# Mark Scheme (Results)

## Summer 2022

Pearson Edexcel GCE
Further Mathematics (8FM0)
Paper 01 Core Pure Mathematics

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## General Marking Guidance

- All candidates must receive the same treatment. Examiners must mark the first candidate in exactly the same way as they mark the last.
- Mark schemes should be applied positively. Candidates must be rewarded for what they have shown they can do rather than penalised for omissions.
- Examiners should mark according to the mark scheme not according to their perception of where the grade boundaries may lie.
- There is no ceiling on achievement. All marks on the mark scheme should be used appropriately.
- All the marks on the mark scheme are designed to be awarded. Examiners should always award full marks if deserved, i.e. if the answer matches the mark scheme. Examiners should also be prepared to award zero marks if the candidate's response is not worthy of credit according to the mark scheme.
- Where some judgement is required, mark schemes will provide the principles by which marks will be awarded and exemplification may be limited.
- When examiners are in doubt regarding the application of the mark scheme to a candidate's response, the team leader must be consulted.
- Crossed out work should be marked UNLESS the candidate has replaced it with an alternative response.

# EDEXCEL GCE MATHEMATICS
## General Instructions for Marking

1. The total number of marks for the paper is 80.

2. The Edexcel Mathematics mark schemes use the following types of marks:

- **M** marks: method marks are awarded for 'knowing a method and attempting to apply it', unless otherwise indicated.
- **A** marks: Accuracy marks can only be awarded if the relevant method (M) marks have been earned.
- **B** marks are unconditional accuracy marks (independent of M marks)
- Marks should not be subdivided.

3. Abbreviations

These are some of the traditional marking abbreviations that will appear in the mark schemes.

- bod – benefit of doubt
- ft – follow through
- the symbol $\sqrt{}$ will be used for correct ft
- cao – correct answer only
- cso - correct solution only. There must be no errors in this part of the question to obtain this mark
- isw – ignore subsequent working
- awrt – answers which round to
- SC: special case
- oe – or equivalent (and appropriate)
- dep – dependent
- indep – independent
- dp decimal places
- sf significant figures
- ∗ The answer is printed on the paper
- ⌐ The second mark is dependent on gaining the first mark

4. For misreading which does not alter the character of a question or materially simplify it, deduct two from any A or B marks gained, in that part of the question affected.

5. Where a candidate has made multiple responses <u>and indicates which response they wish to submit</u>, examiners should mark this response.
If there are several attempts at a question <u>which have not been crossed out</u>, examiners should mark the final answer which is the answer that is the <u>most complete</u>.

6. Ignore wrong working or incorrect statements following a correct answer.

7. Mark schemes will firstly show the solution judged to be the most common response expected from candidates. Where appropriate, alternatives answers are provided in the notes. If examiners are not sure if an answer is acceptable, they will check the mark scheme to see if an alternative answer is given for the method used.

| Question | Scheme | Marks | AOs |
|---|---|---|---|
| 1(a) | (i) $\mathbf{AB}=\begin{pmatrix}4&-1\\7&2\\-5&8\end{pmatrix}\begin{pmatrix}2&3&2\\-1&6&5\end{pmatrix}=\begin{pmatrix}8+1&12-6&8-5\\14-2&21+12&14+10\\-10-8&-15+48&-10+40\end{pmatrix}=\begin{pmatrix}9&6&3\\12&33&24\\-18&33&30\end{pmatrix}$ | M1 | 1.1b |
| | So $\mathbf{AB}-3\mathbf{C}=\begin{pmatrix}9&6&3\\12&33&24\\-18&33&30\end{pmatrix}-\begin{pmatrix}-15&6&3\\12&9&24\\-18&33&6\end{pmatrix}=\begin{pmatrix}24&0&0\\0&24&0\\0&0&24\end{pmatrix}$<br>or<br>$\mathbf{AB}-3\mathbf{C}=\begin{pmatrix}9&6&3\\12&33&24\\-18&33&30\end{pmatrix}+\begin{pmatrix}15&-6&-3\\-12&-9&-24\\18&-33&-6\end{pmatrix}=\begin{pmatrix}24&0&0\\0&24&0\\0&0&24\end{pmatrix}$<br>and states a value for $k$ | M1 | 1.1b |
| | Hence $\mathbf{AB}-3\mathbf{C}-24\mathbf{I}=\mathbf{0}$ so $k=-24$ | A1 | 1.1b |
| | (ii) Need two things<br>One of:<br>• **BA** is a 2×2 matrix<br>• Finds the matrix **BA** (must be a 2×2 matrix)<br>AND<br>One of:<br>• cannot subtract a 3×3 matrix<br>• finds matrix **3C** and comments that they have different dimensions / can't be done<br>• can't subtract matrices of different sizes<br>• 3**C** or **C** is a 3×3 matrix<br>• **BA** needs to be a 3×3 matrix | B1 | 2.4 |
| | | **(4)** | |
| (b)(i) | $\begin{pmatrix}-5&2&1\\4&3&8\\-6&11&2\end{pmatrix}\begin{pmatrix}x\\y\\z\end{pmatrix}=\begin{pmatrix}-14\\3\\7\end{pmatrix}\Rightarrow\begin{pmatrix}x\\y\\z\end{pmatrix}=\begin{pmatrix}-5&2&1\\4&3&8\\-6&11&2\end{pmatrix}^{-1}\begin{pmatrix}-14\\3\\7\end{pmatrix}$<br>Or states $\begin{pmatrix}x\\y\\z\end{pmatrix}=\mathbf{C}^{-1}\begin{pmatrix}-14\\3\\7\end{pmatrix}$<br>Or states $\begin{pmatrix}x\\y\\z\end{pmatrix}=\frac{1}{360}\begin{pmatrix}-82&7&13\\-56&-4&44\\62&43&-23\end{pmatrix}\begin{pmatrix}-14\\3\\7\end{pmatrix}$ | M1 | 1.2 |
| (ii) | $=\frac{1}{360}\begin{pmatrix}-82&7&13\\-56&-4&44\\62&43&-23\end{pmatrix}\begin{pmatrix}-14\\3\\7\end{pmatrix}=\ldots$ | M1 | 1.1b |

| | | | |
|---|---|---|---|
| | $= \begin{pmatrix} -\frac{41}{180} & \frac{7}{360} & \frac{13}{360} \\ -\frac{7}{45} & -\frac{1}{90} & \frac{11}{90} \\ \frac{31}{180} & \frac{43}{360} & -\frac{23}{360} \end{pmatrix} \begin{pmatrix} -14 \\ 3 \\ 7 \end{pmatrix} = \ldots$ <br> $\mathbf{C}^{-1} \begin{pmatrix} -14 \\ 3 \\ 7 \end{pmatrix} = \ldots$ | | |
| | So solution is $x = \frac{7}{2}, y = 3, z = -\frac{5}{2}$ or $(3.5, 3, -2.5)$ | A1 | 1.1b |
| | | **(3)** | |
| | | | **(7 marks)** |

**Notes:**

**(a) (i)**

**M1:** Attempts to find **AB**. Usually this will be done on calculator so answer implies the method. If answer is incorrect allow for at least 6 correct entries or calculations shown.
This mark can be implied by a correct matrix for $\mathbf{AB} - 3\mathbf{C}$ gives the first M1

**M1:** Uses their **AB** and **3C** matrices to find a multiple **I** and states a value for $k$

**A1:** Correct proof with $k = -24$ seen explicitly (may be in equation).

Minimum working required is $\mathbf{AB} - 3\mathbf{C} = \begin{pmatrix} 24 & 0 & 0 \\ 0 & 24 & 0 \\ 0 & 0 & 24 \end{pmatrix}$ gets M1 then states a value for $k$ M1 then $k = -24$ gets A1

**Special case:** If minimum working required is not seen and just $k = -24$ stated then M1 M0 A0 as they have not shown that the value of $k$ works.

**(ii)**

**B1:** Correct explanation referring to the dimensions of **BA** and **C** (or 3**C**) and that they do not match in the equation. They can find both these matrices and then comment they cannot be subtracted.

**(b) Mark (i) and (ii) altogether**

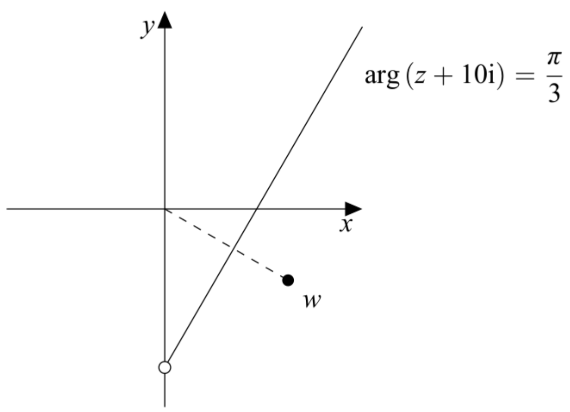
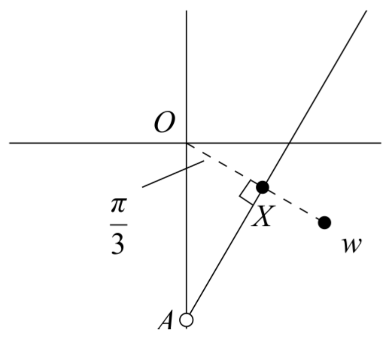
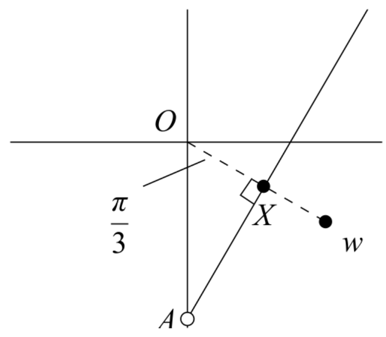
**M1:** States or implies use of the correct method of using the inverse matrix.

**M1:** Carries out the process of multiplying after finding the inverse. May find inverse long hand first. Finding the inverse matrix then writes down an answer gains M1.

**Note:** There is no need to find the inverse matrix. If the inverse matrix is not stated just answers written down then two out of the three correct ordinates imply the M1.

**A1:** Correct solution. Must be clear that $x = \frac{7}{2}, y = 3, z = -\frac{5}{2}$ allow $\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 3.5 \\ 3 \\ -2.5 \end{pmatrix}$

**Note:** If they solve using simultaneous equations only this is M0 M0 A0
If there is no reference to the inverse matrix and correct answers stated this is M0 M0 A0

| Question | Scheme | Marks | AOs |
|---|---|---|---|
| 2(a) | $\lvert w\rvert=\sqrt{\left(4\sqrt{3}\right)^2+\left(-4\right)^2}=8$ | B1 | 1.1b |
| | $\arg w=\arctan\left(\frac{\pm4}{4\sqrt{3}}\right)=\arctan\left(\pm\frac{1}{\sqrt{3}}\right)$ | M1 | 1.1b |
| | $=-\frac{\pi}{6}$ | A1 | 1.1b |
| | So $\left(w=\right)8\left(\cos\left(-\frac{\pi}{6}\right)+\mathrm{i}\sin\left(-\frac{\pi}{6}\right)\right)$ | A1 | 1.1b |
| | | **(4)** | |
| **(b)** |  (i) $w$ in 4th quadrant with either $\left(4\sqrt{3},-4\right)$ seen or $-\frac{\pi}{4}<\arg w<0$ | B1 | 1.1b |
| | (ii) half line with positive gradient emanating from imaginary axis. | M1 | 1.1b |
| | The half line should pass between $O$ and $w$ starting from a point on the imaginary axis below $w$ | A1 | 1.1b |
| | | **(3)** | |
| **(c)** |  $\Delta OAX$ is right angled at $X$ so $OX = 10\sin\frac{\pi}{6} = 5$ (oe) | M1 | 3.1a |
| | So shortest distance is $WX = OW - OX =$ '8' – 5 =… | M1 | 1.1b |
| | So min distance is 3 | A1 | 1.1b |
| | **Alternative 1**  A complete method to find the coordinates of $X$. Finds the equation of the line from $O$ to $w$, $y=-\frac{1}{\sqrt{3}}x$ and the equation of the half line $y=\sqrt{3}x-10$, solves to find the point of intersection $X\left(\frac{5\sqrt{3}}{2},-\frac{5}{2}\right)$ | M1 | 3.1a |
| | Finds the length $WX$ $\sqrt{\left(4\sqrt{3}-\frac{5\sqrt{3}}{2}\right)^2+\left(-4--\frac{5}{2}\right)^2}$ | M1 | 1.1b |
| | So min distance is 3 | A1 | 1.1b |
| | **Alternative 2** | M1 | 3.1a |

| | | | |
|---|---|---|---|
| | Finds the length $AW = \sqrt{\left(4\sqrt{3}-0\right)^2+\left(-4--10\right)^2} = \ldots\left\{\sqrt{84}\right\}$<br>Finds the angle between the horizontal and the line $AW$<br>$= \tan^{-1}\left(\frac{-4--10}{4\sqrt{3}}\right) = \ldots\left\{0.7137\ldots\text{radians or } 40.89\ldots^\circ\right\}$ | | |
| | Finds the length of $WX = \sqrt{84}\times\sin\left(\frac{\pi}{3}-0.7137\right) = \ldots$<br>Or $= \sqrt{84}\times\sin\left(60-40.89\right) = \ldots$ | M1 | 1.1b |
| | So min distance is 3 | A1 | 1.1b |
| | **Alternative 3**<br>Vector equation of the half line $r = \begin{pmatrix}0\\-10\end{pmatrix}+\lambda\begin{pmatrix}1\\\sqrt{3}\end{pmatrix}$<br>$XW = \begin{pmatrix}4\sqrt{3}-\lambda\\-4-\lambda\sqrt{3}-(-10)\end{pmatrix}$<br>Then either<br>$\begin{pmatrix}4\sqrt{3}-\lambda\\6-\lambda\sqrt{3}\end{pmatrix}\bullet\begin{pmatrix}1\\\sqrt{3}\end{pmatrix} = 4\sqrt{3}-\lambda+6\sqrt{3}-3\lambda = 0 \Rightarrow \lambda = \ldots\left\{\frac{5}{2}\sqrt{3}\right\}$<br>$r = \begin{pmatrix}0\\-10\end{pmatrix}+\frac{5}{2}\sqrt{3}\begin{pmatrix}1\\\sqrt{3}\end{pmatrix} = \ldots$<br>Or $XW^2 = \left(4\sqrt{3}-\lambda\right)^2+\left(6-\lambda\sqrt{3}\right)^2 = 48-8\lambda\sqrt{3}+\lambda^2+36-12\lambda\sqrt{3}+3\lambda^2$<br>$xw^2 = 84-20\lambda\sqrt{3}+4\lambda^2$ leading to<br>$\frac{\mathrm{d}\left(XW^2\right)}{\mathrm{d}\lambda} = -20\sqrt{3}+8\lambda = 0 \Rightarrow \lambda = \ldots$ | M1 | 3.1a |
| | Finds the length $WX$ $\sqrt{\left(4\sqrt{3}-\frac{5\sqrt{3}}{2}\right)^2+\left(-4--\frac{5}{2}\right)^2}$<br>Or $XW = \sqrt{\left(4\sqrt{3}-'\frac{5}{2}\sqrt{3}'\right)^2+\left(6-'\frac{5}{2}\sqrt{3}'\sqrt{3}\right)^2}$ | M1 | 1.1b |
| | So min distance is 3 | A1 | 1.1b |
| | | **(3)** | |
| | | | **(10 marks)** |

**Notes:**

**(a)**

**B1:** Correct modulus

**M1:** Attempts the argument. Allow for $\arctan\left(\frac{\pm 4}{\pm 4\sqrt{3}}\right)$ or equivalents using the modulus (may be in wrong quadrant for this mark).

**A1:** Correct argument $-\frac{\pi}{6}$ (must be in fourth quadrant but accept $\frac{11\pi}{6}$ or other difference of $2\pi$ for this mark).

**A1:** Correct expression found for $w$, in the correct form, must have positive $r$= 8 and $\theta = -\frac{\pi}{6}$.

**Note:** using degrees B1 M1 A0 A0

**(b)(i)&(ii)**

**B1:** $w$ plotted in correct quadrant with either the correct coordinate clearly seen or above the line $y = -x$

**M1:** Half line drawn starting on the imaginary axis away from $O$ with positive gradient (need not be labelled)

**A1:** Sketch on **one diagram**– both previous marks must have been scored and the half line should pass between $O$ and $w$ starting from a point on the imaginary axis below $w$. (You may assume it starts at $-10$i unless otherwise stated by the candidate)

**Note:** If candidates draw the loci on separate diagrams the maximum they can score is B1 M1 A0

**(c)**

**M1:** Formulates a correct strategy to find the shortest distance, e.g. uses right angle $OXA$ where $X$ is where the lines meet and proceeds at least as far as $OX$.

**M1:** Full method to achieve the shortest distance, e.g. for $WX = OW - OX$.

**A1: cao** shortest distance is 3

**Alternative 1:**

**M1:** Uses a correct method to find the equation of the line from $O$ to $w$, $y = -\frac{1}{\sqrt{3}}x$ and the equation of the half line $y = \sqrt{3}x - 10$, solves to find the point of intersection $X\left(\frac{5\sqrt{3}}{2}, -\frac{5}{2}\right)$

If the incorrect gradient(s) is used with no valid method seen this is M0

**M1**: Finds the length $WX = \sqrt{\left(\text{their}\,\frac{5\sqrt{3}}{2} - 4\sqrt{3}\right)^2 + \left(\text{their} - \frac{5}{2} - -4\right)^2} = ...$ condone a sign slip in the brackets.

**A1: cao** shortest distance is 3

**Alternative 2:**

**M1:** Uses a correct method to find the length $AW$ and a correct method to find the angle between the horizontal and the line $AW$

**M1**: Finds the length of $WX = \text{their}\sqrt{84} \times \sin\left(\frac{\pi}{3} - \text{their } 0.7137\right) = ...$

**A1: cao** shortest distance is 3

**Alternative 3**

**M1:** Finds the vector equation of the half line, then $XW$.

**Then either:** Sets dot product $XW$ and the line = 0 and solves for $\lambda$. Substitutes their $\lambda$ into the equation of the half line to find the point of intersection.

**Or** finds the length of $XW$ and differentiates, set = 0 and solve for $\lambda$

**M1**: Finds the length $WX = \sqrt{\left(\text{their}\,\frac{5\sqrt{3}}{2} - 4\sqrt{3}\right)^2 + \left(\text{their} - \frac{5}{2} - -4\right)^2} = ...$ condone a sign slip in the brackets.

Or substitutes their value for $\lambda$ into the length of *(d)*

**A1: cao** shortest distance is 3

| Question | Scheme | Marks | AOs |
|---|---|---|---|
| 3(a) | Coordinates of $Q$ are $(8,-3,2)$ | B1 | 2.2a |
| | | (1) | |
| (b) | Coordinates of $R$ are $\begin{pmatrix}\cos 120° & 0 & \sin 120° \\ 0 & 1 & 0 \\ -\sin 120° & 0 & \cos 120°\end{pmatrix}\begin{pmatrix}8\\3\\2\end{pmatrix} = \ldots$ or $\begin{pmatrix}-0.5 & 0 & \frac{\sqrt{3}}{2} \\ 0 & 1 & 0 \\ -\frac{\sqrt{3}}{2} & 0 & -0.5\end{pmatrix}\begin{pmatrix}8\\3\\2\end{pmatrix} = \ldots$ | M1 | 1.1a |
| | So $R$ is $\left(-4+\sqrt{3},\ 3, -4\sqrt{3}-1\right)$ | A1 | 1.1b |
| | | (2) | |
| (c) | Finds the distance $PR = \sqrt{\left(8-'\left(-4+\sqrt{3}\right)'\right)^2 + (3-'3')^2 + \left(2-'\left(-4\sqrt{3}-1\right)'\right)^2}$ Alternatively finds their $\overrightarrow{PR}$ or their $\overrightarrow{RP}$ then applies length of a vector formula. $\sqrt{\left(12-\sqrt{3}\right)^2 + \left(3+4\sqrt{3}\right)^2}$ or $\sqrt{\left(-12+\sqrt{3}\right)^2 + \left(-3-4\sqrt{3}\right)^2}$ | M1 | 2.1 |
| | $= \sqrt{204} \quad \left(= 2\sqrt{51}\right)$ cso | A1 | 1.1b |
| | | (2) | |
| (d) | $\overrightarrow{PR}.\overrightarrow{PQ} = \left(-12+\sqrt{3}, 0, -3-4\sqrt{3}\right).\left(0,-6,0\right) = 0$ hence perpendicular | B1ft | 1.1b |
| | | (1) | |
| (e) | $PQ$ is perpendicular to $PR$ so Area = $\frac{1}{2} \times PQ \times PR$ | M1 | 1.1b |
| | $= \frac{1}{2} \times 6 \times \sqrt{204} = 6\sqrt{51}$ cso | A1 | 1.1b |
| | | (2) | |
| | | **(8 marks)** | |

**Notes:**

**(a)**
**B1:** Coordinates of $Q$ correctly stated, accept as a column vector.

**(b)**
**M1:** Correct attempt to find coordinates of $R$ using the given matrix with $\theta = 120$. Must be multiplying in the correct way round. With no working two correct values or (-2.27, 3, -7.93) implies this mark.
**A1:** Correct exact coordinates as shown in scheme. Accept as a column vector. Cos 120 and sin 120 must have been evaluated.

**(c)**
**M1:** Applies the distance formula with the coordinates of $P$ and their $R$. Alternatively finds the vector $\overrightarrow{PR}$ or $\overrightarrow{RP}$ then applies length of a vector formula.
**A1:** Correct answer following correct coordinates of $R$, must be a surd but need not be fully simplified.

**(d)**

**B1ft:** Shows the dot product is zero between the vectors $\overrightarrow{PR}$ and $\overrightarrow{PQ}$ and draws the conclusion perpendicular. Accept with $\pm$ vectors for each. Follow through as long as the vectors are of the correct form, so $\overrightarrow{PR} = \begin{pmatrix} a \\ 0 \\ b \end{pmatrix}$ and $\overrightarrow{PQ} = \begin{pmatrix} 0 \\ c \\ 0 \end{pmatrix}$

Note They could state if vectors $\overrightarrow{PR}$ and $\overrightarrow{PQ}$ are perpendicular then $\overrightarrow{PR}.\overrightarrow{PQ} = 0$ then shows $\overrightarrow{PR}.\overrightarrow{PQ} = 0$ this is B1

**(e)**

**M1:** Correct method for the area of the triangle, follow through on their coordinates of *R* and *Q*. May see longer methods if they do not realise the triangle is right angled.

**A1:** For $6\sqrt{51}$ cso following correct coordinates of *R*

**Alternative 1**

M1 Complete method to find the correct area

Finding all the lengths $|PQ| = 6, \quad |PR| = \sqrt{240} = 4\sqrt{15}, \quad |QR| = \sqrt{204} = 2\sqrt{51}$

Use cosine rule to find an angle e.g. $\cos PRQ = \dfrac{240 + 204 - 36}{2 \times \sqrt{240} \times \sqrt{204}} = \dfrac{\sqrt{85}}{10}$

leading to $PRQ = 22.7...$ or $\sin PRQ = \sqrt{1 - \left(\dfrac{\sqrt{85}}{10}\right)^2} = ...\left\{\dfrac{\sqrt{15}}{10}\right\}$

Uses the area of the triangle $= \dfrac{1}{2} \times \sqrt{240} \times \sqrt{204} \times \dfrac{\sqrt{15}}{10}$ or $= \dfrac{1}{2} \times \sqrt{240} \times \sqrt{204} \times \sin 22.8$

**A1:** For $6\sqrt{51}$

**Alternative 2**

M1: Uses $\dfrac{1}{2}|\mathbf{a} \times \mathbf{b}|$ to find the required area

e.g. $QP = \begin{pmatrix} 0 \\ 6 \\ 0 \end{pmatrix}$ $RP = \begin{pmatrix} 12 - \sqrt{3} \\ 0 \\ 3 + 4\sqrt{3} \end{pmatrix}$ cross product

$$\begin{vmatrix} 0 & 6 & 0 \\ 12 - \sqrt{3} & 0 & 3 + 4\sqrt{3} \end{vmatrix} = -6\left(12 - \sqrt{3}\right)\mathbf{i} + 6\left(3 + 4\sqrt{3}\right)\mathbf{k}$$

Area $= \dfrac{1}{2}\sqrt{\left(-6\left(12 - \sqrt{3}\right)^2 + \left(6\left(3 + 4\sqrt{3}\right)^2\right)\right)} = \dfrac{1}{2}\sqrt{7344}$

**A1:** For $6\sqrt{51}$

| Question | Scheme | Marks | AOs |
|---|---|---|---|

| | | | |
|---|---|---|---|
| **4(i)** | $\sum \alpha_i = -\frac{5}{3}$ and $\sum \alpha_i \alpha_j = 0$<br>This mark can be awarded if seen in part (ii) or part (iii) | B1 | 3.1a |
| | So $\alpha^2+\beta^2+\gamma^2+\delta^2 = (\alpha+\beta+\gamma+\delta)^2 - 2\left(\sum \alpha_i\alpha_j\right) = ...$ | M1 | 1.1b |
| | $= \frac{25}{9} - 2\times 0 = \frac{25}{9}$ | A1 | 1.1b |
| | | **(3)** | |
| **(ii)** | $\sum \alpha_i\alpha_j\alpha_k = \frac{7}{3}$ and $\prod \alpha_i = 2$ or for $x = \frac{2}{w}$ used in equation<br>This mark can be awarded if seen in part (i) or part (iii) | B1 | 2.2a |
| | So $2\left(\frac{1}{\alpha}+\frac{1}{\beta}+\frac{1}{\gamma}+\frac{1}{\delta}\right) = 2\times\frac{\sum \alpha_i\alpha_j\alpha_k}{\alpha\beta\gamma\delta} = 2\times\frac{'\frac{7}{3}'}{'\frac{6}{3}'}$ or for<br>$3\left(\frac{16}{w^4}\right)+5\left(\frac{8}{w^3}\right)-7\left(\frac{2}{w}\right)+6=0 \Rightarrow 6w^4-14w^3+...=0$ leading to $\frac{14}{6}$ | M1 | 1.1b |
| | $\left(=2\times\frac{7/3}{2}\right)\left(=\frac{14}{6}\right)=\frac{7}{3}$ | A1 | 1.1b |
| | | **(3)** | |
| **(iii)** | $(3-\alpha)(3-\beta)(3-\gamma)(3-\delta) = ...$ expands all four brackets<br>Or equation with these roots is $3(3-x)^4+5(3-x)^3-7(3-x)+6=0$ | M1 | 3.1a |
| | $= 81-27\left(\sum \alpha_i\right)+9\left(\sum \alpha_i\alpha_j\right)-3\left(\sum \alpha_i\alpha_j\alpha_k\right)+\prod \alpha_i$<br>$= 81-27\left(-\frac{5}{3}\right)+9(0)-3\left(\frac{7}{3}\right)+2$<br>Or expands to fourth power and constant terms and attempts product of roots $3x^4+...+3\times3^4+5\times3^3-7\times3+6 \to \prod \alpha_i = \frac{"363"}{3}$ | dM1 | 1.1b |
| | $=121$ | A1 | 1.1b |
| | | **(3)** | |
| | **(9 marks)** | | |

**Notes:**

**(i)**

**B1:** Correct sum and pair sum of roots seen or implied. Must realise the pair sum is zero.

**Note:** These values can be seen anywhere in the candidate's solution

**M1**: Uses correct expression for the sum of squares.

**A1**: $\frac{25}{9}$. Allow this mark from incorrect sign on sum of squares (but they will score B0 if the sign is incorrect).

**(ii)**

**B1:** Correct triple sum and product of roots seen or implied. May be stated in (i). Alternatively, this may be scored for sight of $x = \frac{2}{w}$ used as a transformation in the equation.

**Note:** These values can be seen anywhere in the candidate's solution

**M1:** Substitutes their values into $2 \times \frac{\sum \alpha_i \alpha_j \alpha_k}{\alpha\beta\gamma\delta} = ...$ In the alternative it is for rearranging the equation to a quartic in *w* and uses to find the sum of the roots.

**A1:** $\frac{7}{3}$ Allow this mark from incorrect sign of both triple sum and product (but they will score B0 if the sign is incorrect).

**(iii)**

**M1:** A correct method to find the value used – may recognise structure as scheme, may expand the expression in stages, or may attempt to use a linear transformation (3 – *x*) or e.g. (3 – *w*) in original equation. Condone slips as long as the intention is clear.

**dM1:** Dependent on previous method mark**.** Uses at least 2 values of their sum of roots etc. in their expression. If using a linear shift this is for expanding to find the coefficient of $x^4$ and constant term and attempts product of roots by dividing the constant term by the coefficient of $x^4$.

**A1:** 121.

| Question | Scheme | Marks | AOs |
|---|---|---|---|
| **5(a)** | $\sum_{r=1}^{n}\left(3r^2-17r-25\right)=3\times\frac{n}{6}(n+1)(2n+1)-17\times\frac{1}{2}n(n+1)-\ldots$ | M1 | 1.1b |
| | $=3\times\frac{n}{6}(n+1)(2n+1)-17\times\frac{1}{2}n(n+1)-25n$ | A1 | 1.1b |
| | $=n\left(\frac{1}{2}\left(2n^2+3n+1\right)-\frac{17}{2}(n+1)-25\right)$ <br> or <br> $=\frac{n}{2}\left(\left(2n^2+3n+1\right)-17(n+1)-50\right)$ | M1 | 1.1b |
| | $=n\left(n^2-7n-33\right)$ cso (so $A$ = 7 and $B$ = 33) | A1 cso | 2.1 |
| | | **(4)** | |
| **(b)** | $\sum_{r=1}^{3k} r\tan\left(60r\right)°$ <br> $=\tan\left(60\right)°+2\tan\left(120\right)°+3\tan\left(180\right)°+4\tan\left(240\right)°+5\tan\left(300\right)°$ $+6\tan\left(360\right)°+$ <br> $=\left(\sqrt{3}-2\sqrt{3}+0\right)+\left(4\sqrt{3}-5\sqrt{3}+0\right)+\ldots$ | M1 | 3.1a |
| | Since tan has period 180° we see $\tan\left(60r\right)°$ repeats every three terms and **each group of three terms results in** $-\sqrt{3}$ **as a sum**, so with ***k*** **groups** of terms the sum is $-k\sqrt{3}$ | A1 | 2.4 |
| | | **(2)** | |
| **(c)** | $\sum_{r=5}^{n}\left(3r^2-17r-25\right)=\sum_{r=1}^{n}\left(3r^2-17r-25\right)-\sum_{r=1}^{4}\left(3r^2-17r-25\right)$ | M1 | 1.1b |
| | $=n\left(n^2-7n-33\right)-4\left(4^2-7\times4-33\right)$ <br> $\left(=n\left(n^2-7n-33\right)+180\right)$ | A1 | 1.1b |
| | $\sum_{r=6}^{3n} r\tan\left(60r\right)°=-n\sqrt{3}+2\sqrt{3}$ (allow for $-n\sqrt{3}--2\sqrt{3}$ ) | B1 | 2.2a |
| | $\Rightarrow n\left(n^2-7n-33\right)+180=15\left[-n\sqrt{3}+2\sqrt{3}\right]^2$ <br> $\Rightarrow n^3-7n^2-33n+180=15\left(3n^2-12n+12\right)$ <br> $\Rightarrow n^3-52n^2+147n=0$ | M1 | 3.1a |
| | $\Rightarrow n^3-52n^2+147n=0\Rightarrow n=\ldots$ | M1 | 1.1b |
| | But need $n$ > 5 for sums to be valid, so $n$ = 49 (allow if $n$ = 0 also given but $n$ = 3 must be rejected). | A1 | 2.3 |
| | | **(6)** | |
| | | | **(12 marks)** |

**Notes:**

**(a)**

**M1**: Applies the formulas for sum of integers and sum of squares of integers to the summation.

**A1**: Correct unsimplified expression for the sum, including the $25n$

**M1:** Expands and factors out the $n$ or ½n

**A1:** Correct proof, no errors seen.

**(b)**

**M1:** Writes out first few terms of the sum, at least 3, and identifies the repeating pattern, e.g. through bracketed terms or stating sum repeat every three terms oe.

**A1:** Correct explanation identifying $-\sqrt{3}$ is the sum of each group of three terms, so with $k$ lots of three terms the sum is $-k\sqrt{3}$

**(c)**

**M1:** Applies formula from (a) to left-hand side as a difference of two summations with either 4 or 5 as the limit on the second sum.

**A1:** Correct expression for the left-hand side in terms of $n$

**B1:** Correct expression for the sum on the right-hand side, allow if it arises from lower limit 6 used instead of 5 as the $6^{th}$ term is zero. May subtract the first few terms directly from the work in (b).

**M1:** Both sides expanded and terms gathered to reach a simplified cubic equation for $n$ with no other unknowns (may not have factor of $n$ if errors made, which is fine for the method mark). This mark is not dependent on any previous marks and can be awarded as long as there is an attempt at both sides of the equation and an attempt at squaring their $\sum_{r=6}^{3n} r\tan(60r)°$.

If divides through by $n$ this mark is awarded for a 3TQ

**M1:** Solves their cubic equation, which may be via calculator (so may need to check values). They may divide by $n$ and solve a quadratic. Condone decimal roots truncated or rounded

**A1:** Selects the correct value of $n$ to give 49 as the only non-trivial answer. The value 3 must be rejected as summation on left undefined for this value, but accept if 0 and 49 are given (since both sides evaluate to 0 for $n = 0$ depending on one's interpretation of summations).

| Question | Scheme | Marks | AOs |
|---|---|---|---|
| 6(a) | Need **k** component to be zero at ground, so $0.84+0.8\lambda-\lambda^2=0 \Rightarrow \lambda=...$ | M1 | 1.1b |
| | $\lambda=-\frac{3}{5},\frac{7}{5}$, but $\lambda\geqslant 0$ so $\lambda=\frac{7}{5}$ | A1 | 1.1b |
| | | **(2)** | |
| (b) | Direction is $(9-4.6\times1.4)\mathbf{i}+15\mathbf{j}+(0.8-2\times1.4)$ $=2.56\mathbf{i}+15\mathbf{j}-2\mathbf{k}$ or $\frac{64}{25}\mathbf{i}+15\mathbf{j}-2\mathbf{k}$ | B1ft | 2.2a |
| | | **(1)** | |
| (c) | Direction perpendicular to ground is $a\mathbf{k}$, so angle to perpendicular is given by $(\cos\theta)=\frac{a\mathbf{k}.(2.56\mathbf{i}+15\mathbf{j}-2\mathbf{k})}{a\times\lvert 2.56\mathbf{i}+15\mathbf{j}-2\mathbf{k}\rvert}$ or $\frac{\begin{pmatrix}2.56\\15\\-2\end{pmatrix}\bullet\begin{pmatrix}0\\0\\a\end{pmatrix}}{\begin{vmatrix}2.56\\15\\-2\end{vmatrix}\begin{vmatrix}0\\0\\a\end{vmatrix}}$ or angle between $\begin{pmatrix}2.56\\15\\-2\end{pmatrix}$ and $\begin{pmatrix}2.56\\15\\0\end{pmatrix}$ is given by $(\cos\theta)=\frac{\begin{pmatrix}2.56\\15\\-2\end{pmatrix}\bullet\begin{pmatrix}2.56\\15\\0\end{pmatrix}}{\begin{vmatrix}2.56\\15\\-2\end{vmatrix}\begin{vmatrix}2.56\\15\\0\end{vmatrix}}$ | M1 | 1.1b |
| | $=\frac{-2}{\sqrt{2.56^2+15^2+(-2)^2}}(=-0.130\ldots)$ Or $=\frac{231.5536}{\sqrt{2.56^2+15^2+(-2)^2}\sqrt{2.56^2+15^2+(0)^2}}=0.991...$ | M1 | 1.1b |
| | $90°-\arccos('-0.130...')=-7.48...$ or $\arccos(0.991...)$ | ddM1 | 3.1b |
| | So the tennis ball hits ground at angle of $7.5°$ (1d.p.) cao | A1 | 3.2a |
| | **Alternative** Finds the length of the vector in the **ij** plane $=\sqrt{2.56^2+15^2}$ | M1 | 1.1b |
| | $\tan\theta=\frac{2}{\sqrt{2.56^2+15^2}}$ | M1 | 1.1b |
| | $\theta=\arctan\left(\frac{2}{\sqrt{2.56^2+15^2}}\right)$ or $\theta=90-\arctan\left(\frac{\sqrt{2.56^2+15^2}}{2}\right)$ | ddM1 | 3.1b |

| | | | |
|---|---|---|---|
| | So the tennis ball hits ground at angle of $7.5°$ (1d.p.) | A1 | 3.2a |
| | | **(4)** | |
| **(d)** | In same plane as net when $\mathbf{r}.\mathbf{j}=0$, $\begin{pmatrix}-4.1+9\lambda-2.3\lambda^2\\-10.25+15\lambda\\0.84+0.8\lambda-\lambda^2\end{pmatrix}\bullet\begin{pmatrix}0\\1\\0\end{pmatrix}$ leading to $-10.25+15\lambda=0\Rightarrow\lambda=...$ $\left(=\frac{41}{60}=0.683333...\right)$ | M1 | 3.1b |
| | So is at position $\left(-4.1+9\times\frac{41}{60}-2.3\left(\frac{41}{60}\right)^2\right)\mathbf{i}+0\mathbf{j}+\left(0.84+0.8\times\frac{41}{60}-\left(\frac{41}{60}\right)^2\right)\mathbf{k}$ | M1 | 1.1b |
| | $=$ awrt $0.976\mathbf{i}+$ awrt $0.920\mathbf{k}$ or $=$ awrt $0.976\mathbf{i}+0.92\mathbf{k}$ (to 3 s.f.) or $=$ awrt $0.976\mathbf{i}+\frac{3311}{3600}\mathbf{k}$ | A1 | 1.1b |
| | | **(3)** | |
| **(e)** | Modelling as a line, height of net is 0.9m along its length so as **0.92 > 0.9** the ball **will** pass over the net according to the model. | B1ft | 3.2a |
| | | **(1)** | |
| **(f)** | Identifies a suitable feature of the model that affects the outcome<br>And uses it to draw a compatible conclusion.<br>For example<br>• The ball is **not a particle** and will have **diameter/radius**, therefore it will hit the net and not pass over.<br>• As above, but so the **ball will clip the net** but it's momentum will take it over as it is mostly above the net.<br>• The **model** says that the ball will clear the net by 2cm which may be smaller than the balls **diameter**<br>• The **net will not be a straight line/taut** so will not be 0.9m high, so the ball **will** have enough clearance to **pass** over the net. | M1<br>A1 | 3.2b<br>2.2b |
| | | **(2)** | |
| | | | **(13 marks)** |

**Notes:**

**Accept any alternative vector notations throughout.**

**(a)**

**M1:** Attempts to solve the quadratic from equating the **k** component to zero.

**A1:** Correct value, must select positive root, so accept 1.4 oe.

Correct answer only M1 A1

**(b)**

**B1ft:** For $(2.56,15,-2)$ o.e or follow through $(9-4.6\times'\lambda',15,0.8-2\times'\lambda')$ for their $\lambda$.

**(c)**

**M1:** Recognises the angle between the perpendicular and direction vector is needed, and identifies the perpendicular as $a\mathbf{k}$ for any non-zero $a$ (including 1), and attempts dot product

Alternatively recognises the dot product of $(2.56, 15, -2)$ and $(2.56, 15, 0)$

**M1:** Applies the dot product formula $\frac{a \bullet b}{|a||b|}$ correctly between *any* two vectors, but must have dot product and modulus evaluated.

**ddM1:** Dependent on both previous marks. A correct method to proceed to the required angle, usually $90° - \arccos('-0.130...')$ as shown in scheme but may e.g. use sin $\theta$ instead of cos $\theta$ in formula.

Alternatively is using dot product of $(2.56, 15, -2)$ and $(2.56, 15, 0)$ finds arccos(0.991…)

**A1:** For 7.5° cao

**Alternative**

**M1:** Finds the length of the vector in the **ij** plane.

**M1:** Finds the tan of any angle the

**ddM1:** Dependent on both previous marks. Finds the required angle

**A1:** For 7.5° cao

**(d)**

**M1:** Attempts to find value of $\lambda$ that gives zero **j** component.

**M1:** Uses their value of $\lambda$ in the equation of the path to find position.

**A1:** Correct position.

**(e)**

**B1ft:** States that 0.920 > 0.9 so according to the model the ball will pass over the net. Follow through on their **k** component and draws an appropriate conclusion. May stay the value of k > 0.92

**(f)**

**M1:** There must be some reference to the model to score this mark. See scheme for examples. It is likely to be either the ball is not a particle, or the top of the net is not a straight line. Accept references to the ball crossing a long way from the middle.

Do not accept reasons such as "there may be wind/air resistance" as these are not referencing the given model.

**A1:** For a reasonable conclusion based on their reference to the model.

**For example**

The ball is not a particle; therefore, it will not go over the net is M1A0 as not explained why – needs reference to radius/diameter

| Question | Scheme | Marks | AOs |
|---|---|---|---|
| 7 | For $n=1$: $\begin{pmatrix}1-6\times1 & 9\times1\\ -4\times1 & 1+6\times1\end{pmatrix}=\begin{pmatrix}-5 & 9\\ -4 & 7\end{pmatrix}=\begin{pmatrix}-5 & 9\\ -4 & 7\end{pmatrix}^1$<br>So the statement is true for $n=1$ | B1 | 2.2a |
| | **Assume** true for $\boldsymbol{n=k}$,<br>or<br>**Assume** $\begin{pmatrix}-5 & 9\\ -4 & 7\end{pmatrix}^k=\begin{pmatrix}1-6k & 9k\\ -4k & 1+6k\end{pmatrix}$ | M1 | 2.5 |
| | $\begin{pmatrix}-5 & 9\\ -4 & 7\end{pmatrix}^{k+1}=\begin{pmatrix}-5 & 9\\ -4 & 7\end{pmatrix}^k\times\begin{pmatrix}-5 & 9\\ -4 & 7\end{pmatrix}$ OR $\begin{pmatrix}-5 & 9\\ -4 & 7\end{pmatrix}\times\begin{pmatrix}-5 & 9\\ -4 & 7\end{pmatrix}^k$ | M1 | 2.1 |
| | $=\begin{pmatrix}1-6k & 9k\\ -4k & 1+6k\end{pmatrix}\times\begin{pmatrix}-5 & 9\\ -4 & 7\end{pmatrix}=\begin{pmatrix}-5+30k-36k & 9-54k+63k\\ 20k-4-24k & -36k+7+42k\end{pmatrix}$<br>OR<br>$=\begin{pmatrix}-5 & 9\\ -4 & 7\end{pmatrix}\times\begin{pmatrix}1-6k & 9k\\ -4k & 1+6k\end{pmatrix}=\begin{pmatrix}-5+30k-36k & -45k+9+54k\\ -4+24k-28k & -36k+7+42k\end{pmatrix}$ | M1 | 1.1b |
| | Achieves from fully correct working $=\begin{pmatrix}-5-6k & 9+9k\\ -4-4k & 7+6k\end{pmatrix}$ | A1 | 1.1b |
| | $=\begin{pmatrix}1-6(k+1) & 9(k+1)\\ -4(k+1) & 1+6(k+1)\end{pmatrix}$<br>Hence the result is true for $n=k+1$. Since it is true for $n=1$, and if true for $n=k$ then true for $n=k+1$, thus by mathematical induction the result holds for all $n\in\mathbb{N}$ | A1cso | 2.4 |
| | | **(6)** | |
| | | **(6 marks)** | |

**Notes:**

**(a)**

**B1:** Shows the statement is true for $n=1$. Accept as minimum $\begin{pmatrix}1-6 & 9\\ -4 & 1+6\end{pmatrix}=\begin{pmatrix}-5 & 9\\ -4 & 7\end{pmatrix}$

**M1:** Makes the inductive assumption, **assume** true $\boldsymbol{n=k}$**.** This may appear in the conclusion.

**M1:** A correct statement for $\begin{pmatrix}-5 & 9\\ -4 & 7\end{pmatrix}^{k+1}$ in terms of $\begin{pmatrix}-5 & 9\\ -4 & 7\end{pmatrix}^k$, can be either way round.

Can be implied by $\begin{pmatrix}1-6k & 9k\\ -4k & 1+6k\end{pmatrix}\times\begin{pmatrix}-5 & 9\\ -4 & 7\end{pmatrix}$ or $\begin{pmatrix}-5 & 9\\ -4 & 7\end{pmatrix}\times\begin{pmatrix}1-6k & 9k\\ -4k & 1+6k\end{pmatrix}$

**M1:** Carries out the multiplication correctly, condone sign slips

**A1:** Correct simplified matrix **from fully correct working**

**A1:** Completes the inductive argument by showing clearly the matrix has the correct form (must have ($k$ + 1) factors in terms) or uses the result with $n=k+1$ and shows that their result is the same.

Conclusion conveying **all** three underlined points or equivalent at some point in their argument. Depends on all three M's and A marks but can be scored without the B mark as long as it is stated true for $n$ = 1

| Question | Scheme | Marks | AOs |
|---|---|---|---|
| **8(a)** | $a = 4$ | B1 | 3.3 |
| | | **(1)** | |
| **(b)** | Model A: (i) Widest point will be 4 (cm) from the base | B1 | 3.4 |
| | (ii) Width at widest point is 12 (cm) $\left(2\times('a'+2)\text{ ft}\right)$ | B1ft | 3.4 |
| | Model B: (i) $y = 4 + \dfrac{x^3 - 64x}{100} \Rightarrow \dfrac{dy}{dx} = \dfrac{3x^2 - 64}{100}$ | M1 | 3.1b |
| | $\dfrac{dy}{dx} = 0 \Rightarrow x = \pm\sqrt{\dfrac{64}{3}} = \pm\dfrac{8\sqrt{3}}{3} = \pm\text{awrt}4.62$ | A1 | 1.1b |
| | So max width is a distance $8 - \dfrac{8}{\sqrt{3}} = 8 - \dfrac{8\sqrt{3}}{3} \approx 3.38$ (cm) from base. | A1 | 3.4 |
| | (ii) $y\Big\|_{-4.61..} = 4 + \dfrac{(-4.62...)^3 - 64(-4.62...)}{100} = ...$ | dM1 | 3.4 |
| | $= 5.97...$ so diameter is approximately 11.9 (cm) $\left[2a + 3.94...\text{ ft}\right]$ | A1ft | 3.2a |
| | | **(7)** | |
| **(c)** | Model A and model B both have diameters closed to 12<br>Model B distance from base is closer to 3 than Model A so is more appropriate. | B1ft | 3.5b |
| | | **(1)** | |
| **(d)** | $V_B = \pi\displaystyle\int_{-8}^{8} y^2\,dx = \pi\int_{-8}^{8}\left(4 + \dfrac{x^3 - 64x}{100}\right)^2 dx = ...$ | B1 | 1.1b |
| | $= \dfrac{\{\pi\}}{10000}\displaystyle\int_{(-8)}^{(8)} 400^2 + x^6 + 64^2x^2 + 2\left(400x^3 - 400\times 64x - 64x^4\right)dx$<br>$= \dfrac{\{\pi\}}{10000}\displaystyle\int_{(-8)}^{(8)} 160000 + x^6 + 4096x^2 + 800x^3 - 51200x - 128x^4\ dx$<br>$= \{\pi\}\displaystyle\int_{(-8)}^{(8)} 16 + \dfrac{x^6}{10000} + \dfrac{4096}{10000}x^2 + \dfrac{8}{100}x^3 - \dfrac{512}{100}x - \dfrac{128}{10000}x^4\ dx$<br>$= \{\pi\}\displaystyle\int_{(-8)}^{(8)} 16 + \dfrac{x^6}{1000} + \dfrac{256}{625}x^2 + \dfrac{2}{25}x^3 - \dfrac{128}{25}x - \dfrac{8}{625}x^4\ dx$<br>$= \{\pi\}\displaystyle\int_{(-8)}^{(8)} 16 + \dfrac{8x(x-8)(x+8)}{100} + \left(\dfrac{x(x-8)(x+8)}{100}\right)^2 dx$ | M1 | 1.1b |
| | $= \dfrac{\{\pi\}}{10000}\left[160000x + \dfrac{x^7}{7} + 4096\dfrac{x^3}{3} + 800\dfrac{x^4}{4} - 51200\dfrac{x^2}{2} - 128\dfrac{x^5}{5}\right]_{(-8)}^{(8)}$ | dM1 | 1.1b |

| | | | |
|---|---|---|---|
| | $=\{\pi\}\left[16x+\frac{x^7}{70000}+\frac{256}{1875}x^3+\frac{1}{50}x^4-\frac{64}{25}x^2-\frac{8}{3125}x^5\right]_{(-8)}^{(8)}$ | | |
| | $=\frac{\{\pi\}}{10000}(620583.00....--2258983.01...)\approx\frac{2879566\pi}{10000}$ | M1 | 3.4 |
| | $=\text{awrt }905\left(\text{cm}^3\right)$ cso | A1 | 1.1b |
| | | **(5)** | |
| **(e)** | Compares their volume to 900 or compares their volume + 100 to 1 litre or 1000 and comments appropriately. | B1ft | 3.5a |
| | | **(1)** | |
| | | | **(15 marks)** |

**Notes:**

**Units not required in this question**

**(a)**

**B1:** For $a$ = 4, ignore any reference to units.

**(b)**

**B1:** Correct distance from base for Model A is 4

**B1ft:** Correct width at widest point. Follow through their '$a$', so $2\times('a'+2)$.

**M1:** Attempts the derivative for Model B's equation, reduce any power by 1

**A1:** Sets $\frac{dy}{dx}=0$ and finds correct $x$ coordinate of the stationary point (accept ±)

**A1:** For $8-\frac{8}{\sqrt{3}}$ or awrt 3.38 cso

**dM1:** Dependent on previous M mark. Uses their value of $x$ to find the value of $y$. If no working shown the value of $y$ must come from their $x$ value.

Note using $x$ = 4.62 give y = 2.029…

**A1:** Correct diameter, awrt 11.9 follow through their '$a$', so $[2a+3.94...\text{ ft}]$

Note: Correct answers with no working send to review

**Trial and error approach**

Candidates could score B1 B1 for model A however if working in integers it is unlikely that they will find the correct value for $x$ (they are using $x$ = – 5) not a valid method M0A0A0dM0A0

**(c)**

**B1ft: They must have answers for all parts in (b).** Accept any well-reasoned comment that follows their answers to (b) If the answers are correct, they must conclude that model B is more appropriate.

- If answers for one model are correct ish but other incorrect, or one value is clearly closer For example

| | **Distance (3)** | **Diameter (12)** | **Distance (3)** | **Diameter (12)** |
|---|---|---|---|---|
| **A** | 9.4 | 9.05 | 4 | 6 |
| **B** | 3.38 | 12.06 | 4.62 | 4.06 |
| **Conclusion** | Selects B as distance/diameter closet | | Select A as diameter closest | |

- If distances and diameters are similar selects the model which has the most appropriate value for distance or diameter
For example

| | **Distance (3)** | **Diameter (12)** | **Distance (3)** | **Diameter (12)** |
|---|---|---|---|---|

| | | | | |
|---|---|---|---|---|
| **A** | 0.76 | 6.8 | 4 | 20 |
| **B** | 1.28 | 10.5 | 3.38 | 19.94 |
| **Conclusion** | selects B as the diameter is closet | | Selects B as distance is closet | |

- If all values of the distances and diameters are varied any sensible reason stated for selecting a model.

**(d)**

**B1:** Applies $\pi \int_{-8}^{8} y^2 \, dx$ to the model. Must have $\pi$ and correct limits, with $y$ substituted in.

Alternatively attempts to square $y$ first and then substitute in.

**M1:** Attempts to expand $y^2$ this can be a poor attempt but must include at least a constant and $x^6$ terms as long a clear attempt at $y^2$ (Limits not required for this mark.)

**dM1:** Attempts the integration, must first be rearranged to an integrable form then look for power increasing by at least 1 in at least two terms. (Limits not required for this mark.)

**M1:** Applies correct limits to their integral following an attempt at $y^2$ with at least a constant and $x^6$ terms.

If there is no working shown, allow this method mark if the correct answer appears from a calculator as it implies correct limits have been applied the correct way round. (So M0dM0M1 is possible.)

**A1:** awrt 905 cso note it must come from a fully correct solution

**Note:** For answers that appear from calculator B1M0dM0M1A0 is possible, the question specifies algebraic integration to be used so the integration needs to be seen to score the other marks.

**(e)**

**B1ft:** Compares their volume to 900 or compares their volume + 100 to 1 litre or 1000 and comments appropriately. Correct answer in (d) needs to conclude that it is suitable.

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