

# Model Solutions

**Pearson Edexcel Level 3 GCE**

**Thursday 16 May 2019**

Afternoon

Paper Reference **8FM0-27**

## Further Mathematics

**Advanced Subsidiary**

**Further Mathematics options**

**27: Decision Mathematics 1**

**(Part of options D, F, H and K)**

### You must have:

Mathematical Formulae and Statistical Tables (Green), calculator,  
D1 Answer Book (enclosed)

**Candidates may use any calculator allowed by Pearson regulations. Calculators must not have the facility for symbolic algebra manipulation, differentiation and integration, or have retrievable mathematical formulae stored in them.**

### Instructions

- Use **black** ink or ball-point pen.
- If pencil is used for diagrams/sketches/graphs it must be dark (HB or B).
- **Fill in the boxes** at the top of the answer book with your name, centre number and candidate number.
- Answer **all** questions and ensure that your answers to parts of questions are clearly labelled.
- Answer the questions in the Answer Book provided  
– *there may be more space than you need.*
- You should show sufficient working to make your methods clear. Answers without working may not gain full credit.
- Answers should be given to three significant figures unless otherwise stated.

### Information

- A booklet 'Mathematical Formulae and Statistical Tables' is provided.
- The total mark for this part of the examination is 40. There are 5 questions.
- The marks for each question are shown in brackets  
– *use this as a guide as to how much time to spend on each question.*

### Advice

- Read each question carefully before you start to answer it.
- Try to answer every question.
- Check your answers if you have time at the end.

Turn over ►

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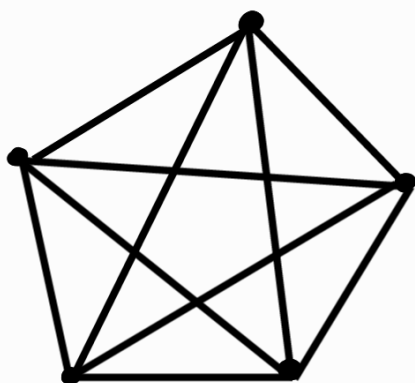


  
Pearson

1. (a) Draw the graph  $K_5$  (1)
- (b) (i) In the context of graph theory explain what is meant by 'semi-Eulerian'.  
 (ii) Draw two semi-Eulerian subgraphs of  $K_5$ , each having five vertices but with a different number of edges. (3)
- (c) Explain why a graph with exactly five vertices with vertex orders 1, 2, 2, 3 and 4 cannot be a tree. (2)

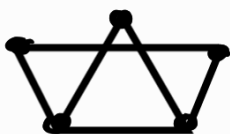
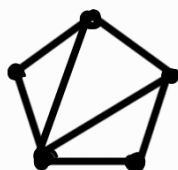
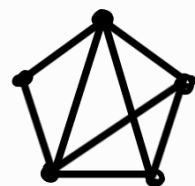
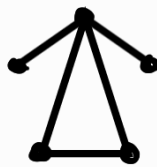
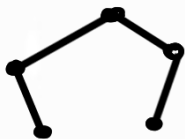
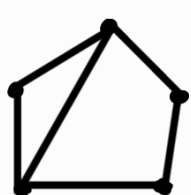
(Total for Question 1 is 6 marks)

a)



bi) A semi-Eulerian graph contains exactly 2 nodes of odd order (and the rest of the nodes of even order).

bii)



(use any 2 of the subgraphs drawn)

c)  $\frac{1+2+2+3+4}{2} = 6$  arcs but a tree on 5 nodes would contain only 4 arcs.

2. The following algorithm produces a numerical approximation for the integral

$$I = \int_A^B x^4 dx$$

- Step 1 Start  
 Step 2 Input the values of A, B and N  
 Step 3 Let  $H = (B - A) / N$   
 Step 4 Let  $C = H / 2$   
 Step 5 Let  $D = 0$   
 Step 6 Let  $D = D + A^4 + B^4$   
 Step 7 Let  $E = A$   
 Step 8 Let  $E = E + H$   
 Step 9 If  $E = B$  go to Step 12  
 Step 10 Let  $D = D + 2 \times E^4$   
 Step 11 Go to Step 8  
 Step 12 Let  $F = C \times D$   
 Step 13 Output F  
 Step 14 Stop

For the case when  $A = 1$ ,  $B = 3$  and  $N = 4$ ,

(a) (i) complete the table in the answer book to show the results obtained at each step of the algorithm.

(ii) State the final output.

(4)

(b) Calculate, to 3 significant figures, the percentage error between the exact value of  $I$  and the value obtained from using the approximation to  $I$  in this case.

(3)

(Total for Question 2 is 7 marks)

ai)

A	B	N	H	C	D	E	F
1	3	4	0.5	0.25	0		
					82	1	
						1.5	
					92.125	2	
					124.125	2.5	
					202.25	3	50.5625

aii) Final Output = 50.5625

$$b) \int_1^3 x^4 dx = \left[ \frac{x^5}{5} \right]_1^3 =$$
$$\left( \frac{3^5}{5} - \frac{1^5}{5} \right) = 48.4$$

$$\left( \frac{50.5625 - 48.4}{48.4} \right) \times 100 = 4.47\% \text{ (3sf)}$$

3.

Activity	Immediately preceding activities
A	-
B	-
C	A
D	A
E	A
F	B, C
G	B, C
H	D
I	D, E, F, G
J	D, E, F, G
K	G

(a) Draw the activity network described in the precedence table above, using activity on arc. Your activity network must contain the minimum number of dummies.

(5)

Every activity shown in the precedence table has the same duration.

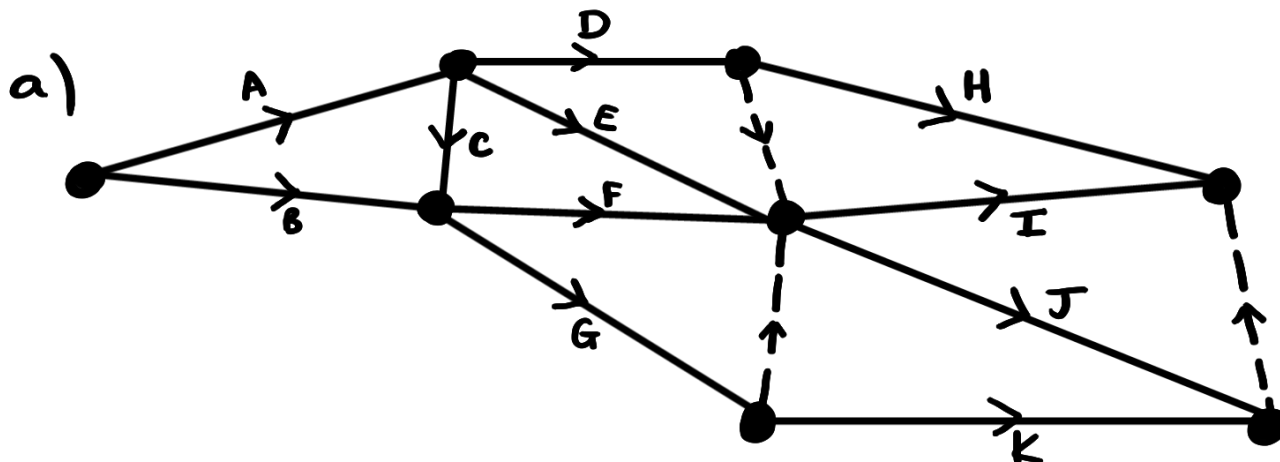
(b) Explain why activity B cannot be critical.

(1)

(c) State which other activities are not critical.

(1)

(Total for Question 3 is 7 marks)



b) Activity F requires activities A, B and C to be completed before F can begin.

The time to complete A and C is double that of B  $\therefore$  B can be delayed waiting for A and C to be completed.

As B has a float, B is not critical.

c) Activities D, E and H.

4.

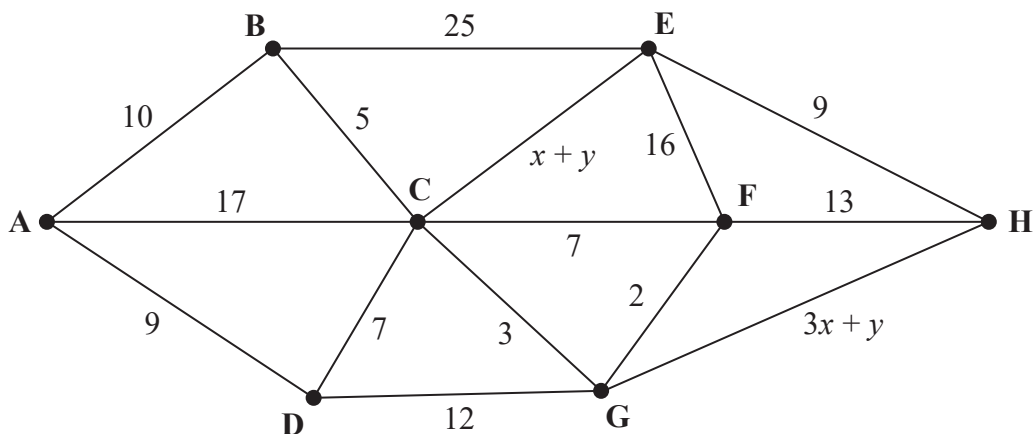


Figure 1

[The total weight of the network is  $135 + 4x + 2y$ ]

The weights on the arcs in Figure 1 represent distances. The weights on the arcs CE and GH are given in terms of  $x$  and  $y$ , where  $x$  and  $y$  are positive constants and  $7 < x + y < 20$

There are three paths from A to H that have the same minimum length.

(a) Use Dijkstra's algorithm to find  $x$  and  $y$ .

(7)

Key:

Vertex	Order of labelling	Final value
Working values		

a cont.) GH  $\Rightarrow$

$$33 - (3x + y) - 18 = 0$$

$$15 - 3x - y = 0$$

$$3x + y = 15 \quad \text{--- ①}$$

Sub ② into ①

$$3(-y + 9) + y = 15$$

$$-3y + 27 + y = 15$$

$$2y = 12$$

$$y = 6$$

$$\therefore \underline{x = 3} \quad (x = -6 + 9)$$

EH  $\Rightarrow$

$$33 - (15 + x + y) - 9 = 0$$

$$9 - x - y = 0$$

$$x + y = 9$$

$$x = -y + 9 \quad \text{--- ②}$$

An inspection route starting at A and finishing at H is found. The route traverses each arc at least once and is of minimum length.

(b) State the arcs that are traversed twice.

(1)

(c) State the number of times that vertex C appears in the inspection route.

(1)

(d) Determine the length of the inspection route.

(1)

(Total for Question 4 is 10 marks)

b) Odd nodes: A, B, D, H

starting at A, finishing at H

Possible pair

$$B(c)D = 5 + 7 = 12$$

$\therefore$  Arcs BC and CD need to be traversed twice.

c) Vertex C would appear 4 times.

$$\begin{aligned} \text{d) } 135 + 4x + 2y + 12 &= 135 + 4(3) + 2(6) + 12 \\ &= \underline{171} \end{aligned}$$



5. Ben is a wedding planner. He needs to order flowers for the weddings that are taking place next month. The three types of flower he needs to order are roses, hydrangeas and peonies.

Based on his experience, Ben forms the following constraints on the number of each type of flower he will need to order.

- At least three-fifths of all the flowers must be roses.
- For every 2 hydrangeas there must be at most 3 peonies.
- The total number of flowers must be exactly 1000

The cost of each rose is £1, the cost of each hydrangea is £5 and the cost of each peony is £4

Ben wants to minimise the cost of the flowers.

Let  $x$  represent the number of roses, let  $y$  represent the number of hydrangeas and let  $z$  represent the number of peonies that he will order.

- (a) Formulate this as a linear programming problem in  $x$  and  $y$  only, stating the objective function and listing the constraints as simplified inequalities with integer coefficients.

$$\text{Minimise } P = x + 5y + 4z$$

(7)

Subject to:

$$\begin{aligned} \cdot 2x &\geq 3y + 3z & \left( x \geq \frac{3}{5}(x+y+z) \Rightarrow \right. \\ & & \left. 5x \geq 3x + 3y + 3z \right) \end{aligned}$$

$$\cdot 3y \geq 2z$$

$$\cdot x + y + z = 1000$$

$$\Rightarrow z = 1000 - y - x \quad (\text{substitute into constraints and objective})$$

$$\begin{aligned} P_{\min} &= x + 5y + 4(1000 - y - x) \\ &= 4000 + y - 3x \end{aligned}$$

$$\begin{aligned} 3y &\geq 2(1000 - y - x) \\ 3y &\geq 2000 - 2y - 2x \end{aligned}$$

$$\cdot 5y + 2x \geq 2000$$

$$\begin{aligned} 2x &\geq 3y + 3(1000 - y - x) \\ 2x &\geq 3y + 3000 - 3y - 3x \end{aligned}$$

$$5x \geq 3000$$

$$\cdot x \geq 600$$

$$\therefore P_{\min} = 4000 + y - 3x$$

Subject to:

$$\cdot 5y + 2x \geq 2000$$

$$\cdot x \geq 600$$

$$y, z \geq 0$$

Ben decides to order the minimum number of roses that satisfy his constraints.

(b) (i) Calculate the number of each type of flower that he will order to minimise the cost of the flowers.

(ii) Calculate the corresponding total cost of this order.

(3)

(Total for Question 5 is 10 marks)

TOTAL FOR DECISION MATHEMATICS 1 IS 40 MARKS

END

$$\text{bi) } x + y + z = 1000$$

$$\text{As } x \geq 600, \quad \underline{x = 600}$$

$$y + z = 400$$

$\therefore$  600 roses,  
160 hydrangeas,  
240 peonies

$$5y + 2x \geq 2000$$

$$5y + 2(600) = 2000$$

$$5y = 800$$

$$\underline{y = 160}$$

$$\therefore 600 + 160 + z = 1000$$

$$\underline{z = 240}$$

$$\text{bii) } P_{\min} = 4000 + y - 3x$$

$$= 4000 + 160 - (3 \times 600)$$

$$= \underline{\underline{\pounds 2360}}$$

