Please check the examination details below before entering your candidate information

Candidate surname	Other names

Centre Number | Candidate Number

Pearson Edexcel Level 3 GCE

Monday 5 June 2023

Afternoon (Time: 1 hour 30 minutes)

Paper reference **9FM0/02**

Further Mathematics

Advanced

PAPER 2: Core Pure Mathematics 2

You must have:
Mathematical Formulae and Statistical Tables (Green), calculator

Total Marks

Candidates may use any calculator permitted by Pearson regulations. Calculators must not have the facility for symbolic algebraic manipulation, differentiation and integration, or have retrievable mathematical formulae stored in them.

Instructions

- Use **black** ink or ball-point pen.
- If pencil is used for diagrams/sketches/graphs it must be dark (HB or B).
- **Fill in the boxes** at the top of this page with your name, centre number and candidate number.
- Answer **all** questions and ensure that your answers to parts of questions are clearly labelled.
- Answer the questions in the spaces provided – *there may be more space than you need.*
- You should show sufficient working to make your methods clear. Answers without working may not gain full credit.
- Inexact answers should be given to three significant figures unless otherwise stated.

Information

- A booklet 'Mathematical Formulae and Statistical Tables' is provided.
- There are 9 questions in this question paper. The total mark for this paper is 75.
- The marks for **each** question are shown in brackets – *use this as a guide as to how much time to spend on each question.*

Advice

- Read each question carefully before you start to answer it.
- Try to answer every question.
- Check your answers if you have time at the end.

Turn over

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1.

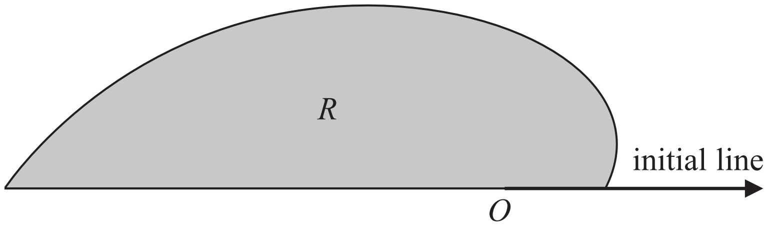

Figure 1

Figure 1 shows a sketch of the curve with polar equation

$$r = 2\sqrt{\sinh\theta + \cosh\theta} \qquad 0 \leqslant \theta \leqslant \pi$$

The region R, shown shaded in Figure 1, is bounded by the initial line, the curve and the line with equation $\theta = \pi$

Use algebraic integration to determine the exact area of R giving your answer in the form $pe^q - r$ where p, q and r are real numbers to be found.

(4)

$$R = \frac{1}{2}\int_0^{\pi} r^2\, d\theta$$

$$= \frac{1}{2}\int_0^{\pi} 4(\sinh\theta + \cosh\theta)\, d\theta \quad 1$$

$$= 2\left[\cosh\theta + \sinh\theta\right]_0^{\pi} \quad 1$$

$$= 2\left[\cosh\pi + \sinh\pi - \cosh 0 - \sinh 0\right]$$

$$= 2\left(\frac{e^{\pi} + e^{-\pi}}{2} + \frac{e^{\pi} - e^{-\pi}}{2} - 1 - 0\right) \quad 1$$

$$= 2e^{\pi} - 2 \quad 1$$

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Question 1 continued

(Total for Question 1 is 4 marks)

2. (a) Write down the Maclaurin series of e^x, in ascending power of x, up to and including the term in x^3

(1)

(b) Hence, without differentiating, determine the Maclaurin series of

$$e^{(e^x - 1)}$$

in ascending powers of x, up to and including the term in x^3, giving each coefficient in simplest form.

(5)

a) $e^x = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!}$ 1

b) $e^{e^x - 1} = 1 + (e^x - 1) + \frac{(e^x - 1)^2}{2!} + \frac{(e^x - 1)^3}{3!}$ 1

$= 1 + \left(1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} - 1\right) + \frac{1}{2}\left(1 + x + \frac{x^2}{2!} + \cdots - 1\right)^2$

$+ \frac{1}{6}\left(1 + x + \cdots - 1\right)^3$ 1

can ignore higher powered terms as only need expansion up to x^3

$= 1 + x + \frac{x^2}{2} + \frac{x^3}{6} + \frac{1}{2}\left(x + \frac{x^2}{2}\right)^2 + \frac{1}{6}(x)^3$

$= 1 + x + \frac{x^2}{2} + \frac{x^3}{6} + \frac{1}{2}\left(x^2 + x^3 + \cdots\right) + \frac{1}{6}x^3$

$= 1 + x + \left(\frac{1}{2} + \frac{1}{2}\right)x^2 + \left(\frac{1}{6} + \frac{1}{2} + \frac{1}{6}\right)x^3$ 1

$= 1 + x + x^2 + \frac{5x^3}{6}$ 1 1

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Question 2 continued

(Total for Question 2 is 6 marks)

3.
$$\mathbf{M} = \begin{pmatrix} -2 & 5 \\ 6 & k \end{pmatrix}$$

where k is a constant.

Given that

$$\mathbf{M}^2 + 11\mathbf{M} = a\mathbf{I}$$

where a is a constant and $\mathbf{I}$ is the 2×2 identity matrix,

(a) (i) determine the value of a

(ii) show that $k = -9$

(3)

(b) Determine the equations of the invariant lines of the transformation represented by $\mathbf{M}$.

(6)

(c) State which, if any, of the lines identified in (b) consist of fixed points, giving a reason for your answer.

(1)

a) (i)

$$M^2 + 11M = \begin{pmatrix} -2 & 5 \\ 6 & k \end{pmatrix}\begin{pmatrix} -2 & 5 \\ 6 & k \end{pmatrix} + 11\begin{pmatrix} -2 & 5 \\ 6 & k \end{pmatrix}$$

$$= \begin{pmatrix} 34 & 5k-10 \\ 6k-12 & k^2+30 \end{pmatrix} + \begin{pmatrix} -22 & 55 \\ 66 & 11k \end{pmatrix} = \begin{pmatrix} a & 0 \\ 0 & a \end{pmatrix} \quad 1$$

consider top-left entry: $34 - 22 = a$

$a = 12$ 1

(ii) consider bottom-left entry: $6k - 12 + 66 = 0$

$6k = -54$

$k = -9$ 1

b) invariant line satisfies

$$\begin{pmatrix} -2 & 5 \\ 6 & -9 \end{pmatrix}\begin{pmatrix} x \\ mx+c \end{pmatrix} = \begin{pmatrix} X \\ mX+c \end{pmatrix}$$

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Question 3 continued

$$\begin{pmatrix} -2x + 5(mx+c) \\ 6x - 9(mx+c) \end{pmatrix} = \begin{pmatrix} X \\ mX + c \end{pmatrix} \quad 1$$

first row: $-2x + 5mx + 5c = X$
second row: $6x - 9mx - 9c = mX + c$

sub first row into second row to eliminate X:

$$6x - 9mx - 9c = m(-2x + 5mx + 5c) + c$$
$$= -2mx + 5m^2x + 5mc + c \quad 1$$

$$(5m^2 + 7m - 6)x + (5m+10)c = 0 \quad 1$$

$\Rightarrow 5m^2 + 7m - 6 = 0$ AND $(5m+10)c = 0$
$(m+2)(5m-3) = 0$ $\qquad m = -2$ or $c = 0$
$m = -2$ or $m = \frac{3}{5}$ 1

if $m = -2$, c can be anything (since no matter what, both terms go to 0)

$\therefore y = -2x + c$ is an invariant line 1

if $m = \frac{3}{5}$, c has to $= 0$ for both terms to go to 0.

$\therefore y = \frac{3}{5}x$ is an invariant line. 1

c) $$\begin{pmatrix} -2 & 5 \\ 6 & -9 \end{pmatrix}\begin{pmatrix} x \\ 3x/5 \end{pmatrix} = \begin{pmatrix} -2x + 3x \\ 6x - 27x/5 \end{pmatrix} = \begin{pmatrix} x \\ 3x/5 \end{pmatrix}$$

so $y = \frac{3x}{5}$ contains fixed points 1

Question 3 continued

$$\begin{pmatrix} -2 & 5 \\ 6 & -9 \end{pmatrix}\begin{pmatrix} x \\ -2x+c \end{pmatrix} = \begin{pmatrix} -2x-10x+5c \\ 6x+18x-9c \end{pmatrix} = \begin{pmatrix} -12x+5c \\ 24x-9c \end{pmatrix}$$

contains a fixed point $(x=0, y=0)$ only if $c=0$

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Question 3 continued

(Total for Question 3 is 10 marks)

4. (a) Sketch the polar curve C, with equation

$$r = 3 + \sqrt{5}\cos\theta \qquad 0 \leqslant \theta \leqslant 2\pi$$

On your sketch clearly label the pole, the initial line and the value of r at the point where the curve intersects the initial line.

(2)

The tangent to C at the point A, where $0 < \theta < \frac{\pi}{2}$, is parallel to the initial line.

(b) Use calculus to show that at A

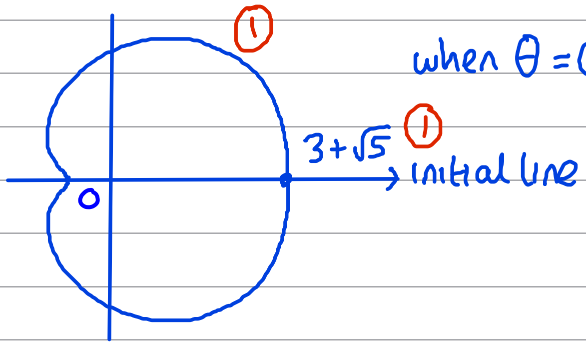
$$\cos\theta = \frac{1}{\sqrt{5}}$$

(4)

(c) Hence determine the value of r at A.

(1)

a)



when $\theta = 0$, $r = 3 + \sqrt{5}\cos 0$
$= 3 + \sqrt{5}$

b) parallel to initial line $\Rightarrow \frac{dy}{d\theta} = 0$

$y = r\sin\theta$
$= (3 + \sqrt{5}\cos\theta)\sin\theta$
$= 3\sin\theta + \frac{\sqrt{5}}{2}\sin 2\theta$

$\sin 2\theta = 2\sin\theta\cos\theta$

$\frac{dy}{d\theta} = 3\cos\theta + \sqrt{5}\cos 2\theta$ 2

$3\cos\theta + \sqrt{5}(2\cos^2\theta - 1) = 0$
$2\sqrt{5}\cos^2\theta + 3\cos\theta - \sqrt{5} = 0$

$\cos\theta = \frac{-3 \pm \sqrt{3^2 - 4(2\sqrt{5})(-\sqrt{5})}}{4\sqrt{5}}$ 1

$\cos\theta = \frac{-3 \pm 7}{4\sqrt{5}}$

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Question 4 continued

given that $0 < \theta < \frac{\pi}{2}$. In this region, $\cos\theta$ is positive,

so take positive root : $\cos\theta = \frac{-3+7}{4\sqrt{5}} = \frac{1}{\sqrt{5}}$ (1)

c) when $\cos\theta = \frac{1}{\sqrt{5}}$, $r = 3 + \sqrt{5}\left(\frac{1}{\sqrt{5}}\right) = 4$. (1)

Question 4 continued

Question 4 continued

(Total for Question 4 is 7 marks)

5. The points representing the complex numbers $z_1 = 35 - 25\text{i}$ and $z_2 = -29 + 39\text{i}$ are opposite vertices of a regular hexagon, H, in the complex plane.

The centre of H represents the complex number α

(a) Show that $\alpha = 3 + 7\text{i}$

(2)

Given that $\beta = \frac{1+\text{i}}{64}$

(b) show that

$$\beta(z_1 - \alpha) = 1$$

(2)

The vertices of H are given by the roots of the equation

$$(\beta(z - \alpha))^6 = 1$$

(c) (i) Write down the roots of the equation $w^6 = 1$ in the form $re^{\text{i}\theta}$

(1)

(ii) Hence, or otherwise, determine the position of the other four vertices of H, giving your answers as complex numbers in Cartesian form.

(4)

a) z_1 and z_2 are opposite vertices so their midpoint is the centre of the hexagon.

$\alpha = \frac{z_1 + z_2}{2} = \frac{35 - 25i - 29 + 39i}{2} = 3 + 7i$ as required (1) (1)

b) $\beta(z_1 - \alpha) = \left(\frac{1+i}{64}\right)(35 - 25i - (3 + 7i))$

$= \frac{1}{64}(1+i)(32 - 32i) = \frac{1}{64}(32 - 32i + 32i - 32i^2)$ (1)

$= \frac{1}{64}(32 - 32i + 32i + 32) = \frac{1}{64}(64) = 1$ as required (1)

c)(i) $w^6 = 1 = e^{2k\pi i}$

$w = e^{\frac{k\pi i}{3}}, k = 0, 1, 2, 3, 4, 5$ (1)

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Question 5 continued

(ii) $(\beta(z-\alpha))^6 = 1 = e^{2k\pi i}$

$\beta(z-\alpha) = e^{\frac{k\pi i}{3}}$, $k = 0,1,2,3,4,5$

$z-\alpha = \dfrac{e^{\frac{k\pi i}{3}}}{\beta}$

$z = \dfrac{e^{\frac{k\pi i}{3}}}{\beta} + \alpha$ (1)

$\dfrac{1}{\beta} = \dfrac{64}{1+i} = \dfrac{64(1-i)}{(1+i)(1-i)} = \dfrac{64(1-i)}{2} = 32(1-i)$

$\therefore z = 32(1-i)\left(\cos\frac{k\pi}{3} + i\sin\frac{k\pi}{3}\right) + (3+7i)$ (1)

when $k=0$, $z = 35-25i$

when $k=1$, $z = (19+16\sqrt{3}) + (-9+16\sqrt{3})i$

when $k=2$, $z = (-13+16\sqrt{3}) + (23+16\sqrt{3})i$ (1)

when $k=3$, $z = -29+39i$

when $k=4$, $z = (-13-16\sqrt{3}) + (23-16\sqrt{3})i$

when $k=5$, $z = (19-16\sqrt{3}) + (-9-16\sqrt{3})i$ (1)

Question 5 continued

Question 5 continued

(Total for Question 5 is 9 marks)

6. Given that

$$y = e^{2x} \sinh x$$

prove by induction that for $n \in \mathbb{N}$

$$\frac{d^n y}{dx^n} = e^{2x}\left(\frac{3^n + 1}{2}\sinh x + \frac{3^n - 1}{2}\cosh x\right)$$

(6)

base case $n=1$

$$\frac{dy}{dx} = 2e^{2x}\sinh x + e^{2x}\cosh x$$

$$= e^{2x}(2\sinh x + \cosh x) \quad ①$$

$$= e^{2x}\left(\frac{3^1+1}{2}\sinh x + \frac{3^1-1}{2}\cosh x\right)$$

so the result is true for $n=1$. ①

assume true for $n=k$:

$$\frac{d^k y}{dx^k} = e^{2x}\left(\frac{3^k+1}{2}\sinh x + \frac{3^k-1}{2}\cosh x\right)$$

$$\frac{d^{k+1} y}{dx^{k+1}} = 2e^{2x}\left(\frac{3^k+1}{2}\sinh x + \frac{3^k-1}{2}\cosh x\right)$$

$$+ e^{2x}\left(\frac{3^k+1}{2}\cosh x + \frac{3^k-1}{2}\sinh x\right) \quad ①$$

$$= e^{2x}\left(\left(\frac{2(3^k+1)}{2} + \frac{3^k-1}{2}\right)\sinh x + \left(\frac{2(3^k-1)}{2} + \frac{3^k+1}{2}\right)\cosh x\right)$$

$$= e^{2x}\left(\frac{3\times 3^k+1}{2}\sinh x + \frac{3\times 3^k-1}{2}\cosh x\right) \quad ①$$

$$= e^{2x}\left(\frac{3^{k+1}+1}{2}\sinh x + \frac{3^{k+1}-1}{2}\cosh x\right) \quad ①$$

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Question 6 continued

Hence the result is also true for $n=k+1$, so if true for $n=k$ then true for $n=k+1$. Since true for $n=1$, true for all positive integers n. 1

(Total for Question 6 is 6 marks)

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Turn over

7. **In this question you must show all stages of your working.**

Solutions relying entirely on calculator technology are not acceptable.

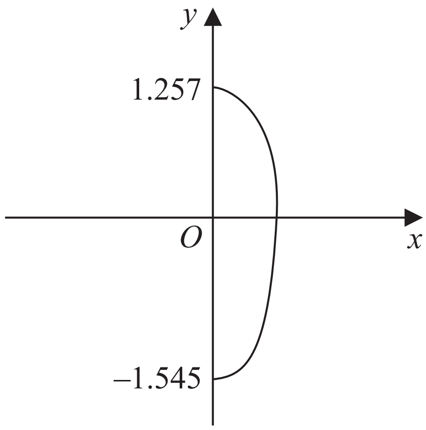

Figure 2

John picked 100 berries from a plant.

The largest berry picked was approximately 2.8 cm long.

The shape of this berry is modelled by rotating the curve with equation

$$16x^2 + 3y^2 - y\cos\left(\frac{5}{2}y\right) = 6 \qquad x \geqslant 0$$

shown in Figure 2, about the y-axis through 2π radians, where the units are cm.

Given that the y intercepts of the curve are –1.545 and 1.257 to four significant figures,

(a) use algebraic integration to determine, according to the model, the volume of this berry.

(6)

Given that the 100 berries John picked were then squeezed for juice,

(b) use your answer to part (a) to decide whether, in reality, there is likely to be enough juice to fill a 200 cm^3 cup, giving a reason for your answer.

(2)

a) $V = \pi \int_{-1.545}^{1.257} x^2 \, dy$ 1

find x^2: $16x^2 + 3y^2 - y\cos(2.5y) = 6$

$x = \frac{1}{16}(6 - 3y^2 + y\cos(2.5y))$

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Question 7 continued

find $\frac{\pi}{16}\int(6-3y^2+y\cos(2.5y))\,dy$

$\int y\cos(2.5y)\,dy = 0.4y\sin(2.5y) - \int 0.4\sin(2.5y)\,dy$

$u = y \quad v' = \cos(2.5y)$
$u' = 1 \quad v = 0.4\sin(2.5y)$

$= 0.4y\sin(2.5y) + 0.16\cos(2.5y)$ (1)

$\therefore \frac{\pi}{16}\int(6-3y^2+y\cos(2.5y))\,dy$

(1)
$= \frac{\pi}{16}\left(6y - y^3 + 0.4y\sin(2.5y) + 0.16\cos(2.5y)\right)$ (1)

adding limits:

$\pi\int_{-1.545}^{1.257} x^2\,dy = \frac{\pi}{16}\left[6y - y^3 + 0.4y\sin(2.5y) + 0.16\cos(2.5y)\right]_{-1.545}^{1.257}$ (1)

$= \frac{\pi}{16}(5.3954... - (-6.1101...))$

$= 2.26\text{cm}^3$ (3sf) (1)

b) Max volume of 100 berries $= 100 \times 2.26\text{cm}^3 = 226\text{cm}^3$ (1)

but not all of the berry can become juice and not all berries will be as big as the largest, so the berries are not likely to produce 200cm^3 of juice. (1)

Question 7 continued

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Question 7 continued

(Total for Question 7 is 8 marks)

8. Given that a cubic equation has three distinct roots that all lie on the same straight line in the complex plane,

(a) describe the possible lines the roots can lie on.

(2)

$$f(z) = 8z^3 + bz^2 + cz + d$$

where b, c and d are real constants.

The roots of $f(z)$ are distinct and lie on a straight line in the complex plane.

Given that one of the roots is $\frac{3}{2} + \frac{3}{2}i$

(b) state the other two roots of $f(z)$

(1)

$$g(z) = z^3 + Pz^2 + Qz + 12$$

where P and Q are real constants, has 3 distinct roots.

The roots of $g(z)$ lie on a different straight line in the complex plane than the roots of $f(z)$

Given that

- $f(z)$ and $g(z)$ have one root in common
- one of the roots of $g(z)$ is -4

(c) (i) write down the value of the common root,

(1)

(ii) determine the value of the other root of $g(z)$

(3)

(d) Hence solve the equation $f(z) = g(z)$

(4)

a) if all 3 roots are real, then all roots lie on the real axis. 1
If 2 roots are complex then they have the same real part (i.e. $a \pm bi$) so for all roots to lie on a straight line it must be a vertical line (i.e. $Re(z) = a$) 1

b) complex conjugate $= \frac{3}{2} - \frac{3}{2}i$ 1

third root must be real and have same real part $= \frac{3}{2}$ 1

Question 8 continued

c) the roots of $g(z)$ do not lie on $\text{Re}(z) = \frac{3}{2}$

if the common root of $f(z)$ and $g(z) = \frac{3}{2} \pm \frac{3}{2}i$,
then $\frac{3}{2} \mp \frac{3}{2}i$ will also be a root of $g(z)$.

Then the roots will not lie on a straight line.
So the shared root is $z = \frac{3}{2}$. 1

1

$$g(z) = \left(z - \frac{3}{2}\right)(z+4)(z+\alpha) = z^3 + Pz^2 + Qz + 12$$

where α is the 3rd root

consider constant term: $-\frac{3}{2} \times 4 \times \alpha = 12$ 1
$\alpha = -2$ 1

d) $f(z) = 8\left(z - \frac{3}{2}\right)\left(z - \frac{3}{2} - \frac{3}{2}i\right)\left(z - \frac{3}{2} + \frac{3}{2}i\right)$

$= 8\left(z - \frac{3}{2}\right)\left(z^2 - 3z + \frac{9}{2}\right)$ 1

the coefficient of z^3 in $f(z)$ is 8, so must multiply by 8 here

$f(z) = g(z)$:

$$8\left(z - \frac{3}{2}\right)\left(z^2 - 3z + \frac{9}{2}\right) = \left(z - \frac{3}{2}\right)(z+4)(z-2) \quad 1$$

$\div \left(z - \frac{3}{2}\right)$, $z = 3/2$ is a solution

$$8\left(z^2 - 3z + \frac{9}{2}\right) = (z+4)(z-2)$$

$$8z^2 - 24z + 36 = z^2 - 2z + 4z - 8$$

$$7z^2 - 26z + 44 = 0 \quad 1$$

Question 8 continued

$$z = \frac{26 \pm \sqrt{26^2 - 4(7)(44)}}{2(7)}$$

$$= \frac{13 \pm \sqrt{139}\,i}{7}$$

solutions are $z = \frac{3}{2}, \frac{13 \pm \sqrt{139}\,i}{7}$ (1)

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Question 8 continued

(Total for Question 8 is 11 marks)

9. A patient is treated by administering an antibiotic intravenously at a constant rate for some time.

Initially there is none of the antibiotic in the patient.

At time t minutes after treatment began

- the concentration of the antibiotic in the blood of the patient is x mg/ml
- the concentration of the antibiotic in the tissue of the patient is y mg/ml

The concentration of antibiotic in the patient is modelled by the equations

$$\frac{dx}{dt} = 0.025y - 0.045x + 2 \quad 1$$

$$\frac{dy}{dt} = 0.032x - 0.025y \quad 2$$

(a) Show that

$$40\,000\frac{d^2y}{dt^2} + 2800\frac{dy}{dt} + 13y = 2560$$

(3)

(b) Determine, according to the model, a general solution for the concentration of the antibiotic in the patient's tissue at time t minutes after treatment began.

(5)

(c) Hence determine a particular solution for the concentration of the antibiotic in the tissue at time t minutes after treatment began.

(4)

To be effective for the patient the concentration of antibiotic in the tissue must eventually reach a level between 185 mg/ml and 200 mg/ml.

(d) Determine whether the rate of administration of the antibiotic is effective for the patient, giving a reason for your answer.

(2)

a) Method: find x and $\dot{x}$ in terms of y and $\dot{y}$

second equation: $\dot{y} = 0.032x - 0.025y$

$\Rightarrow 0.032x = \dot{y} + 0.025y$

$x = 31.25\dot{y} + 0.78125y$

$\dot{x} = 31.25\ddot{y} + 0.78125\dot{y}$ 1

sub into 1:

$31.25\ddot{y} + 0.78125\dot{y} = 0.025y - 0.045(31.25\dot{y} + 0.78125y) + 2$

1

$31.25\ddot{y} + 2.1875\dot{y} + \frac{13}{1280}y = 2$

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Question 9 continued

$\times 1280$: $40{,}000\ddot{y} + 2800\dot{y} + 13y = 2560$ as required (1)

b) auxillary equation: $40{,}000m^2 + 2800m + 13 = 0$ (1)

$$(200m+1)(200m+13) = 0$$

$$m = -\frac{1}{200} \quad \text{or} \quad m = -\frac{13}{200}$$

(1)

complementary function: $y = Ae^{-\frac{t}{200}} + Be^{-\frac{13t}{200}}$ (1)

particular integral: let $y = \lambda$
$\dot{y} = 0$
$\ddot{y} = 0$

$\Rightarrow 40{,}000(0) + 2800(0) + 13\lambda = 2560$
$\lambda = \frac{2560}{13}$ (1)

general solution: $y = Ae^{-\frac{t}{200}} + Be^{-\frac{13t}{200}} + \frac{2560}{13}$ (1)

c) from stem, initially there is no antibiotic.
$\therefore t=0, y=0, x=0$

$0 = Ae^0 + Be^0 + \frac{2560}{13} \Rightarrow A + B = -\frac{2560}{13}$ (1)

Need another equation in A and B.
sub $t=0$, $x=0$ and $y=0$ into (2):

$\dot{y} = 0.032 \times 0 - 0.025 \times 0$ (1)
$= 0$

differentiate general solution and sub in $t=0, y=0, \dot{y}=0$:

Question 9 continued

$\dot{y} = -\frac{A}{200} e^{-\frac{t}{200}} - \frac{13B}{200} e^{-\frac{13t}{200}}$

$0 = -\frac{A}{200} e^{0} - \frac{13B}{200} e^{0}$

$0 = A + 13B$ (1)

solve simultaneously for A and B: $A = -\frac{640}{3}$ $B: \frac{640}{39}$

particular solution: $y = -\frac{640}{3} e^{-\frac{t}{200}} + \frac{640}{39} e^{-\frac{13t}{200}} + \frac{2560}{13}$ (1)

d) as $t \to \infty$, $e^{-kt} \to 0$ so $y \to \frac{2560}{13} \approx 196.92$ (1)

so rate of administration is sufficient to reach the required level. (1)

Question 9 continued

Question 9 continued

(Total for Question 9 is 14 marks)

TOTAL FOR PAPER IS 75 MARKS