be earned by the four workers.

(a) Explain how the table should be modified.

(1)

(b) Reducing rows first, use the Hungarian algorithm to obtain an allocation which maximises the total earnings, stating how each table was formed.

(7)

(c) Formulate the problem as a linear programming problem. You must define your decision variables and make your objective function and constraints clear.

(5)

(Total for Question 3 is 13 marks)

3. (a) subtract each entry from a constant (easier to work with smaller numbers)

e.g. 37

typically pick the largest number

	P	Q	R	S
A	32	32	33	35
B	28	35	31	37
C	35	29	33	36
D	36	30	36	33

$\boxed{37-x}$

	P	Q	R	S
A	5	5	4	2
B	9	2	6	0
C	2	8	4	1
D	1	7	1	4

REDUCE ROWS

smallest number in row.

	P	Q	R	S
A	3	3	2	0
B	9	2	6	0
C	1	7	3	0
D	0	6	0	3

← reduce row A by 2
← no reduction for row B as smallest number in row is zero
← reduce row C by 1
← reduce row D by 1

REDUCE COLUMNS

	P	Q	R	S
A	3	1	2	0
B	9	0	6	0
C	1	5	3	0
D	0	4	0	3

reduce column Q by 2. no reduction for any other column.

	P	Q	R	S
A	3	1	2	0
B	9	0	6	0
C	1	5	3	0
D	0	4	0	3

We need 3 lines to cover all the zeros, hence solution is not optimal.
- augment by 1.

(subtract 1 to all and add 1 to the where the lines

	P	Q	R	S
A	2	1	1	0
B	8	0	5	0
C	0	5	2	0
D	0	5	0	4

now optimal
A to S, B to Q, C to P, D to R.

DO NOT WRITE IN THIS AREA

Question 3 continued

	P	Q	R	S
A				
B				
C				
D				

	P	Q	R	S
A				
B				
C				
D				

(3c) let $x_{ij} = \begin{cases} 1 & \text{if worker } i \text{ does task } j \\ 0 & \text{otherwise} \end{cases}$

where $i \in \{A, B, C, D\}$ and $j \in \{P, Q, R, S\}$

original entry in AP box

minimise $5x_{AP} + 5x_{AQ} + 4x_{AR} + 2x_{AS} + 9x_{BP} + 2x_{BQ} + 6x_{BR} + 2x_{CP}$
$+ 8x_{CQ} + 4x_{CR} + x_{CS} + x_{DP} + 7x_{DQ} + x_{DR} + 4x_{DS}$

Subject to:

$\sum x_{iP} = 1$, $\sum x_{iQ} = 1$, $\sum x_{iR} = 1$, $\sum x_{iS} = 1$

$\sum x_{Aj} = 1$, $\sum x_{Bj} = 1$, $\sum x_{Cj} = 1$, $\sum x_{Dj} = 1$

(Total for Question 3 is 13 marks)

4. A game uses a standard pack of 52 playing cards.

A player gives 5 tokens to play and then picks a card. If they pick a 2, 3, 4, 5 or 6 then they gain 15 tokens. If any other card is picked they lose.

If they lose, the card is replaced and they can choose to pick again for another 5 tokens. This time if they pick either an ace or a king they gain 40 tokens. If any other card is picked they lose.

Daniel is deciding whether to play this game.

(a) Draw a decision tree to model Daniel's possible decisions and the possible outcomes.

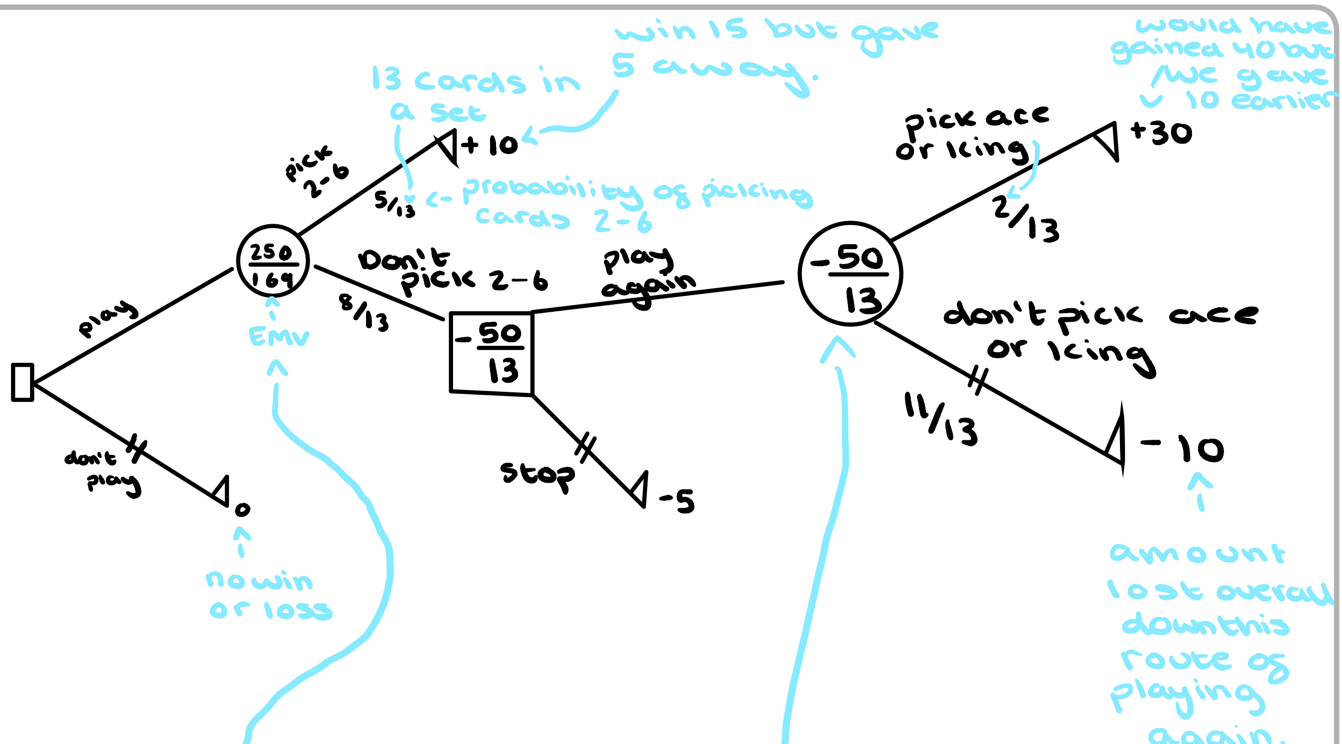
(6)

(b) Calculate Daniel's optimal EMV and state the optimal strategy indicated by the decision tree.

(2)

(Total for Question 4 is 8 marks)

4.



$$\frac{5}{13} \times 10 + \left(\frac{8}{13} \times -\frac{50}{13}\right) = \frac{250}{169}$$

$$\frac{2}{13} \times 30 + \left(\frac{11}{13} \times -10\right)$$

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Question 4 continued

(4b) EMV is 1.48 (tokens) per game (correct to 3sf)

greatest EMV value

Analysis: Play the game and if the player doesn't pick 2-6 on first go they should play again.

(Total for Question 4 is 8 marks)

5.

	B plays 1	B plays 2	B plays 3	B plays 4
A plays 1	4	−2	3	2
A plays 2	3	−1	2	0
A plays 3	−1	2	0	3

A two person zero-sum game is represented by the pay-off matrix for player A given above.

(a) Explain, with justification, how this matrix may be reduced to a 3×3 matrix.

(2)

(b) Find the play-safe strategy for each player and verify that there is no stable solution to this game.

(4)

The game is formulated as a linear programming problem for player A.

The objective is to maximise $P = V$, where V is the value of the game to player A.

One of the constraints is that $p_1 + p_2 + p_3 \leqslant 1$, where p_1, p_2, p_3 are the probabilities that player A plays 1, 2, 3 respectively.

(c) Formulate the remaining constraints for this problem. Write these constraints as inequalities.

(3)

The Simplex algorithm is used to solve the linear programming problem.

The solution obtained is $p_1 = 0$, $p_2 = \frac{3}{7}$, $p_3 = \frac{4}{7}$

(d) Calculate the value of the game to player A.

(3)

(Total for Question 5 is 12 marks)

5.

	B plays 1	B plays 2	B plays 3	B plays 4
A plays 1	4	–2	3	2
A plays 2	3	–1	2	0
A plays 3	–1	2	0	3

(a) column 2 dominates column 4.

Because $2 > -2$

$0 > -1$

$3 > 2$

column 2 smaller so dominates

(player B loses less (-ve number means B gains since this is a payoff matrix for A.)

largest min value

(b) row minima: $-2, -1, -1$, max is -1

Column maxima: $4, 2, 3$, min is 2

min of max values

ignore column 4 as has been dominated

Playsafe is A plays row 2 or 3 — rows including maximin

B plays column 2

column including minimax

Row maximum $(-1) \neq$ column minimax (2) so solution is not stable.

(c)

e.g. add 2 to all entries to make all positive

$$\begin{pmatrix} 4 & -2 & 3 \\ 3 & -1 & 2 \\ -1 & 2 & 0 \end{pmatrix} \longrightarrow \begin{pmatrix} 6 & 0 & 5 \\ 5 & 1 & 4 \\ 1 & 4 & 2 \end{pmatrix}$$

column values where $p_1, p_2 \& p_3$ are the probabilities A plays 1,2,3 respectively.

Subject to $V - 6p_1 - 5p_2 - p_3 \leq 0$

$V - p_2 - 4p_3 \leq 0$

$V - 5p_1 - 4p_2 - 2p_3 \leq 0$

DO NOT WRITE IN THIS AREA

Question 5 continued

(d) substitute $p_1 = 0$, $p_2 = 3/7$, $p_3 = 4/7$ into inequalities from part (c) to obtain,

$v \leq 6(0) + 5(3/7) + 4/7 = 15/7 + 4/7 = 19/7$

$v \leq 3/7 + 4(4/7) = 3/7 + 16/7 = 19/7$

$v \leq 5(0) + 4(3/7) + 2(4/7) = 12/7 + 8/7 = 20/7$

so $v \leq 19/7, 19/7, 20/7$ therefore $v = \frac{19}{7}$

lowest value

so value of game to player A $= \frac{19}{7} - 2$

$= \frac{5}{7}$

(Total for Question 5 is 12 marks)

6.

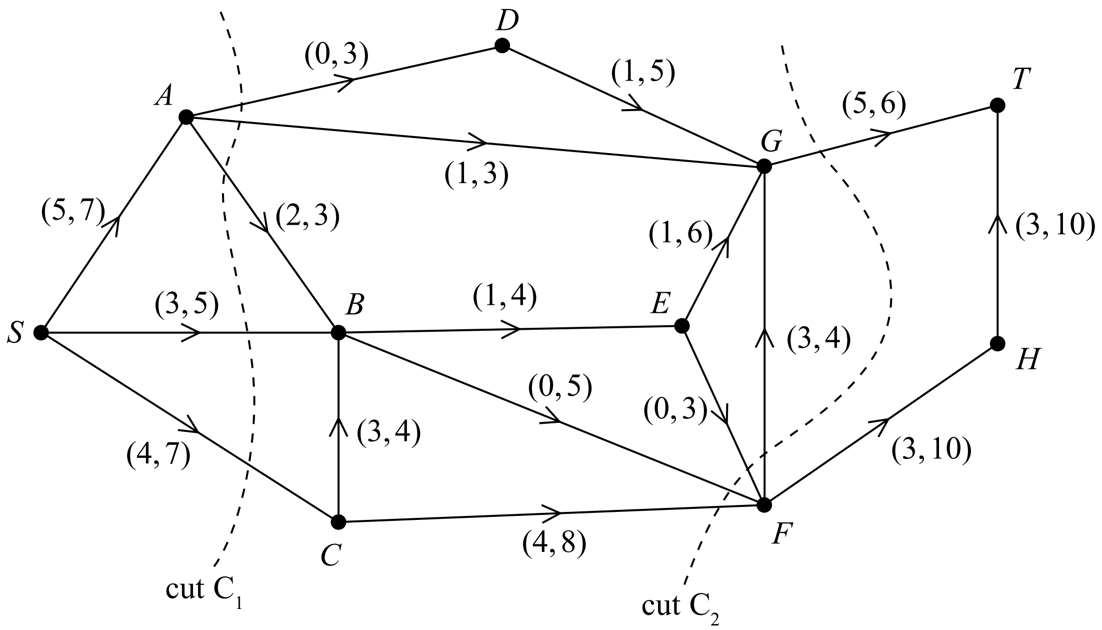

Figure 1

Figure 1 shows a capacitated, directed network. The number on each arc (x, y) represents the lower (x) capacity and upper (y) capacity of that arc.

(a) Calculate the value of the cut C_1 and cut C_2

(2)

(b) Explain why the flow through the network must be at least 12 and at most 16

(1)

(c) Explain why arcs DG, AG, EG and FG must all be at their lower capacities.

(1)

(d) Determine a maximum flow pattern for this network and draw it on Diagram 1 in the answer book. You do not need to use the labelling procedure.

(2)

(e) (i) State the value of the maximum flow through the network.

(ii) Explain why the value of the maximum flow is equal to the value of the minimum flow through the network.

(3)

Node E becomes blocked and no flow can pass through it. To maintain the maximum flow through the network the upper capacity of exactly one arc is increased.

(f) Explain how it is possible to maintain the maximum flow found in (d).

(3)

(Total for Question 6 is 12 marks)

6.

upper (y) capacity of the arcs the cut goes through.

$C_1 = 3+3+3+5+7$
$= 21$

arrow going into the cut

$C_2 = 8+5+3+6-3$
$= 19$

x values

(b) The minimum flow out of the source S is at least 5+3+4 = 12 and the maximum flow into the sink T is 6+10 = 16

y values

(c) The minimum flow into G is 1+1+1+3 = 6 but the maximum flow out of G is 6 therefore the arcs into G must be at their lower capacities.

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Question 6 continued

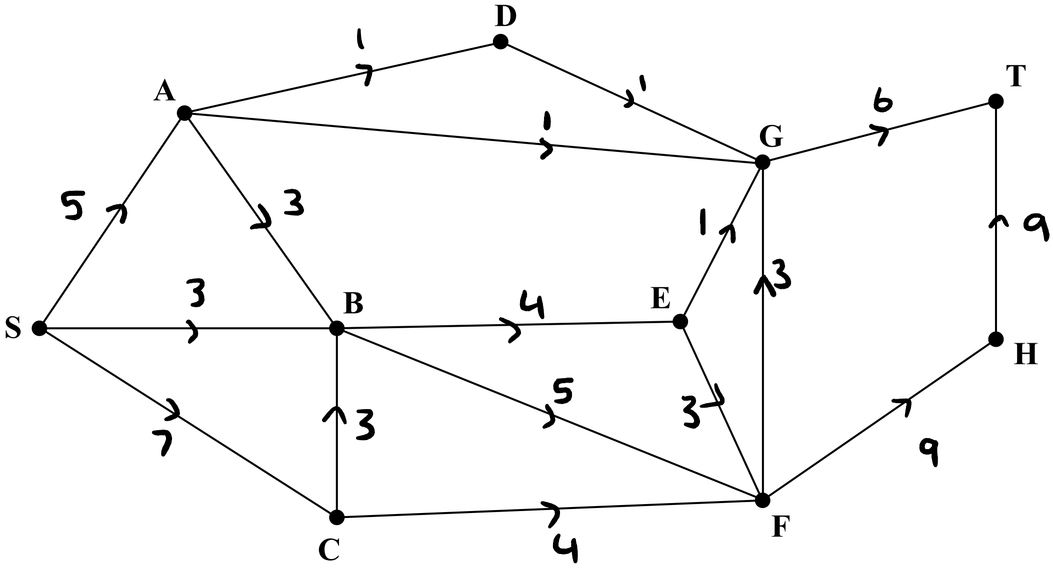

Diagram 1

(bei) Maximum flow is 15

(beii) The minimum flow out of the source is 12 but the flow out of C is at least 3+4=7. Therefore, the minimum flow through the network is 5+3+4+3=15 which is equal to the maximum flow.

5+3+4

(bf) Increase the upper capacity of arc BF to at least 9 and therefore increase the flow in this arc to 9.
Therefore, increase the flow in FH and HT to 10.
The flow in GT decreases to 5 and all other arcs are unchanged.

(Total for Question 6 is 12 marks)

7. A company assembles boats.

They can assemble up to five boats in any one month, but if they assemble more than three they will have to hire additional space at a cost of £800 per month.

The company can store up to two boats at a cost of £350 each per month.

The overhead costs are £1500 in any month in which work is done.

Boats are delivered at the end of each month. There are no boats in stock at the beginning of January and there must be none in stock at the end of May.

The order book for boats is

Month	January	February	March	April	May
Number ordered	3	2	6	3	4

Use dynamic programming to determine the production schedule which minimises the costs to the company. Show your working in the table provided in the answer book and state the minimum production cost.

(Total for Question 7 is 12 marks)

TOTAL FOR PAPER IS 75 MARKS

so for May use value 4 from Q.

7.

works backwards

number in stock at start of month

number made in month

must have 0 left in stock at end of May.

store 2 boats at £350 each

Stage	State	Action	Dest	Value
May	2	2	0	700 + 1500 = 2200 *
	1	3	0	350 + 1500 = 1850 *
	0	4	0	1500 + 800 = 2300 *
April	2	1	0	700 + 1500 + 2300 = 4500
	2	2	1	700 + 1500 + 1850 = 4050 *
	2	3	2	700 + 1500 + 2200 = 4400
	1	2	0	350 + 1500 + 2300 = 4150
	1	3	1	350 + 1500 + 1850 = 3700 *
	1	4	2	350 + 1500 + 800 + 2200 = 4850
	0	3	0	1500 + 2300 = 3800 *
	0	4	1	1500 + 800 + 1850 = 4150
	0	5	2	1500 + 800 + 2200 = 4500
March	2	4	0	700 + 1500 + 800 + 3800 = 6800
	2	5	1	700 + 1500 + 800 + 3700 = 6700 *
	1	5	0	350 + 1500 + 800 + 3800 = 6450 *
Feb	2	1	1	700 + 1500 + 6450 = 8650 *
	2	2	2	700 + 1500 + 6700 = 8900
	1	2	1	350 + 1500 + 6450 = 8300 *
	1	3	2	350 + 1500 + 6700 = 8550
	0	3	1	1500 + 6450 = 7950 *
	0	4	2	1500 + 800 + 6700 = 9000
Jan	0	3	0	1500 + 7950 = 9450 *
	0	4	1	1500 + 800 + 8300 = 10600
	0	5	2	1500 + 800 + 8650 = 10950

making more than 3 - as sell 3 in April.

how many left over

use as 0 left over for start of next month.

pick the smallest cost for each state to use in March.

max to assemble is 5.

can only store max 2 boats

only 0 allowed at start of Jan

Month	Jan	Feb	March	April	May
Number made	3	3	5	3	4

work backwards through the final star

minimum production cost = £9450

Month	January	February	March	April	May
Number assembled					

Question 7 continued

(Total for Question 7 is 12 marks)

TOTAL FOR PAPER IS 75 MARKS