MODEL ANSWERS

# Pearson Edexcel Level 3 GCE

## Monday 24 June 2019

Morning (Time: 1 hour 30 minutes) | Paper Reference **9FM0/3D**

# Further Mathematics

## Advanced

## Paper 3D: Decision Mathematics 1

**You must have:**
Mathematical Formulae and Statistical Tables (Green), calculator,
Decision Mathematics Answer Book (enclosed)

**Candidates may use any calculator permitted by Pearson regulations. Calculators must not have the facility for algebraic manipulation, differentiation and integration, or have retrievable mathematical formulae stored in them.**

### Instructions

- Use **black** ink or ball-point pen.
- If pencil is used for diagrams/sketches/graphs it must be dark (HB or B).
- Write your answers for this paper in the Decision Mathematics answer book provided.
- **Fill in the boxes** at the top of the answer book with your name, centre number and candidate number.
- Do not return the question paper with the answer book.
- Answer **all** questions and ensure that your answers to parts of questions are clearly labelled.
- Answer the questions in the spaces provided
  – *there may be more space than you need.*
- You should show sufficient working to make your methods clear. Answers without working may not gain full credit.
- Answers should be given to three significant figures unless otherwise stated.

### Information

- A booklet 'Mathematical Formulae and Statistical Tables' is provided.
- There are 7 questions in this question paper. The total mark for this paper is 75.
- The marks for **each** question are shown in brackets
  – *use this as a guide as to how much time to spend on each question.*

### Advice

- Read each question carefully before you start to answer it.
- Try to answer every question.
- Check your answers if you have time at the end.

*Turn over* ▶

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**Answer ALL questions. Write your answers in the answer book provided.**

**1.** 2.1 1.7 3.0 1.9 3.2 1.2 3.3 1.4 1.5 0.2

(a) Use the first-fit bin packing algorithm to determine how the numbers listed above can be packed into bins of size 5

**(2)**

The list of numbers is now to be sorted into descending order.

(b) Perform a quick sort on the original list to obtain the sorted list. You should show the result of each pass and identify your pivots clearly.

**(4)**

For a list of $n$ numbers, the quick sort algorithm has, on average, order $n \log n$.

Given that it takes 2.32 seconds to run the algorithm when $n = 450$

(c) calculate approximately how long it will take, to the nearest tenth of a second, to run the algorithm when $n = 11\,250$. You should make your method and working clear.

**(2)**

**(Total for Question 1 is 8 marks)**

a) Bin 1: 2.1, 1.7, 1.2 (full)

Bin 2: 3.0, 1.9

Bin 3: 3.2, 1.4, 0.2

Bin 4: 3.3, 1.5

let: ○ - pivot □ - ordered

always pick number to the right of the middle (when the total number of data is even eg. 10).

b)

- 2.1 1.7 3.0 1.9 3.2 1.2 3.3 1.4 1.5 0.2
- 2.1 1.7 3.0 1.9 3.2 3.3 1.4 1.5 1.2 0.2
- 3.3 3.2 2.1 1.7 3.0 1.9 1.4 1.5 1.2 0.2
- 3.3 3.2 2.1 3.0 1.9 1.7 1.4 1.5 1.2 0.2

- 3.3 3.2 3.0 2.1 1.9 1.7 1.5 1.4 1.2 0.2

- 3.3 3.2 3.0 2.1 1.9 1.7 1.5 1.4 1.2 0.2

Sort Complete

c) $$\frac{2.32\,(11250\log(11250))}{450\log(450)} = 88.6 \text{ seconds}$$

**2.**

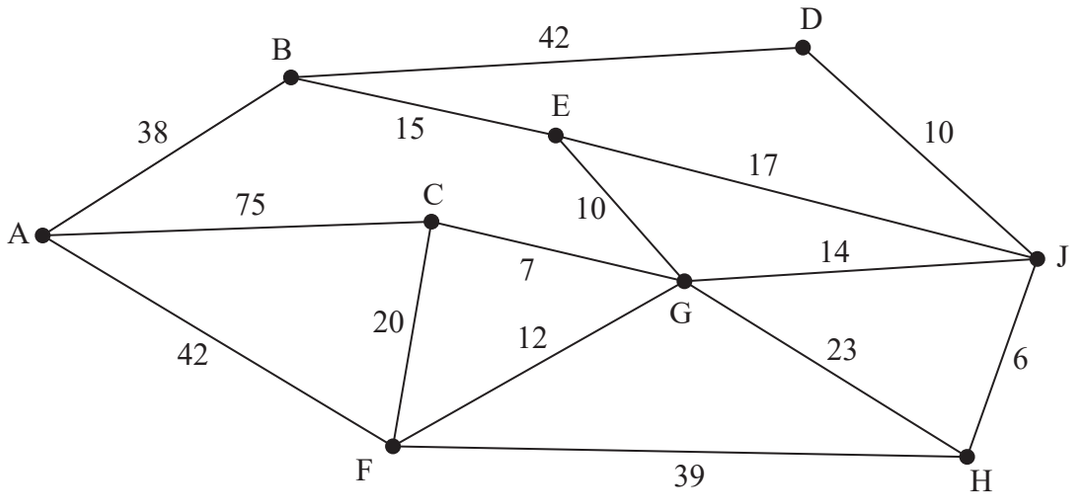



**Figure 1**

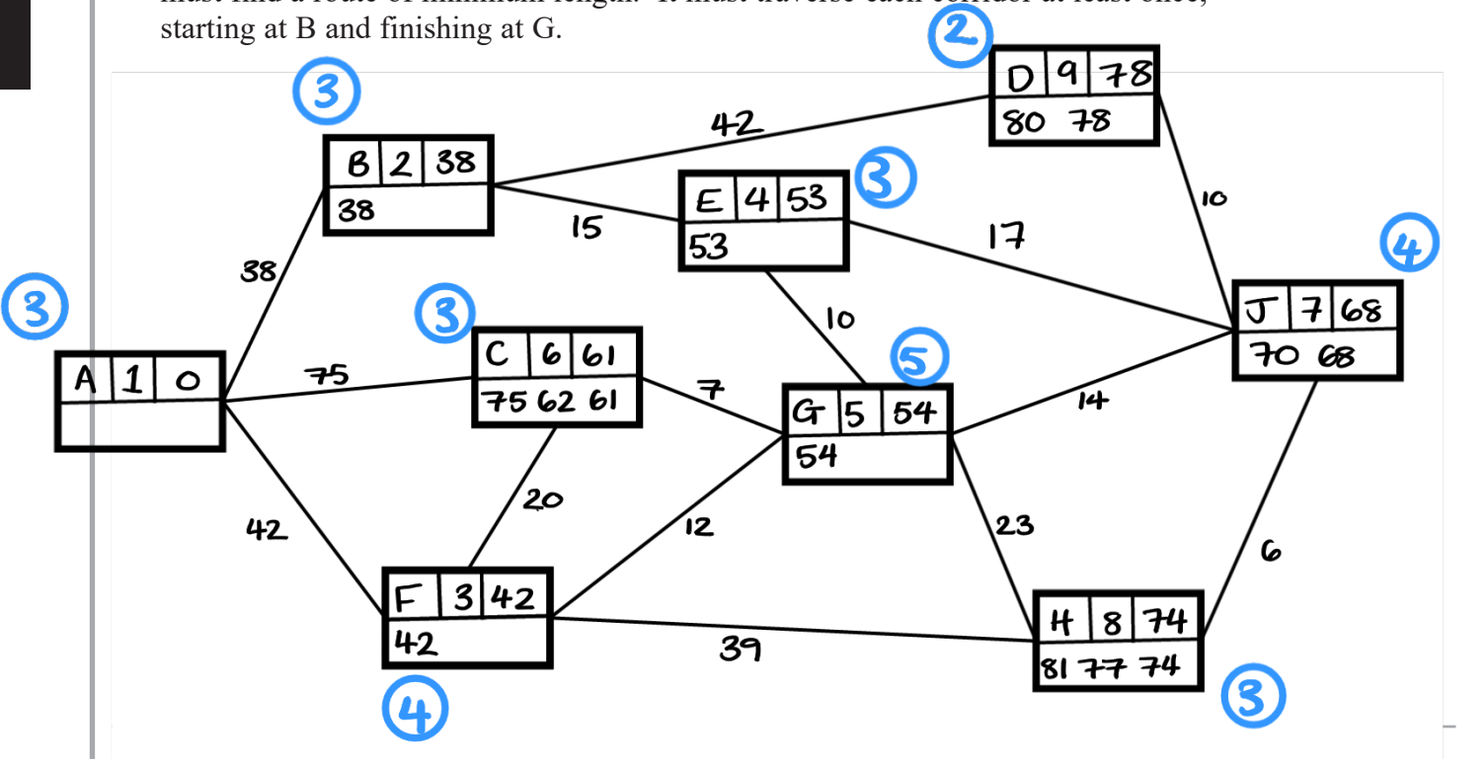
[*The total weight of the network is* 370]

Figure 1 represents a network of corridors in a building. The number on each arc represents the length, in metres, of the corresponding corridor.

(a) Use Dijkstra's algorithm to find the shortest path from A to D, stating the path and its length.

**(6)**

On a particular day, Naasir needs to check the paintwork along each corridor. Naasir must find a route of minimum length. It must traverse each corridor at least once, starting at B and finishing at G.




Path : AFGJD

Length of Path : 78 m

On a particular day, Naasir needs to check the paintwork along each corridor. Naasir must find a route of minimum length. It must traverse each corridor at least once, starting at B and finishing at G.

(b) Use an appropriate algorithm to find the arcs that will need to be traversed twice. You must make your method and working clear.

**(4)**

Odd nodes: A, B, C, E, G, H

Starting at B, Finishing at G

Possible pairs:

- A(FG)C + E(J)H = 61 + 23 = 84
- A(B)E + C(GJ)H = 53 + 27 = 80 *shortest route*
- A(FGJ)H + C(G)E = 74 + 17 = 91

∴ Repeating arcs: AB, BE, CG, GJ and JH

(c) Find the length of Naasir's route.

**(1)**

370 + 80 = 450 m

On a different day, all the corridors that start or finish at B are closed for redecorating. Naasir needs to check all the remaining corridors and may now start at any vertex and finish at any vertex. A route is required that excludes all those corridors that start or finish at B.

(d) (i) Determine the possible starting and finishing points so that the length of Naasir's route is minimised. You must give reasons for your answer.

(ii) Find the length of Naasir's new route.

**(3)**

i) odd nodes: C, D, G, H

C(GJ)D = 31
G(J)H = 20
CG = 7 *shortest*
D(J)H = 16
C(GJ)H = 27
D(J)G = 24

The shortest path between any two nodes is CG ∴ repeat CG. This means route should start at D and finish at H (or vice versa).

ii) New route: 370 − 38 − 42 − 15 + 7 = 282 m

**(Total for Question 2 is 14 marks)**

**3.**

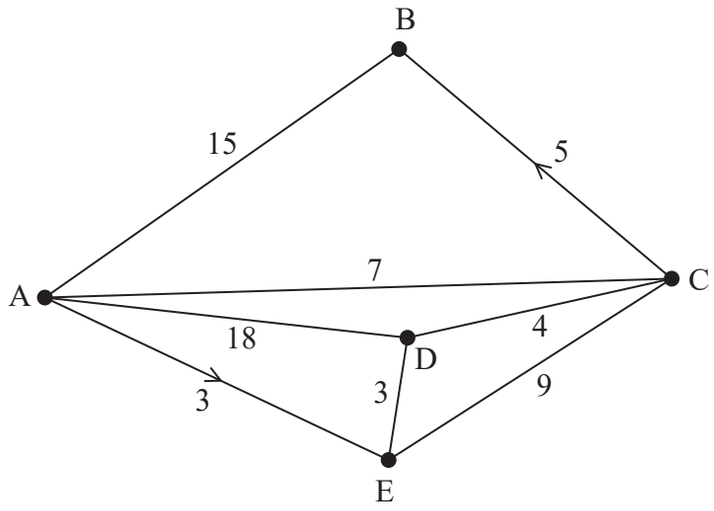



**Figure 2**

The network in Figure 2 shows the direct roads linking five villages, A, B, C, D and E.
The number on each arc represents the length, in miles, of the corresponding road.
The roads from A to E and from C to B are one-way, as indicated by the arrows.

(a) Complete the initial distance and route tables for the network provided in the answer book.

**(2)**

**3.** (a)

Initial distance table

| | A | B | C | D | E |
|---|---|---|---|---|---|
| A | – | 15 | 7 | 18 | 3 |
| B | 15 | – | ∞ | ∞ | ∞ |
| C | 7 | 5 | – | 4 | 9 |
| D | 18 | ∞ | 4 | – | 3 |
| E | ∞ | ∞ | 9 | 3 | – |

Initial route table

| | A | B | C | D | E |
|---|---|---|---|---|---|
| A | A | B | C | D | E |
| B | A | B | C | D | E |
| C | A | B | C | D | E |
| D | A | B | C | D | E |
| E | A | B | C | D | E |

(b) Perform the first three iterations of Floyd's algorithm. You should show the distance table and the route table after each of the three iterations.

**(5)**

(b) 1st iteration

Distance table

| | A | B | C | D | E |
|---|---|---|---|---|---|
| A | – | 15 | 7 | 18 | 3 |
| B | 15 | – | [22] | [33] | [18] |
| C | 7 | 5 | – | 4 | 9 |
| D | 18 | [33] | 4 | – | 3 |
| E | ∞ | ∞ | 9 | 3 | – |

Route table

| | A | B | C | D | E |
|---|---|---|---|---|---|
| A | A | B | C | D | E |
| B | A | B | A | A | A |
| C | A | B | C | D | E |
| D | A | A | C | D | E |
| E | A | B | C | D | E |

2nd iteration

Distance table

| | A | B | C | D | E |
|---|---|---|---|---|---|
| A | – | 15 | 7 | 18 | 3 |
| B | 15 | – | 22 | 33 | 18 |
| C | 7 | 5 | – | 4 | 9 |
| D | 18 | 33 | 4 | – | 3 |
| E | ∞ | ∞ | 9 | 3 | – |

Route table

| | A | B | C | D | E |
|---|---|---|---|---|---|
| A | A | B | C | D | E |
| B | A | B | A | A | A |
| C | A | B | C | D | E |
| D | A | A | C | D | E |
| E | A | B | C | D | E |

3rd iteration

Distance table

| | A | B | C | D | E |
|---|---|---|---|---|---|
| A | – | [12] | 7 | [11] | 3 |
| B | 15 | – | 22 | [26] | 18 |
| C | 7 | 5 | – | 4 | 9 |
| D | [11] | [9] | 4 | – | 3 |
| E | [16] | [14] | 9 | 3 | – |

Route table

| | A | B | C | D | E |
|---|---|---|---|---|---|
| A | A | C | C | C | E |
| B | A | B | A | C | A |
| C | A | B | C | D | E |
| D | C | C | C | D | E |
| E | C | C | C | D | E |

After five iterations of Floyd's algorithm the final distance table and partially completed final route table are shown below.

Distance table

| | A | B | C | D | E |
|---|---|---|---|---|---|
| A | – | 12 | 7 | 6 | 3 |
| B | 15 | – | 22 | 21 | 18 |
| C | 7 | 5 | – | 4 | 7 |
| D | 11 | 9 | 4 | – | 3 |
| E | 14 | 12 | 7 | 3 | – |

Route table

| | A | B | C | D | E |
|---|---|---|---|---|---|
| A | A | | | | |
| B | A | B | | | |
| C | A | B | C | | |
| D | C | C | C | D | |
| E | D | D | D | D | E |

(c) (i) Explain how the partially completed final route table can be used to find the shortest route from E to A.

(ii) State this route.

(3)

i) Start at E (5th row) and read across to the A (1st column), there is a D there so the route from E to A is via D.

Now consider both E to D and D to A. For E reading across to the D (4th column), there is a D indicating that the shortest path from E to D is ED. For D reading across to the A (1st column), there is a C

i cont.) indicating that the shortest path from D to A is via C.

ii) EDCA

Mabintou decides to use the distance table to try to find the shortest cycle that passes through each vertex. Starting at D, she applies the nearest neighbour algorithm to the final distance table.

(d) (i) State the cycle obtained using the nearest neighbour algorithm.

(ii) State the length of this cycle.

(iii) Interpret the cycle in terms of the actual villages visited.

(iv) Prove that Mabintou's cycle is not optimal.

**(4)**

**(Total for Question 3 is 14 marks)**

i) $D_3 E_7 C_5 B_{15} A_6 D$

| | A | B | C | D | E |
|---|---|---|---|---|---|
| A | – | 12 | 7 | 6 | 8 |
| B | 15 | – | 22 | 21 | 18 |
| C | 7 | 5 | – | 4 | 7 |
| D | 11 | 9 | 4 | – | 3 |
| E | 14 | 12 | 7 | 8 | – |

ii) 3 + 7 + 5 + 15 + 6 = 36 miles

iii) D - E - D - C - B - A - E - D

iv) for example, the cycle A - E - D - C - B - A has a length of 30 miles < 36 miles so Mabintou's route is not optimal.

**4.**

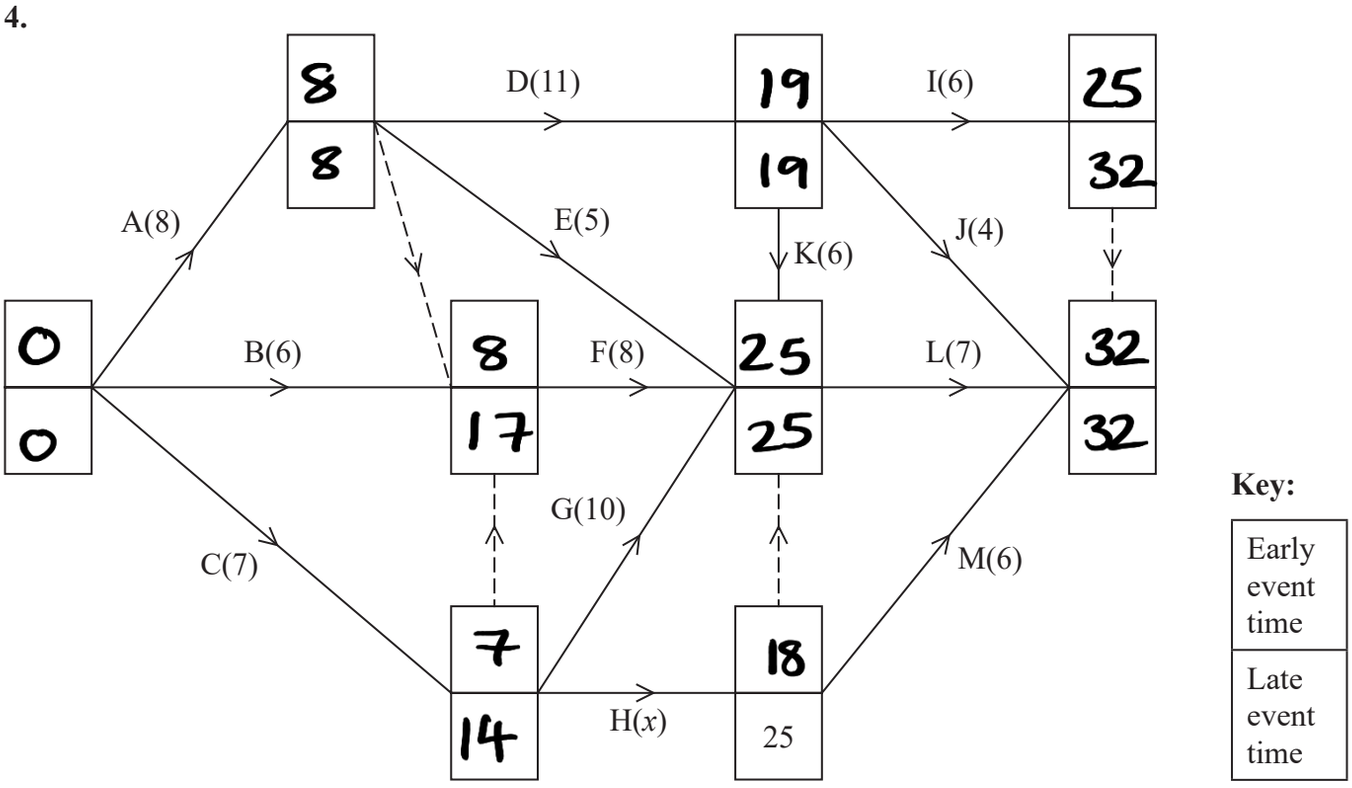



**Figure 3**

The network in Figure 3 shows the activities that need to be undertaken to complete a project. Each activity is represented by an arc and the duration of the activity, in days, is shown in brackets. The early event times and late event times are to be shown at each vertex and one late event time has been completed for you.

The total float of activity H is 7 days.

(a) Explain, with detailed reasoning, why $x = 11$

**(2)**

$25 - x - 7 = 7$

$18 - x = 7$

$11 = x$

(Early event time at the end of activity C is 7

$\therefore$ float on activity H is $25 - 7 - x$).

(b) Determine the missing early event times and late event times, and hence complete Diagram 1 in your answer book.

**(3)**

See on diagram above.

Each activity requires one worker and the project must be completed in the shortest possible time using as few workers as possible.

(c) Calculate a lower bound for the number of workers needed to complete the project in the shortest possible time.

**(1)**

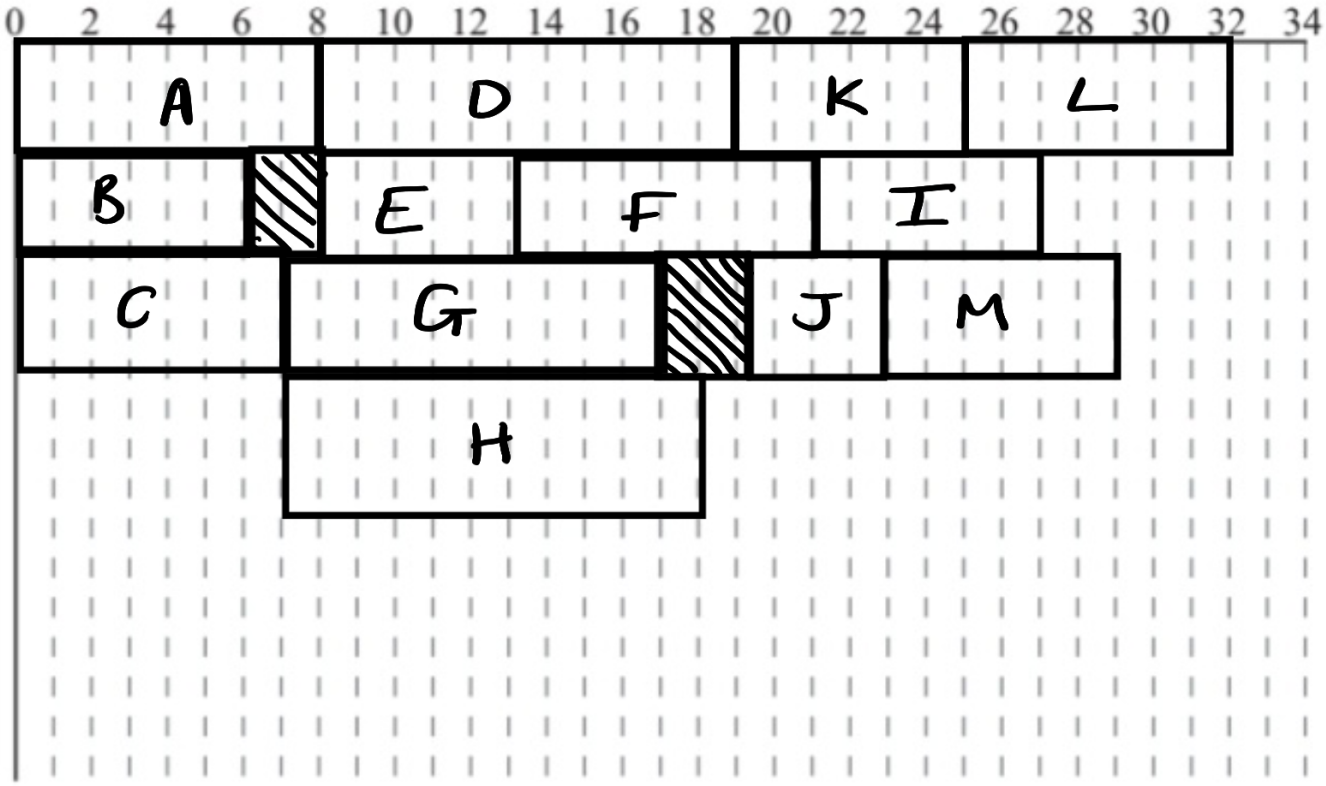
$(8 + 6 + 7 + 11 + 5 + 8 + 10 + 11 + 6 + 6 + 4 + 7 + 6 = 95)$

$\frac{95}{32} = 2.968\ldots \approx 3$ workers.

(d) Schedule the activities using Grid 1 in the answer book.

**(3)**

**(Total for Question 4 is 9 marks)**




**Grid 1**

**5.**

| Activity | Immediately preceding activities |
|---|---|
| A | – |
| B | – |
| C | – |
| D | A |
| E | C |
| F | B, C, D |
| G | A |
| H | B, C, D |
| I | B, C, D, G |
| J | B, C, D, G |
| K | E, H |

(a) Draw the activity network described in the precedence table above, using activity on arc. Your activity network must contain only the minimum number of dummies.

**(5)**

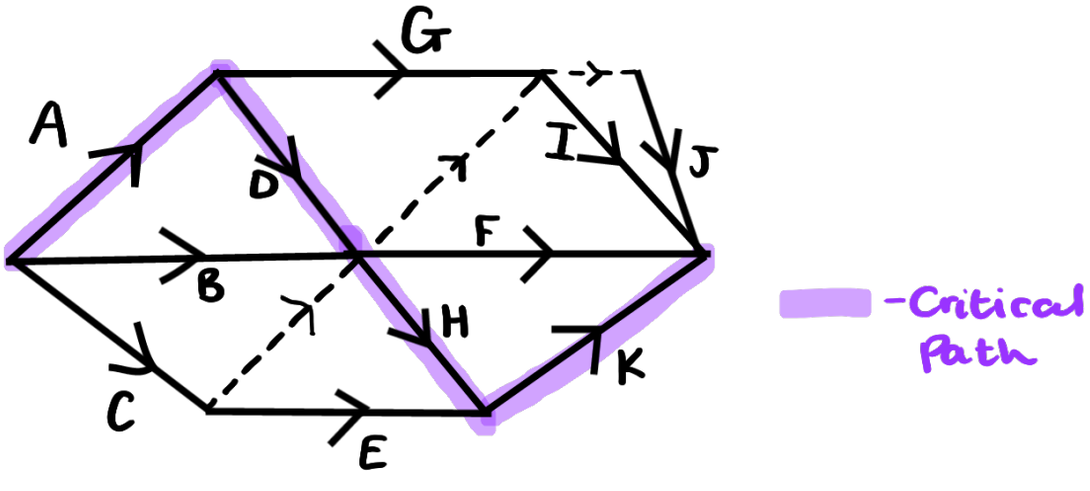
Given that all the activities shown in the precedence table have the same duration,

(b) state the critical path for the network.

**(1)**

**(Total for Question 5 is 6 marks)**

a)




b) Critical path: ADHK

**6.** A linear programming problem in $x$, $y$ and $z$ is described as follows.

Maximise $P = 2x + 2y - z$

subject to $3x + y + 2z \leqslant 30$

$x - y + z \geqslant 8$

$4y + 2z \geqslant 15$

$x, y, z \geqslant 0$

(a) Explain why the Simplex algorithm cannot be used to solve this linear programming problem.

**(1)**

Simplex can only be applied when the non-negativity constraints are $\leqslant$.

(b) Set up the initial tableau for solving this linear programming problem using the big-M method.

**(7)**

- $3x+y+2z+s_1 = 30$
- $x-y+z-s_2+a_1 = 8 \Rightarrow a_1 = 8-x+y-z+s_2$
- $4y+2z-s_3+a_2 = 15 \Rightarrow a_2 = 15-4y-2z+s_3$

$a_1 + a_2 = 8-x+y-z+s_2+15-4y-2z+s_3$

$= 23-x-3y-3z+s_2+s_3$

$P = 2x+2y-z-M(a_1+a_2)$

$P = 2x+2y-z-M(23-x-3y-3z+s_2+s_3)$

$= 2x+2y-z-23M+Mx+3My+3Mz-Ms_2-s_3$

$P = -23M+(2+M)x+(2+3M)y+(-1+3M)z-Ms_2-Ms_3$

- $\Rightarrow P-(2+M)x-(2+3M)y-(-1+3M)z+Ms_2+Ms_3 = -23M$

(b)

| b.v. | $x$ | $y$ | $z$ | $s_1$ | $s_2$ | $s_3$ | $a_1$ | $a_2$ | Value |
|---|---|---|---|---|---|---|---|---|---|
| $s_1$ | 3 | 1 | 2 | 1 | 0 | 0 | 0 | 0 | 30 |
| $a_1$ | 1 | −1 | 1 | 0 | −1 | 0 | 1 | 0 | 8 |
| $a_2$ | 0 | 4 | 2 | 0 | 0 | −1 | 0 | 1 | 15 |
| $P$ | $-(2+M)$ | $-(2+3M)$ | $-(3M-1)$ | 0 | $M$ | $M$ | 0 | 0 | $-23M$ |

After a first iteration of the big-M method, the tableau is

| b.v. | $x$ | $y$ | $z$ | $s_1$ | $s_2$ | $s_3$ | $a_1$ | $a_2$ | Value |
|---|---|---|---|---|---|---|---|---|---|
| $s_1$ | 3 | 0 | 1.5 | 1 | 0 | 0.25 | 0 | −0.25 | 26.25 |
| $a_1$ | 1 | 0 | 1.5 | 0 | −1 | −0.25 | 1 | 0.25 | 11.75 |
| $y$ | 0 | 1 | 0.5 | 0 | 0 | −0.25 | 0 | 0.25 | 3.75 |
| $P$ | $-(2 + M)$ | 0 | $2 - 1.5M$ | 0 | $M$ | $-0.5 + 0.25M$ | 0 | $0.5 + 0.75M$ | $7.5 - 11.75M$ |

(c) State the value of each variable after the first iteration.

**(1)**

(d) Explain why the solution given by the first iteration is not feasible.

**(1)**

Taking the most negative entry in the profit row to indicate the pivot column,

(e) obtain the most efficient pivot for a second iteration. You must give reasons for your answer.

**(2)**

c) $S_1 = 26.25$, $a_1 = 11.75$, $y = 3.75$, $x = z = S_2 = S_3 = a_2 = 0$

d) $a_1 = 11.75$, which is an artificial variable. For it to be a feasible solution, it must be zero.

e) Most negative value in the objective row is $2-1.5M$ so the pivot is a value from the z-column.

$\frac{26.25}{1.5} = 17.5$ $\quad$ $\frac{3.75}{0.5} = 7.5$ ⇐ lowest value

$\frac{11.75}{1.5} = 7.83\dot{}$

As $\frac{3.75}{0.5}$ is less than both $\frac{11.75}{1.5}$ and $\frac{26.25}{1.5}$, the 0.5 in row y is the pivot.

**(Total for Question 6 is 12 marks)**

**7.** A shop sells two types of watch, analogue watches and digital watches.

The shop manager knows that, each month, she should order at least 60 watches in total. In addition, at most 80% of the watches she orders must be digital.

Let $x$ be the number of analogue watches ordered and let $y$ be the number of digital watches ordered.

(a) Write down inequalities, in terms of $x$ and $y$, to model these constraints.

**(2)**

$x + y \geq 60$

$y \leq \frac{4}{5}(x+y)$ $\quad 80\% \Rightarrow \frac{8}{10} \Rightarrow \frac{4}{5}$

Two further constraints are

$$y + 3x \geqslant 140$$

$$4y + x \geqslant 80$$

(b) Represent all these constraints on Diagram 1 in the answer book. Hence determine, and label, the feasible region, $R$.

**(4)**

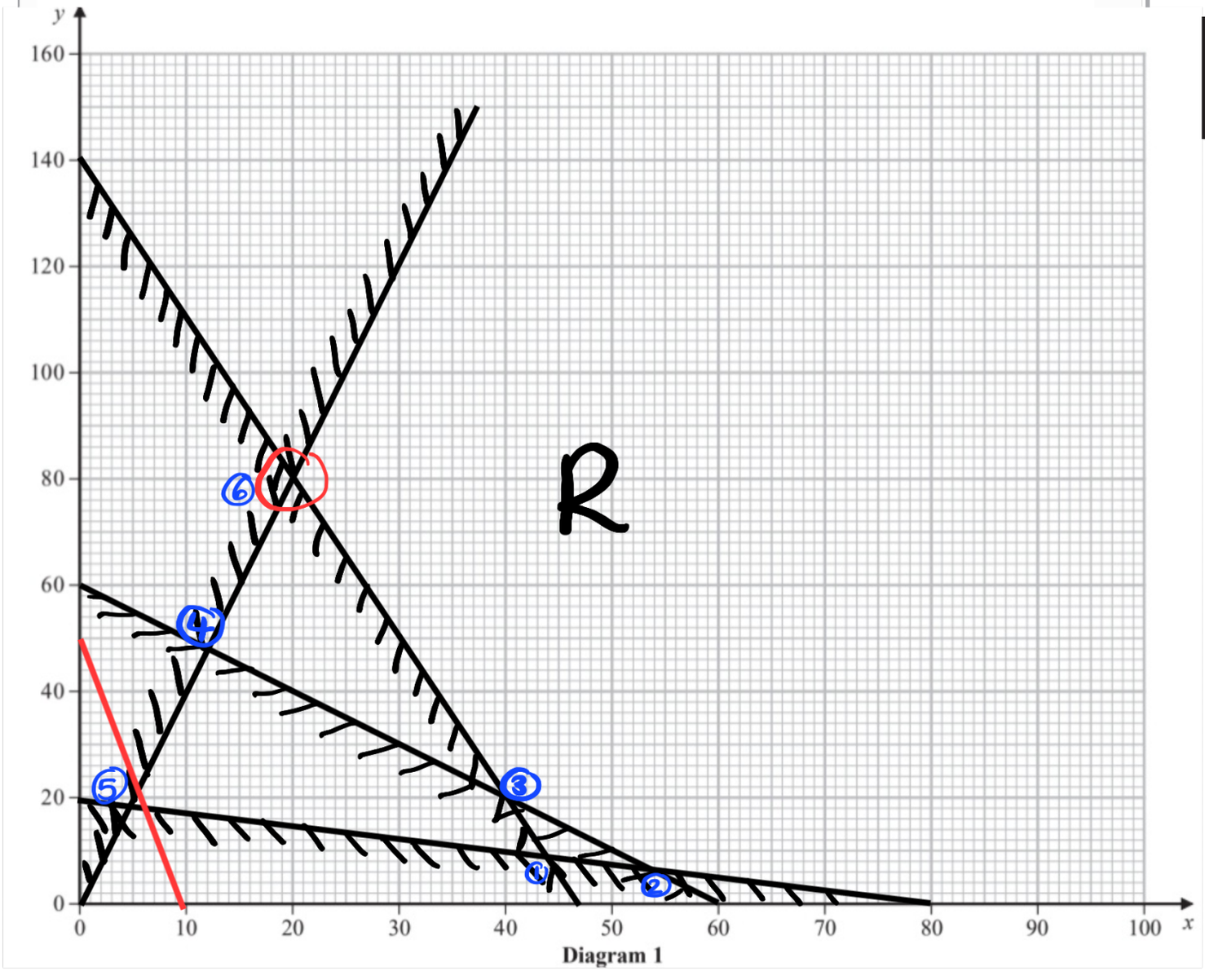



**Diagram 1**

The cost to the shop of ordering an analogue watch is five times the cost of ordering a digital watch. The shop manager wishes to minimise the total cost.

(c) Determine the number of each type of watch the shop manager should order. You must make your method clear. (3)

Objective line drawn

Point testing

$P_{min} = K(5x+y)$ (using $50x+10y$)

(20, 80)

So 20 analogue watches and 80 digital watches

① $4y+x=80$

$y+3x=140$

$y=-3x+140$

$4(-3x+140)+x=80$

$-11x+560=80$

$480=11x$

$x=\frac{480}{11}$, $y=\frac{100}{11}$

② $x+y=60$

$4y+x=80$

$x=\frac{160}{3}$, $y=\frac{20}{3}$

③ $x+y=60$

$y+3x=140$

$x=40$, $y=20$

④ $y=\frac{4}{5}(x+y)$

$5y=4x+4y$

$y=4x$

$x+y=60$

$x=12$, $y=48$

⑤ $y=4x$

$4y+x=80$

$x=\frac{80}{17}$, $y=\frac{320}{17}$

⑥ $y=4x$

$y+3x=140$

$x=20$, $y=80$

Putting it into $P_{min}$ $(5x+y)$

① $5\left(\frac{480}{11}\right)+\frac{100}{11}=\frac{2500}{11}$

② $5\left(\frac{160}{3}\right)+\frac{20}{3}=\frac{820}{3}$

③ $5(40)+20=220$

④ $5(12)+48=108$

⑤ $5\left(\frac{80}{17}\right)+\frac{320}{17}=\frac{720}{17}$

⑥ $5(20)+80=180$ ⇐

∴ (20, 80) so 20 analogue watches and 80 digital watches

Given that the minimum total cost of ordering the watches is £4455

(d) determine the cost of ordering one analogue watch and the cost of ordering one digital watch. You must make your method clear.

**(3)**

**(Total for Question 7 is 12 marks)**

$20a + 80d = 4455$

$a = 5d$

Leading to $a = 123.75$ and $d = 24.75$ so an analogue watch costs £123.75 and a digital watch costs £24.75.

**TOTAL FOR PAPER IS 75 MARKS**