Model Solutions

Please check the examination details below before entering your candidate information

Candidate surname	Other names

Pearson Edexcel Level 3 GCE

Centre Number | Candidate Number

Monday 11 May 2020

Afternoon (Time: 1 hour 40 minutes) | Paper Reference **8FM0/01**

Further Mathematics

Advanced Subsidiary
Paper 1: Core Pure Mathematics

You must have:
Mathematical Formulae and Statistical Tables (Green), calculator

Total Marks

Candidates may use any calculator allowed by Pearson regulations. Calculators must not have the facility for symbolic algebra manipulation, differentiation and integration, or have retrievable mathematical formulae stored in them.

Instructions

- Use **black** ink or ball-point pen.
- If pencil is used for diagrams/sketches/graphs it must be dark (HB or B).
- **Fill in the boxes** at the top of this page with your name, centre number and candidate number.
- Answer **all** questions and ensure that your answers to parts of questions are clearly labelled.
- Answer the questions in the spaces provided – *there may be more space than you need.*
- You should show sufficient working to make your methods clear. Answers without working may not gain full credit.
- Inexact answers should be given to three significant figures unless otherwise stated.

Information

- A booklet 'Mathematical Formulae and Statistical Tables' is provided.
- There are 10 questions in this question paper. The total mark for this paper is 80.
- The marks for **each** question are shown in brackets – *use this as a guide as to how much time to spend on each question.*

Advice

- Read each question carefully before you start to answer it.
- Try to answer every question.
- Check your answers if you have time at the end.

Turn over

Pearson

1. A system of three equations is defined by

$$\begin{aligned} kx + 3y - \quad z &= 3 \\ 3x - \quad y + \quad z &= -k \\ -16x - ky - kz &= k \end{aligned}$$

where k is a positive constant.

Given that there is no unique solution to all three equations,

(a) show that $k = 2$

(2)

Using $k = 2$

(b) determine whether the three equations are consistent, justifying your answer.

(3)

(c) Interpret the answer to part (b) geometrically.

(1)

a) $\begin{vmatrix} K & 3 & -1 \\ 3 & -1 & 1 \\ -16 & -K & -K \end{vmatrix} = K(K+K) - 3(-3K+16) - 1(-3K-16)$

$= 2K^2 + 9K - 48 + 3K + 16$

$= 2K^2 + 12K - 32 = 0$

$K^2 + 6K - 16 = 0$

$(K+8)(K-2) = 0$

$K = -8$ (reject as K is positive)

$\underline{K = 2}$

b) $2x + 3y - z = 3$ — 1

$3x - y + z = -2$ — 2

$-16x - 2y - 2z = 2$ — 3

1 + 3x 2

$(2x + 3y - z) + (9x - 3y + 3z) = 3 + (-6)$

$11x + 2z = -3$ — 4

2x 2 − 3

$(6x - 2y + 2z) - (-16x - 2y - 2z) = -4 - 2$

$22x + 4z = -6$ — 5

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Question 1 continued

b continued)

4 x 2 = 5 The two equations are a linear multiple of eachother ∴ the equations are consistent.

c) The three planes form a sheaf.

(Total for Question 1 is 6 marks)

2. Given that

$$z_1 = 2 + 3\mathrm{i}$$
$$|z_1 z_2| = 39\sqrt{2}$$
$$\arg(z_1 z_2) = \frac{\pi}{4}$$

where z_1 and z_2 are complex numbers,

(a) write z_1 in the form $r(\cos\theta + \mathrm{i}\sin\theta)$

Give the exact value of r and give the value of θ in radians to 4 significant figures.

(2)

(b) Find z_2 giving your answer in the form $a + \mathrm{i}b$ where a and b are integers.

(6)

a) $|z_1| = \sqrt{2^2 + 3^2} = \sqrt{13}$

$\arg z = \tan^{-1}\left(\frac{3}{2}\right) = 0.98279\ldots = 0.9828$ (4sf).

$z_1 = \sqrt{13}\left(\cos(0.9828) + i\sin(0.9828)\right)$

b) $|z_1 z_2| = |z_1| \times |z_2| = 39\sqrt{2}$

$\sqrt{13} \times |z_2| = 39\sqrt{2}$

$|z_2| = 3\sqrt{26}$

$\arg(z_1 z_2) = \arg(z_1) + \arg(z_2) = \frac{\pi}{4}$

$\tan^{-1}\left(\frac{3}{2}\right) + \arg(z_2) = \frac{\pi}{4}$

$\frac{\pi}{4} - \tan^{-1}\left(\frac{3}{2}\right) = -0.19739\ldots = \arg(z_2)$

$z_2 = 3\sqrt{26}\left(\cos(-0.197\ldots) + i\sin(-0.197\ldots)\right)$

$= 15 - 3i$

Question 2 continued

(Total for Question 2 is 8 marks)

3.

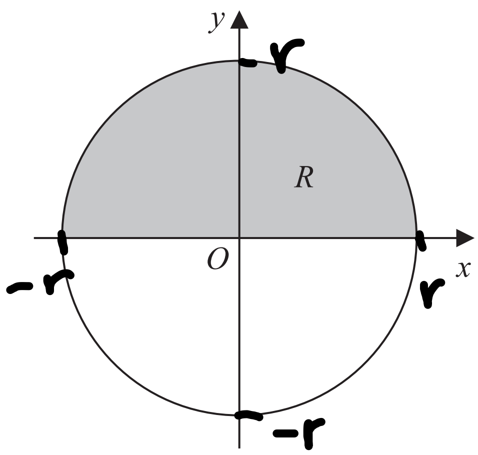

Figure 1

Figure 1 shows a circle with radius r and centre at the origin.
The region R, shown shaded in Figure 1, is bounded by the x-axis and the part of the circle for which $y > 0$
The region R is rotated through 360° about the x-axis to create a sphere with volume V

Use integration to show that $V = \frac{4}{3}\pi r^3$

(5)

$x^2+y^2=r^2 \rightarrow y^2 = r^2-x^2$

$V = \pi \int_{-r}^{r} r^2 - x^2 \, dx$

$= \pi \left[r^2 x - \frac{x^3}{3} \right]_{-r}^{r}$

$= \pi \left[\left(r^3 - \frac{r^3}{3} \right) - \left(-r^3 + \frac{r^3}{3} \right) \right]$

$= \pi \left[2r^3 - \frac{2r^3}{3} \right] = \pi \left(\frac{6r^3}{3} - \frac{2r^3}{3} \right)$

$\therefore V = \frac{4\pi r^3}{3}$ (as required)

Question 3 continued

(Total for Question 3 is 5 marks)

4. **All units in this question are in metres.**

A lawn is modelled as a plane that contains the points $L(-2, -3, -1)$, $M(6, -2, 0)$ and $N(2, 0, 0)$, relative to a fixed origin O.

(a) Determine a vector equation of the plane that models the lawn, giving your answer in the form $\mathbf{r} = \mathbf{a} + \lambda\mathbf{b} + \mu\mathbf{c}$

(3)

(b) (i) Show that, according to the model, the lawn is perpendicular to the vector $\begin{pmatrix} 1 \\ 2 \\ -10 \end{pmatrix}$

(ii) Hence determine a Cartesian equation of the plane that models the lawn.

(4)

There are two posts set in the lawn.
There is a washing line between the two posts.
The washing line is modelled as a straight line through points at the top of each post with coordinates $P(-10, 8, 2)$ and $Q(6, 4, 3)$.

(c) Determine a vector equation of the line that models the washing line.

(2)

(d) State a limitation of one of the models.

(1)

The point $R(2, 5, 2.75)$ lies on the washing line.

(e) Determine, according to the model, the shortest distance from the point R to the lawn, giving your answer to the nearest cm.

(2)

Given that the shortest distance from the point R to the lawn is actually 1.5 m,

(f) use your answer to part (e) to evaluate the model, explaining your reasoning.

(1)

a) $\overrightarrow{LM} = \begin{pmatrix} 6 \\ -2 \\ 0 \end{pmatrix} - \begin{pmatrix} -2 \\ -3 \\ -1 \end{pmatrix} = \begin{pmatrix} 8 \\ 1 \\ 1 \end{pmatrix}$

$\overrightarrow{LN} = \begin{pmatrix} 2 \\ 0 \\ 0 \end{pmatrix} - \begin{pmatrix} -2 \\ -3 \\ -1 \end{pmatrix} = \begin{pmatrix} 4 \\ 3 \\ 1 \end{pmatrix}$

$\underline{r} = \begin{pmatrix} 2 \\ 0 \\ 0 \end{pmatrix} + \lambda\begin{pmatrix} 8 \\ 1 \\ 1 \end{pmatrix} + \mu\begin{pmatrix} 4 \\ 3 \\ 1 \end{pmatrix}$

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Question 4 continued

bi) Method 1

Using the 'cross product':

$$\begin{vmatrix} i & j & k \\ 8 & 1 & 1 \\ 4 & 3 & 1 \end{vmatrix} = \begin{pmatrix} 1-3 \\ -(8-4) \\ 24-4 \end{pmatrix} = \begin{pmatrix} -2 \\ -4 \\ 20 \end{pmatrix}$$

$$\begin{pmatrix} -2 \\ -4 \\ 20 \end{pmatrix} \times -\frac{1}{2} = \begin{pmatrix} 1 \\ 2 \\ -10 \end{pmatrix}$$ (as required)

Method 2

$$\begin{pmatrix} 8 \\ 1 \\ 1 \end{pmatrix} \cdot \begin{pmatrix} x \\ y \\ z \end{pmatrix} = 8x + y + z = 0$$

$$\begin{pmatrix} 4 \\ 3 \\ 1 \end{pmatrix} \cdot \begin{pmatrix} x \\ y \\ z \end{pmatrix} = 4x + 3y + z = 0$$

Using simultaneous equations:

let $x = 1$

$8 + y + z = 0$

$y + z = -8$ — ①

$4 + 3y + z = 0$

$3y + z = -4$ — ②

① − ②

$y + z - (3y + z) = -8 - (-4)$

$-2y = -4$

$y = 2$

$z = -10$

$\therefore$ Perpendicular vector $= \begin{pmatrix} 1 \\ 2 \\ -10 \end{pmatrix}$ (as required)

Question 4 continued

bii) $\begin{pmatrix} x \\ y \\ z \end{pmatrix} \cdot \begin{pmatrix} 1 \\ 2 \\ -10 \end{pmatrix} = x + 2y - 10z$

when $x = 2,\ y = 0,\ z = 0$ using $N \begin{pmatrix} 2 \\ 0 \\ 0 \end{pmatrix}$

$2 + 2(0) - 10(0) = \begin{pmatrix} 1 \\ 2 \\ -10 \end{pmatrix} \cdot \begin{pmatrix} 2 \\ 0 \\ 0 \end{pmatrix}$

$x + 2y - 10z = 2$

c) $\overrightarrow{PQ} = \begin{pmatrix} 6 \\ 4 \\ 3 \end{pmatrix} - \begin{pmatrix} -10 \\ 8 \\ 2 \end{pmatrix} = \begin{pmatrix} 16 \\ -4 \\ 1 \end{pmatrix}$

$\underline{r} = \begin{pmatrix} 6 \\ 4 \\ 3 \end{pmatrix} + t \begin{pmatrix} 16 \\ -4 \\ 1 \end{pmatrix}$

d) · The lawn will not be flat
· The washing line will not be straight

e) Using equation: $\dfrac{|n_1\alpha + n_2\beta + n_3\gamma + d|}{\sqrt{n_1^2 + n_2^2 + n_3^2}}$

$= \dfrac{|(2\times1) + (5\times2) + (2.75\times-10) - 2|}{\sqrt{1^2 + 2^2 + (-10)^2}}$

$= 1.71\text{ m}$

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Question 4 continued

f) 1.71m is significantly different to 1.5
∴ the model is not good.

or

$\frac{1.71 - 1.5}{1.5} \times 100 = 14\%$ error

↖ this is a large % error
so the model is bad.

(Total for Question 4 is 13 marks)

5.

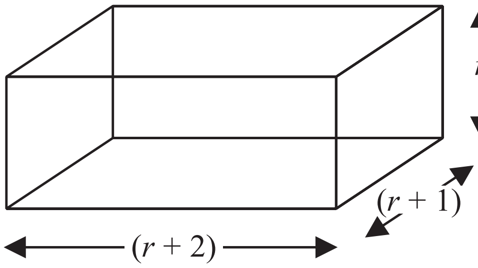

Figure 2

A block has length $(r + 2)$ cm, width $(r + 1)$ cm and height r cm, as shown in Figure 2.

In a set of n such blocks, the first block has a height of 1 cm, the second block has a height of 2 cm, the third block has a height of 3 cm and so on.

(a) Use the standard results for $\sum_{r=1}^{n} r^3, \sum_{r=1}^{n} r^2$ and $\sum_{r=1}^{n} r$ to show that the **total** volume, V, of all n blocks in the set is given by

$$V = \frac{n}{4}(n+1)(n+2)(n+3) \qquad n \geqslant 1$$

(5)

Given that the total volume of all n blocks is

$$(n^4 + 6n^3 - 11\,710)\ \text{cm}^3$$

(b) determine how many blocks make up the set.

(2)

a) Volume $= r(r+1)(r+2)$

$\therefore \sum_{r=1}^{n} r^3 + 3r^2 + 2r = \sum_{r=1}^{n} r^3 + 3\sum_{r=1}^{n} r^2 + 2\sum_{r=1}^{n} r$

$= \frac{1}{4}n^2(n+1)^2 + 3 \times \frac{1}{6}n(n+1)(2n+1) + 2 \times \frac{n}{2}(n+1)$

$= \frac{n}{4}(n+1)\left(n(n+1) + 2(2n+1) + 4\right)$

$= \frac{n}{4}(n+1)\left(n^2 + n + 4n + 2 + 4\right)$

$= \frac{n}{4}(n+1)\left(n^2 + 5n + 6\right)$

$V = \frac{n}{4}(n+1)(n+2)(n+3)$ (as required)

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Question 5 continued

b) $\frac{n}{4}(n+1)(n+2)(n+3) = n^4 + 6n^3 - 11710$

$\frac{1}{4}n^4 + \frac{3}{2}n^3 + \frac{11}{4}n^2 + \frac{3}{2}n = n^4 + 6n^3 - 11710$

$\frac{3}{4}n^4 + \frac{9}{2}n^3 - \frac{11}{4}n^2 - \frac{3}{2}n - 11710 = 0$ $(\times 4)$

$3n^4 + 18n^3 - 11n^2 - 6n - 46840 = 0$

There are 10 blocks

$\therefore n = 10$

Question 5 continued

Question 5 continued

(Total for Question 5 is 7 marks)

6. (i)

$$\mathbf{A} = \begin{pmatrix} 2 & a \\ a-4 & b \end{pmatrix}$$

where a and b are non-zero constants.

Given that the matrix **A** is self-inverse,

(a) determine the value of b and the possible values for a.

(5)

The matrix **A** represents a linear transformation M.

Using the smaller value of a from part (a),

(b) show that the invariant points of the linear transformation M form a line, stating the equation of this line.

(3)

(ii)

$$\mathbf{P} = \begin{pmatrix} p & 2p \\ -1 & 3p \end{pmatrix}$$

where p is a positive constant.

The matrix **P** represents a linear transformation U.
The triangle T has vertices at the points with coordinates (1, 2), (3, 2) and (2, 5).
The area of the image of T under the linear transformation U is 15

(a) Determine the value of p.

(4)

The transformation V consists of a stretch scale factor 3 parallel to the x-axis with the y-axis invariant followed by a stretch scale factor –2 parallel to the y-axis with the x-axis invariant. The transformation V is represented by the matrix **Q**.

(b) Write down the matrix **Q**.

(2)

Given that U followed by V is the transformation W, which is represented by the matrix **R**,

(c) find the matrix **R**.

(2)

ia) $\begin{pmatrix} 2 & a \\ a-4 & b \end{pmatrix}\begin{pmatrix} 2 & a \\ a-4 & b \end{pmatrix} = \begin{pmatrix} 4+a^2-4a & 2a+ab \\ 2a-8+ab-4b & a^2-4a+b^2 \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$

$\Rightarrow 4+a^2-4a=1$

$a^2-4a+3=0$

$(a-3)(a-1)=0$

$a=3$, $a=1$

$2a+ab=0$

when $a=1$ or when $a=3$

$b=-2$

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Question 6 continued

ib) using $a=1$, $b=-2$

$$\begin{pmatrix} 2 & 1 \\ -3 & -2 \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} x \\ y \end{pmatrix}$$

$2x+y=x$
$x+y=0$
$y=-x$

iia) Area of triangle $T=3$

$\det P = (3p \times p) - (-1 \times 2p)$
$= 3p^2 + 2p = \frac{15}{3}$

$3p^2+2p-5=0$

$(3p+5)(p-1)=0$

$p=-\frac{5}{3}$ (reject)

$p=1$

iib) $Q = \begin{pmatrix} 3 & 0 \\ 0 & -2 \end{pmatrix}$

iic) $\begin{pmatrix} 3 & 0 \\ 0 & -2 \end{pmatrix}\begin{pmatrix} p & 2p \\ -1 & 3p \end{pmatrix} = R$

$\therefore R = \begin{pmatrix} 3 & 0 \\ 0 & -2 \end{pmatrix}\begin{pmatrix} 1 & 2 \\ -1 & 3 \end{pmatrix} = \begin{pmatrix} 3 & 6 \\ 2 & -6 \end{pmatrix}$

Question 6 continued

Question 6 continued

(Total for Question 6 is 16 marks)

7.

$$f(z) = z^4 + az^3 + bz^2 + cz + d$$

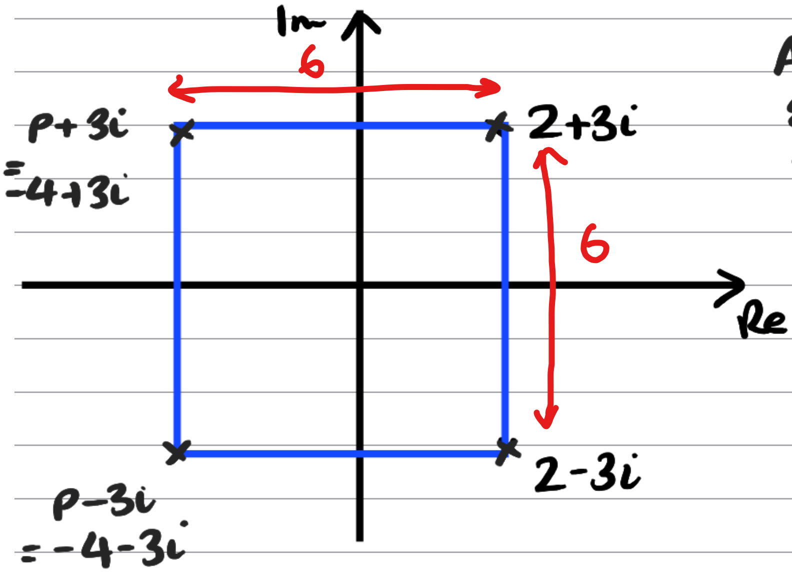
where a, b, c and d are real constants.

The equation $f(z) = 0$ has complex roots z_1, z_2, z_3 and z_4
When plotted on an Argand diagram, the points representing z_1, z_2, z_3 and z_4 form the vertices of a square, with one vertex in each quadrant.
Given that $z_1 = 2 + 3i$, determine the values of a, b, c and d.

(6)

$2-3i$ is also a root, as it is the conjugate pair of $2+3i$.

$(z-(2-3i))(z-(2+3i))$
$= (z-2+3i)(z-2-3i)$
$= z^2-2z-3iz-2z+4+6i+3iz-6i-9i^2$
$= z^2-4z+13$



As it forms a square, the other roots are $-4+3i$ and $-4-3i$

$(z-(-4+3i))(z-(-4-3i))$
$=(z+4-3i)(z+4+3i)$
$= z^2+4z+3iz+4z+16+12i-3iz-12i-9i^2$
$= z^2+8z+25$

$(z^2-4z+13)(z^2+8z+25)$
$= z^4+8z^3+25z^2-4z^3-32z^2-100z+13z^2+104z+325$
$= z^4+4z^3+6z^2+4z+325$

$a=4$, $b=6$, $c=4$, $d=325$

P 6 2 6 8 5 A 0 2 0 2 8

Question 7 continued

(Total for Question 7 is 6 marks)

8. Prove by induction that, for $n \in \mathbb{Z}^+$

$$f(n) = 2^{n+2} + 3^{2n+1}$$

is divisible by 7

(6)

• when $n=1$

$f(1) = 2^{1+2} + 3^{2(1)+1}$

$= 2^3 + 3^3$

$= 35$

$= 5(7)$

shows the statement is true for $n=1$ as $5(7) = 35$

• Assume $f(k)$ is divisible by 7.

(true for $n=k$)

$f(k) = 2^{k+2} + 3^{2k+1}$ (is divisible by 7).

when $n = k+1$

$f(k+1) = 2^{k+1+2} + 3^{2(k+1)+1}$

$= 2 \times 2^{k+2} + 3^2 \times 3^{2k+1}$

$f(k+1) - f(k) = (2 \times 2^{k+1} + 3^2 \times 3^{2k+1}) - (2^{k+1} + 3^{2k+1})$

$= 2^{k+1} + 8 \times 3^{2k+1}$

$= (2^{k+1} + 3^{2k+1}) + 7 \times 3^{2k+1}$

$= f(k) + 7 \times 3^{2k+1}$

$\therefore f(k+1) = 2f(k) + 7 \times 3^{2k+1}$

If true for $n=k$ then true for $n=k+1$ and as it is true for $n=1$, the statement is true for all positive integers n.

Question 8 continued

(Total for Question 8 is 6 marks)

9. The cubic equation

$$3x^3 + x^2 - 4x + 1 = 0$$

has roots α, β, and γ.

Without solving the cubic equation,

(a) determine the value of $\frac{1}{\alpha} + \frac{1}{\beta} + \frac{1}{\gamma}$ **(3)**

(b) find a cubic equation that has roots $\frac{1}{\alpha}$, $\frac{1}{\beta}$ and $\frac{1}{\gamma}$, giving your answer in the form $x^3 + ax^2 + bx + c = 0$, where a, b and c are integers to be determined. **(3)**

a) $\alpha + \beta + \gamma = -\frac{b}{a} = -\frac{1}{3}$

$\alpha\beta + \beta\gamma + \alpha\gamma = \frac{c}{a} = -\frac{4}{3}$

$\alpha\beta\gamma = \frac{-d}{a} = -\frac{1}{3}$

$\frac{1}{\alpha} + \frac{1}{\beta} + \frac{1}{\gamma} = \frac{\beta\gamma + \alpha\gamma + \alpha\beta}{\alpha\beta\gamma} = \frac{-\frac{4}{3}}{-\frac{1}{3}} = 4$

b) $\frac{1}{\alpha} + \frac{1}{\beta} + \frac{1}{\gamma} = -\frac{b}{a} = 4$

$\frac{1}{\alpha\beta} + \frac{1}{\beta\gamma} + \frac{1}{\alpha\gamma} = \frac{\gamma + \alpha + \beta}{\alpha\beta\gamma} = \frac{c}{a} = \frac{-\frac{1}{3}}{-\frac{1}{3}}$

$= 1$

$\frac{1}{\alpha\beta\gamma} = \frac{-d}{a} = \frac{1}{-\frac{1}{3}} = -3$

$x^3 - 4x^2 + x + 3 = 0$

$a = -4$, $b = 1$, $c = 3$.

Question 9 continued

(Total for Question 9 is 6 marks)

10. Given that there are two distinct complex numbers z that satisfy

$$\{z : |z - 3 - 5\text{i}| = 2r\} \cap \left\{z : \arg(z - 2) = \frac{3\pi}{4}\right\}$$

determine the exact range of values for the real constant r.

(7)

$(x-3)^2 + (y-5)^2 = (2r)^2$
and
$y = -x+2$

So $(x-3)^2 + (-x+2-5)^2 = 4r^2$

$(x-3)^2 + (-x-3)^2 = 4r^2$

$x^2 - 6x + 9 + x^2 + 6x + 9 = 4r^2$
$2x^2 + 18 = 4r^2$
$2x^2 + 0x + 18 - 4r^2 = 0$

$b^2 - 4ac > 0$
$0^2 - (4 \times 2 \times (18 - 4r^2)) > 0$
$0 > 144 - 32r^2$
$32r^2 > 144$
$r^2 > \frac{9}{2}$
$r > \frac{3\sqrt{2}}{2}$

max value of r
$(2r)^2 = 5^2 + (3-2)^2$
$4r^2 = 26$
$r^2 = \frac{26}{4}$
$r = \frac{\sqrt{26}}{2}$
$r < \frac{\sqrt{26}}{2}$

$\therefore \frac{3\sqrt{2}}{2} < r < \frac{\sqrt{26}}{2}$

Question 10 continued

Question 10 continued

(Total for Question 10 is 7 marks)

TOTAL FOR CORE PURE MATHEMATICS IS 80 MARKS