MODEL ANSWERS

Pearson Edexcel Level 3 GCE

Thursday 14 May 2020

Afternoon | Paper Reference **8FM0/27**

Further Mathematics

Advanced Subsidiary
Further Mathematics options
27: Decision Mathematics 1
(Part of options D, F, H and K)

You must have:
Mathematical Formulae and Statistical Tables (Green), calculator, D1 Answer Book (enclosed)

Candidates may use any calculator allowed by Pearson regulations. Calculators must not have the facility for symbolic algebra manipulation, differentiation and integration, or have retrievable mathematical formulae stored in them.

Instructions

- Use **black** ink or ball-point pen.
- If pencil is used for diagrams/sketches/graphs it must be dark (HB or B).
- **Fill in the boxes** at the top of the answer book with your name, centre number and candidate number.
- Answer **all** questions and ensure that your answers to parts of questions are clearly labelled.
- Answer the questions in the Answer Book provided
  – *there may be more space than you need.*
- You should show sufficient working to make your methods clear. Answers without working may not gain full credit.
- Inexact answers should be given to three significant figures unless otherwise stated.
- Do not return the question paper with the D1 Answer Book.

Information

- A booklet 'Mathematical Formulae and Statistical Tables' is provided.
- The total mark for this part of the examination is 40. There are 4 questions.
- The marks for each question are shown in brackets
  – *use this as a guide as to how much time to spend on each question.*

Advice

- Read each question carefully before you start to answer it.
- Try to answer every question.
- Check your answers if you have time at the end.

Turn over ▶

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Pearson

1. 3.7 2.5 5.4 1.9 2.7 3.2 3.1 2.7 4.2 2.0

(a) Use the first-fit bin packing algorithm to determine how the numbers listed above can be packed into bins of size 8.5

(3)

The first-fit bin packing algorithm is to be used to pack n numbers into bins. The number of comparisons is used to measure the order of the first-fit bin packing algorithm.

(b) By considering the worst case, determine the order of the first-fit bin packing algorithm in terms of n. You must make your method and working clear.

(3)

(Total for Question 1 is 6 marks)

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a) size 8.5

Bin 1: 3.7, 2.5, 1.9

Bin 2: 5.4, 2.7

Bin 3: 3.2, 3.1, 2.0

Bin 4: 2.7, 4.2

b) In the worst case the 2nd number must be compared with the 1st number so 1 comparison, then the 3rd number must be compared with the 1st and 2nd numbers so 2 comparisons ... and so on.

∴ in total the number of comparisons =

$1 + 2 + 3 + \ldots + (n-1)$

$1 + 2 + 3 + \ldots + (n-1) = \frac{1}{2}(n-1)n \quad \Rightarrow \sum_{r=1}^{n-1} \frac{n}{2}(n-1)$

∴ $\frac{1}{2}(n-1)n$ is a quadratic order (n^2)

2.

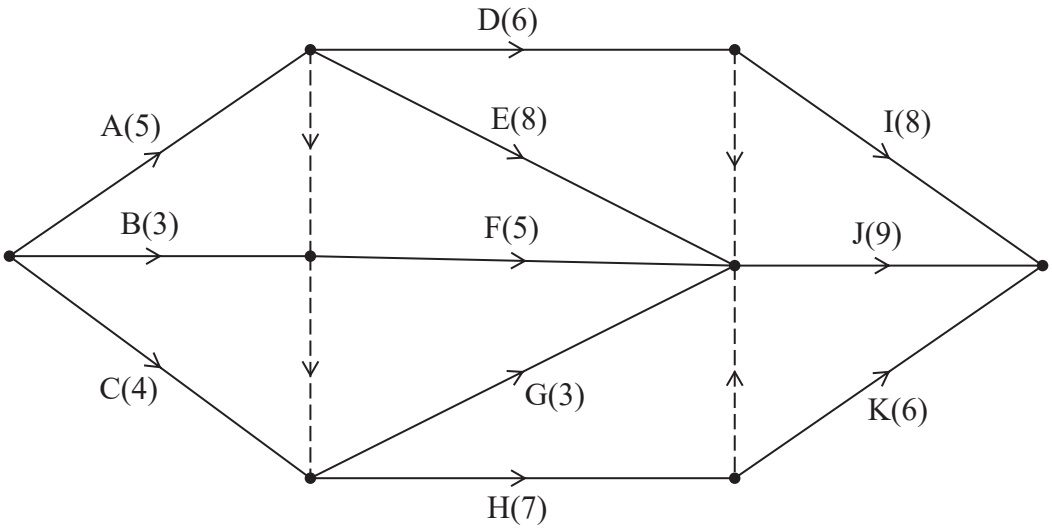


Figure 1

A project is modelled by the activity network shown in Figure 1. The activities are represented by the arcs. The number in brackets on each arc gives the time, in hours, to complete the corresponding activity. Each activity requires one worker. The project is to be completed in the shortest possible time.

(a) Complete the precedence table in the answer book.

(2)

(b) Complete Diagram 1 in the answer book to show the early event times and the late event times.

(3)

(c) (i) State the minimum project completion time.

(ii) List the critical activities.

(2)

(d) Calculate the maximum number of hours by which activity H could be delayed without affecting the shortest possible completion time of the project. You must make the numbers used in your calculation clear.

(1)

(e) Calculate a lower bound for the number of workers needed to complete the project in the minimum time. You must show your working.

(2)

(f) Draw a cascade chart for this project on Grid 1 in the answer book.

(3)

(g) Using the answer to (f), explain why it is not possible to complete the project in the shortest possible time using the number of workers found in (e).

(1)

(Total for Question 2 is 14 marks)

2. (a)

Activity	Immediately preceding activities
A	–
B	–
C	–
D	A

Activity	Immediately preceding activities
E	A
F	A, B
G	A, B, C
H	A, B, C

Activity	Immediately preceding activities
I	D
J	D, E, F, G, H
K	H

b)

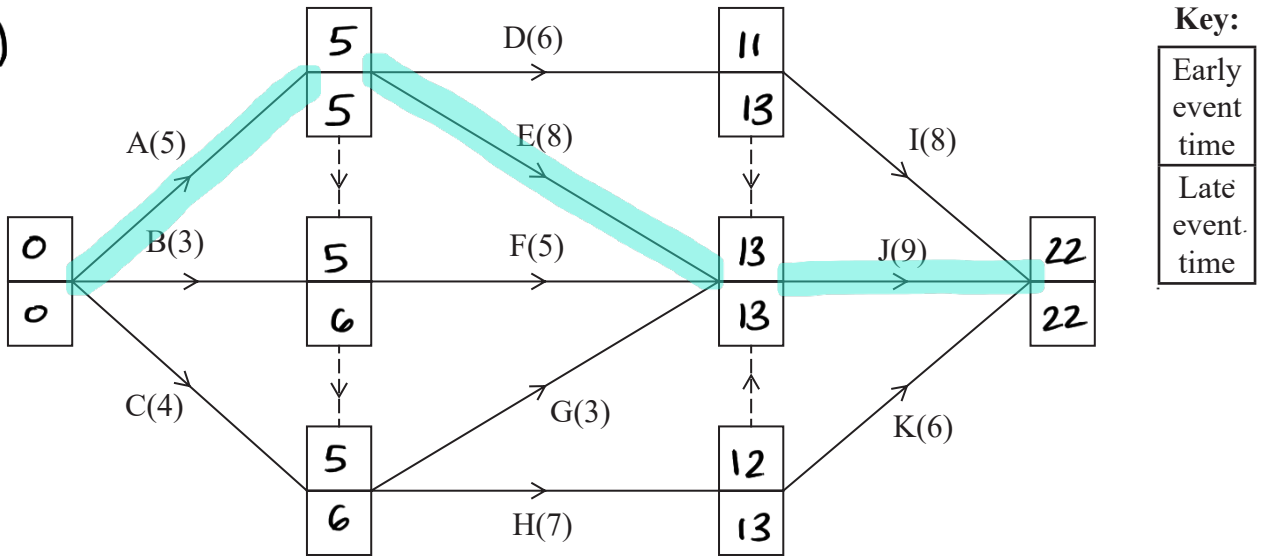


Diagram 1

ci) Minimum time = 22 hours

cii) Critical activities: A, E, J

d) H float = 13 − 7 − 5 = 1 hour

∴ H could be delayed by 1 hour.

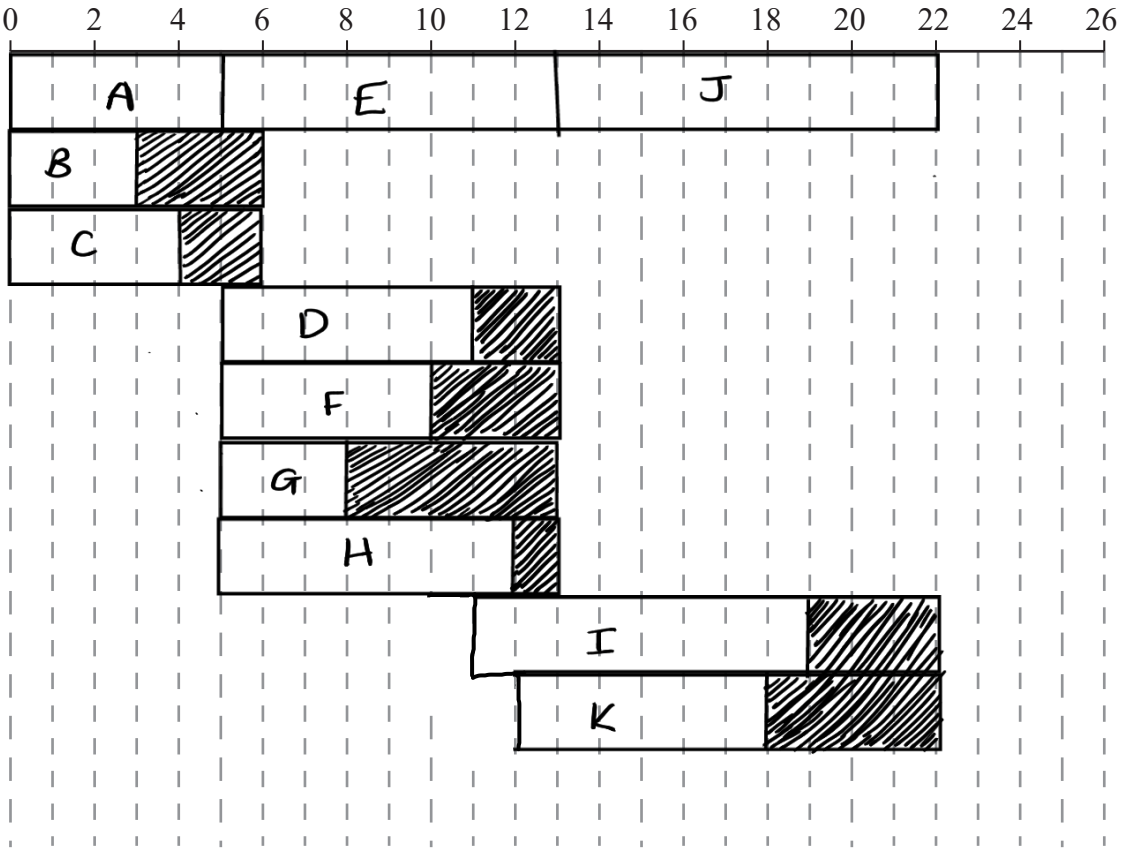
e) $\frac{5+3+4+6+8+5+3+7+8+9+6}{22}$

$= \frac{32}{11} = 2.90$

∴ Lower bound of 3 workers.

Question 2 continued

f)



Grid 1

g) At time 8.5 hours, activities E, D, F, H must be happening so not possible to complete with only 3 workers.

(Total for Question 2 is 14 marks)

3.

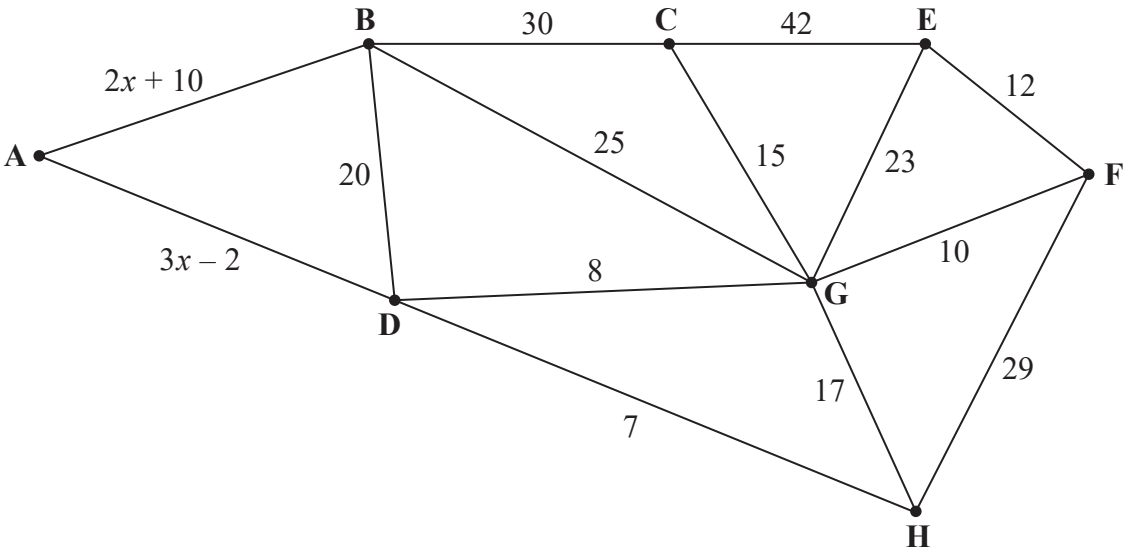


Figure 2

[*The weight of the network is* $5x + 246$]

(a) Explain why it is not possible to draw a graph with an odd number of vertices of odd valency.

(2)

Figure 2 represents a network of 14 roads in a town. The expression on each arc gives the time, in minutes, to travel along the corresponding road.

Prim's algorithm, starting at A, is applied to the network. The order in which the arcs are selected is AD, DH, DG, FG, EF, CG, BD. It is given that the order in which the arcs are selected is unique.

(b) Using this information, find the smallest possible range of values for x, showing your working clearly.

(3)

A route that minimises the total time taken to traverse each road at least once is required. The route must start and finish at the same vertex.

Given that the time taken to traverse this route is 318 minutes,

(c) use an appropriate algorithm to determine the value of x, showing your working clearly.

(6)

(Total for Question 3 is 11 marks)

3.

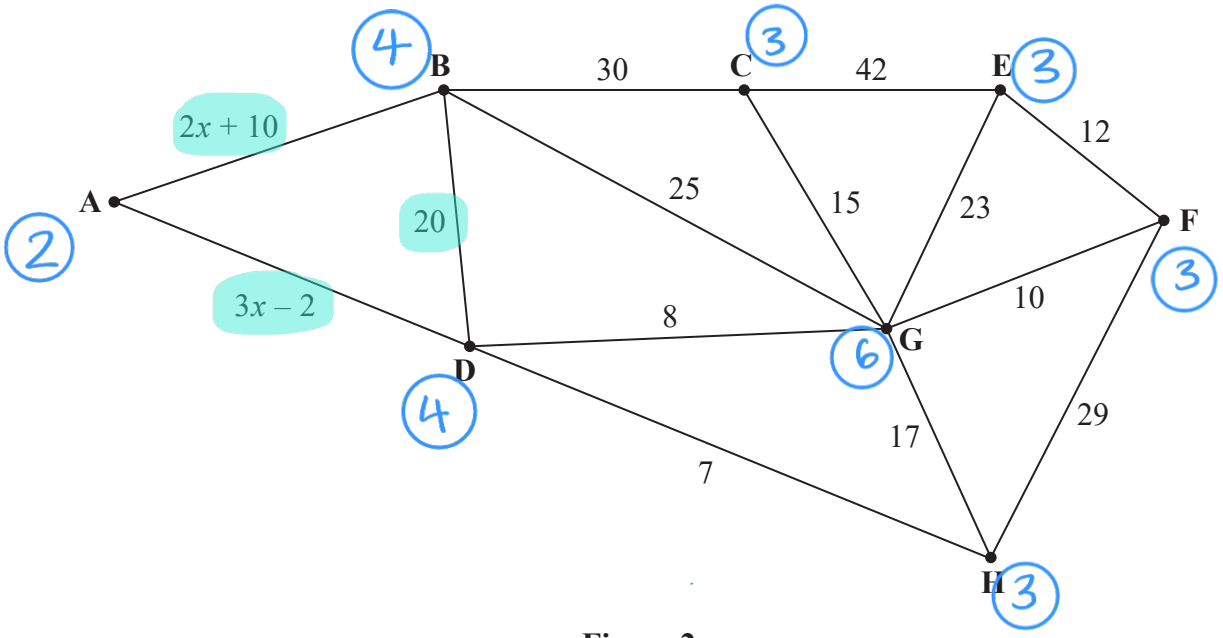


Figure 2

[*The weight of the network is* $5x + 246$]

a) The sum of the orders of all the nodes = 2 x number of arcs.
This implies that the sum of the orders of all the nodes is even ∴ there must be an even (or zero) number of nodes of odd order.
(cannot have an odd number of nodes of odd order)

b) $AB > AD$
$2x + 10 > 3x - 2$
$12 > x$

$AB > BD$
$2x + 10 > 20$
$2x > 10$
$x > 5$

c) odd nodes: C, E, F, H
Possible pairs:
C(GF)E + F(GD)H = 37 + 25 = 62
C(G)F + E(FGD)H = 25 + 37 = 62
C(GD)H + EF = 30 + 12 = 42 * shortest *

∴ $5x + 246 + 42 = 318$
$5x = 30$
$x = 6$

4.

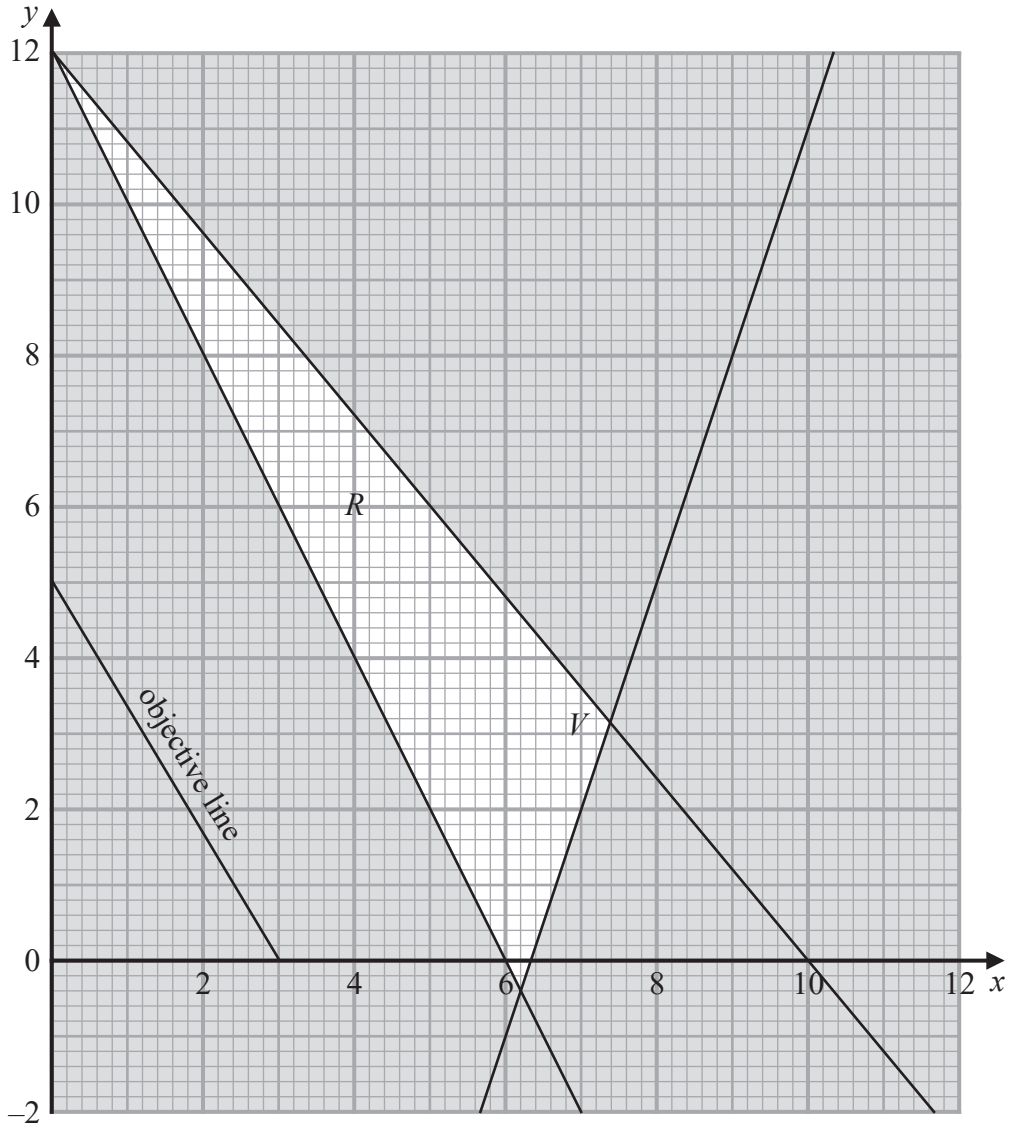


Figure 3

Figure 3 shows the constraints of a linear programming problem in x and y, where R is the feasible region. Figure 3 also shows an objective line for the problem and the optimal vertex, which is labelled as V.

The value of the objective at V is 556

Express the linear programming problem in algebraic form. List the constraints as simplified inequalities with integer coefficients and determine the objective.

(9)

(Total for Question 4 is 9 marks)

TOTAL FOR DECISION MATHEMATICS 1 IS 40 MARKS

END

4.

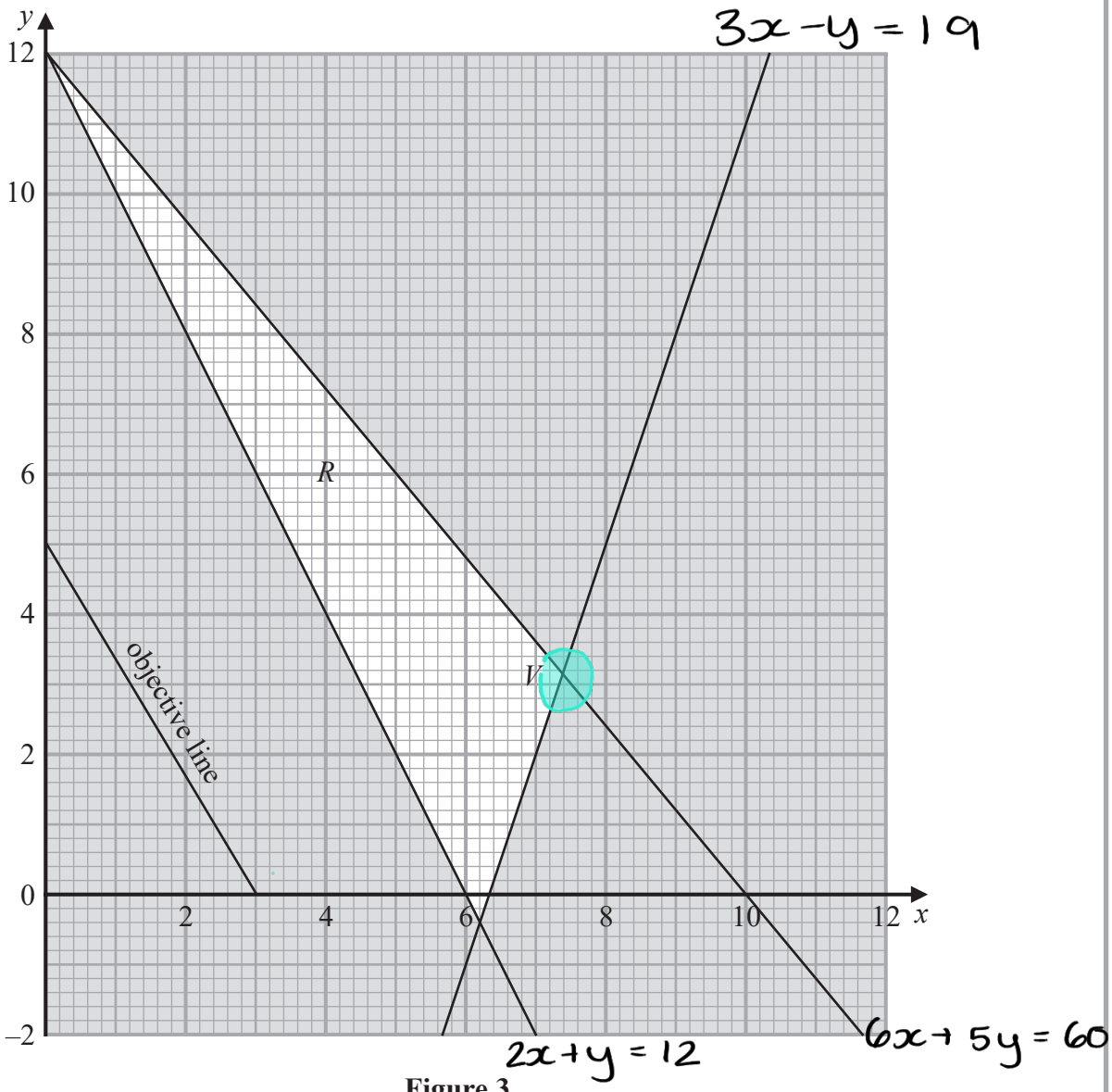


Figure 3

- $2x + y \geq 12$
- $6x + 5y \leq 60$
- $3x - y \leq 19$

Solving V:

cv. $3x - y = 19$ — (1)

$6x + 5y = 60$ — (2)

$2 \times (1) - (2)$

$6x - 2y - (6x + 5y) = 38 - 60$

$-7y = -22$

$y = \frac{22}{7}$ $x = \frac{155}{21}$

$V\left(\frac{155}{21}, \frac{22}{7}\right)$

$P_{max} = k(5x + 3y)$

when $x = \frac{155}{21}$, $y = \frac{22}{7}$ and $P_{max} = 556$

$556 = k\left(5 \times \frac{155}{21} + 3 \times \frac{22}{7}\right)$

$k = 12$

$\therefore P_{max} = 60x + 36y$

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