Pearson Edexcel

Mark Scheme (Results)

Summer 2019

Pearson Edexcel GCE Further Mathematics

Further Core 1 (9FM0) Paper 1

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General Marking Guidance

- All candidates must receive the same treatment. Examiners must mark the first candidate in exactly the same way as they mark the last.
- Mark schemes should be applied positively. Candidates must be rewarded for what they have shown they can do rather than penalised for omissions.
- Examiners should mark according to the mark scheme not according to their perception of where the grade boundaries may lie.
- There is no ceiling on achievement. All marks on the mark scheme should be used appropriately.
- All the marks on the mark scheme are designed to be awarded. Examiners should always award full marks if deserved, i.e. if the answer matches the mark scheme. Examiners should also be prepared to award zero marks if the candidate's response is not worthy of credit according to the mark scheme.
- Where some judgement is required, mark schemes will provide the principles by which marks will be awarded and exemplification may be limited.
- When examiners are in doubt regarding the application of the mark scheme to a candidate's response, the team leader must be consulted.
- Crossed out work should be marked UNLESS the candidate has replaced it with an alternative response.

EDEXCEL GCE MATHEMATICS

General Instructions for Marking

1. The total number of marks for the paper is 75.

2. The Edexcel Mathematics mark schemes use the following types of marks:

- **M** marks: method marks are awarded for 'knowing a method and attempting to apply it', unless otherwise indicated.
- **A** marks: Accuracy marks can only be awarded if the relevant method (M) marks have been earned.
- **B** marks are unconditional accuracy marks (independent of M marks)
- Marks should not be subdivided.

3. Abbreviations

These are some of the traditional marking abbreviations that will appear in the mark schemes.

- bod – benefit of doubt
- ft – follow through
- the symbol $\sqrt{}$ will be used for correct ft
- cao – correct answer only
- cso - correct solution only. There must be no errors in this part of the question to obtain this mark
- isw – ignore subsequent working
- awrt – answers which round to
- SC: special case
- oe – or equivalent (and appropriate)
- dep – dependent
- indep – independent
- dp decimal places
- sf significant figures
- ∗ The answer is printed on the paper
- □ The second mark is dependent on gaining the first mark

4. For misreading which does not alter the character of a question or materially simplify it, deduct two from any A or B marks gained, in that part of the question affected.

5. Where a candidate has made multiple responses and indicates which response they wish to submit, examiners should mark this response.
   If there are several attempts at a question which have not been crossed out, examiners should mark the final answer which is the answer that is the most complete.

6. Ignore wrong working or incorrect statements following a correct answer.

7. Mark schemes will firstly show the solution judged to be the most common response expected from candidates. Where appropriate, alternatives answers are provided in the notes. If examiners are not sure if an answer is acceptable, they will check the mark scheme to see if an alternative answer is given for the method used.

General Principles for Core Mathematics Marking

(*But note that specific mark schemes may sometimes override these general principles*)

Method mark for solving 3 term quadratic:

1. Factorisation

$(x^2 + bx + c) = (x + p)(x + q)$, where $|pq| = |c|$, leading to $x = \ldots$

$(ax^2 + bx + c) = (mx + p)(nx + q)$, where $|pq| = |c|$ and $|mn| = |a|$, leading to $x = \ldots$

2. Formula

Attempt to use the correct formula (with values for *a*, *b* and *c*)

3. Completing the square

Solving $x^2 + bx + c = 0$: $\left(x \pm \frac{b}{2}\right)^2 \pm q \pm c = 0, \quad q \neq 0$, leading to $x = \ldots$

Method marks for differentiation and integration:

1. Differentiation

Power of at least one term decreased by 1. $(x^n \to x^{n-1})$

2. Integration

Power of at least one term increased by 1. $(x^n \to x^{n+1})$

Use of a formula

Where a method involves using a formula that has been learnt, the advice given in recent examiners' reports is that the formula should be quoted first.

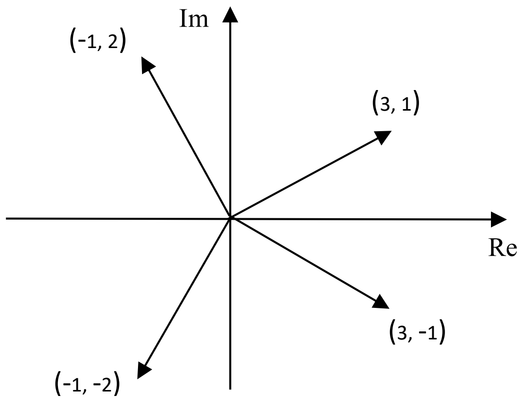
Normal marking procedure is as follows:

Method mark for quoting a correct formula and attempting to use it, even if there are small errors in the substitution of values.

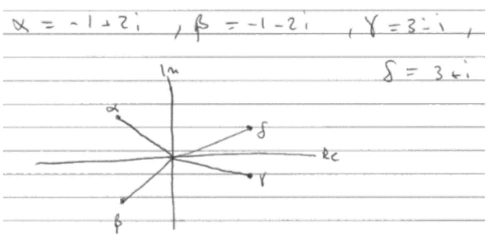
Where the formula is not quoted, the method mark can be gained by implication from correct working with values but may be lost if there is any mistake in the working.

Exact answers

Examiners' reports have emphasised that where, for example, an exact answer is asked for, or working with surds is clearly required, marks will normally be lost if the candidate resorts to using rounded decimals.

Question	Scheme	Marks	AOs
1(a)	$z=-1-2\text{i}$ or $z=3+\text{i}$	M1	1.2
	$z=-1-2\text{i}$ and $z=3+\text{i}$	A1	1.1b
		B1	1.1b
		B1	1.1b
		(4)	
(b) **Way 1**	$(z-(-1+2\text{i}))(z-(-1-2\text{i}))=\ldots$ or $(z-(3+\text{i}))(z-(3-\text{i}))=\ldots$ $\text{f}(z)=(z-(-1+2\text{i}))(z-(-1-2\text{i}))(z-(3+\text{i}))(z-(3-\text{i}))=\ldots$	M1	3.1a
	z^2+2z+5 or $z^2-6z+10$ e.g. $\text{f}(z)=(z^2+2z+5)(\ldots)$	A1	1.1b
	z^2+2z+5 and $z^2-6z+10$ $\text{f}(z)=(z^3+z^2(-1-\text{i})+z(-1+2\text{i})-15-5\text{i})(\ldots)$	A1	1.1b
	$\text{f}(z)=(z^2+2z+5)(z^2-6z+10)$ Expands the brackets to forms a quartic	M1	3.1a
	$\text{f}(z)=z^4-4z^3+3z^2-10z+50$ or States $a=-4, b=3, c=-10, d=50$	A1	1.1b
		(5)	

Question	Scheme	Marks	AOs
Way 2	sum roots $= \alpha + \beta + \gamma + \delta = (-1+2\text{i})+(-1-2\text{i})+(3+\text{i})+(3-\text{i}) = ...$ pair sum $= \alpha\beta + \alpha\gamma + \alpha\delta + \beta\gamma + \beta\delta + \gamma\delta$ $= (-1+2\text{i})(-1-2\text{i})+(-1+2\text{i})(3-\text{i})+(-1+2\text{i})(3+\text{i})+(-1-2\text{i})(3-\text{i})$ $+(-1-2\text{i})(3+\text{i})+(3+\text{i})(3-\text{i}) = ...$ triple sum $= \alpha\beta\gamma + \alpha\beta\delta + \beta\gamma\delta + \alpha\gamma\delta$ $= (-1+2\text{i})(-1-2\text{i})(3-\text{i})+(-1+2\text{i})(-1-2\text{i})(3+\text{i})+(-1+2\text{i})(3+\text{i})(3-\text{i})$ $+(-1-2\text{i})(3+\text{i})(3-\text{i}) = ...$ Product $= \alpha\beta\gamma\delta = (-1+2\text{i})(-1-2\text{i})(3-\text{i})(3+\text{i}) = ...$	M1	3.1a
	sum = 4, pair sum = 3, triple sum = 10 and product = 50	A1 A1	1.1b 1.1b
	$a = -(\text{their sum roots}) = -4$ $b = +(\text{their pair sum}) = 3$ $c = -(\text{triple sum}) = -10$ $d = +(\text{product}) = 50$	M1 A1	3.1a 1.1b
		(5)	
Way 3	$\text{f}(z) = (-1+2\text{i})^4 + a(-1+2\text{i})^3 + b(-1+2\text{i})^2 + c(-1+2\text{i}) + d = 0$ $\text{f}(z) = (3+\text{i})^4 + a(3+\text{i})^3 + b(3+\text{i})^2 + c(3+\text{i}) + d = 0$ Leading to $-7+11a-3b-c+d=0 \qquad 24-2a-4b+2c=0$ $28+18a+8b+3c+d=0 \qquad 96+26a+6b+c=0$	M1 A1 A1	3.1a 1.1b 1.1b
	Solves their simultaneous equation to find a value for one of the constants	M1	3.1a
	$a=-4, b=3, c=-10, d=50$	A1	1.1b
		(5)	
			(9 marks)

Notes
(a) M1: Identifies at least one correct complex conjugate as another root (can be seen/implied by Argand diagram) A1: Both complex conjugate roots identified correctly (can be seen/implied by Argand diagram) For the next two marks allow either a cross, dot or line drawn where the end point is labelled with the correct coordinate, corresponding complex number or clearly plotted with correct numbers labelled on the axis or indication of the correct coordinates by use of scale markers. Condone (3, i) etc. The axes do not need to be labelled with Re and Im. B1: One complex conjugate pair correctly plotted. B1: Both complex conjugate pair correctly plotted. The $3 \pm \mathrm{i}$ must be closer to the real axes than the $-1 \pm 2\mathrm{i}$ **If there is no indication of the coordinates, scale or complex numbers on the Argand diagram this is B0 B0.** **Do accept correct labelling e.g.**  (b) **<u>Way 1</u>** M1: Correct strategy for forming at least one of the quadratic factors. Follow through their roots. A1: At least one correct simplified quadratic factor. A1: Both simplified quadratic factors correct or a correct simplified cubic factor M1: A complete strategy to find values for a, b, c and d e.g. uses their quadratic factors or cubic and linear factor to form a quartic. A1: Correct quartic in terms of z or correct values for a, b, c and d stated. **<u>Way 2</u>** M1: Correct strategy for finding at least three of the sum roots, pair sum, triple sum and product. Follow through their roots. This can be implied by at least three correct values for the sum roots, pair sum, triple sum and product with no working shown. If the calculations are not shown for the sums and product and they have at least two incorrect values this is M0. A1: At least two correct values for the sum roots, pair sum, triple sum or product. A1: All correct values for the sum, pair sum, triple sum and product. M1: Must have real values of a, b , c and d and use $a = -$their sum roots, $b =$ their pair sum, $c = -$their triple sum and $d =$ their product. A1: Correct quartic in terms of z or correct values for a, b, c and d stated. **<u>Way 3</u>** M1: Substitutes two roots into $\mathrm{f}(z) = 0$ and equates coefficients to form 4 equations A1: At least two correct equations. A1: All four correct equations

M1: Solve their four equation (using calculator) to find at least one value. This will need checking if incorrect equations used. A1: Correct quartic in terms of z or correct values for a, b, c and d stated. **Note:** Correct answer only will score 5/5

Question	Scheme	Marks	AOs
2	$\dfrac{8x-12}{(2x^2+3)(x+1)}=\dfrac{Ax+B}{2x^2+3}+\dfrac{C}{x+1}$	M1	3.1a
	$8x-12=(Ax+B)(x+1)+C(2x^2+3)$ E.g. $x=-1\Rightarrow C=-4, x=0\Rightarrow B=0, x=1\Rightarrow A=8$ Or Compares coefficients and solves $(A+2C=0 \quad A+B=8 \quad B+3C=-12)$ $\Rightarrow A=..., B=..., C=...$	dM1	1.1b
	$A=8 \quad B=0 \quad C=-4$	A1	1.1b
	$\displaystyle\int\left(\frac{8x}{2x^2+3}-\frac{4}{x+1}\right)\mathrm{d}x=2\ln(2x^2+3)-4\ln(x+1)$	A1ft	1.1b
	$2\ln(2x^2+3)-4\ln(x+1)=\ln\left(\dfrac{(2x^2+3)^2}{(x+1)^4}\right)$ or $2\ln(2x^2+3)-4\ln(x+1)=2\ln\left(\dfrac{(2x^2+3)}{(x+1)^2}\right)$	M1	2.1
	$\lim\limits_{x\to\infty}\left\{\ln\dfrac{(2x^2+3)^2}{(x+1)^4}\right\}=\ln 4$ or $\lim\limits_{x\to\infty}\left\{2\ln\dfrac{(2x^2+3)}{(x+1)^2}\right\}=2\ln 2$	B1	2.2a
	$\Rightarrow \displaystyle\int_0^{\infty}\frac{8x-12}{(2x^2+3)(x+1)}\mathrm{d}x=\ln\frac{4}{9}$ cao	A1	1.1b
		(7)	
			(7 marks)

Notes

M1: Selects the correct form for partial fractions.
dM1: Full method for finding values for all three constants. Dependent on having the correct form for the partial fractions. Allow slips as long as the intention is clear.
A1: Correct constants or partial fractions.

A1ft: Integrates $\int \frac{px}{2x^2+3} - \frac{q}{x+1} \mathrm{d}x = \frac{p}{4}\ln\left(2x^2+3\right) - q\ln\left(x+1\right)$ and no extra terms M1: Combines two algebraic log terms correctly. B1: Correct upper limit for $x \to \infty$ by recognising the dominant terms. (Simply replacing x with ∞ scores B0). This can be implied. A1: Deduces the correct value for the improper integral in the correct form, cao A0 for $2\ln\frac{2}{3}$ Correct answer with no working seen is no marks. **Note:** Incorrect partial fraction form, $\frac{A}{2x^2+3} + \frac{B}{x+1}$ or $\frac{Ax}{2x^2+3} + \frac{B}{x+1}$ the maximum it can score is M0M0A0A0M1B1A0

Question	Scheme	Marks	AOs
3(a)(i)	$2(0.4+a)=1.2$ or $0.4+a=0.6$ or $0.4+a\cos 0=0.6$ $\Rightarrow a = ...$	M1	3.4
	$a = 0.2$ * cso	A1*	1.1b
		(2)	
(b)	Area of rectangle is $1.2\times 0.6(=0.72)$	B1	1.1b
	Area enclosed by curve = $\frac{1}{2}\int (0.4+0.2\cos 2\theta)^2 (\mathrm{d}\theta)$	M1	3.1a
	$(0.4+0.2\cos 2\theta)^2 = 0.16+0.16\cos 2\theta + 0.04\cos^2 2\theta$ $= 0.16+0.16\cos 2\theta + 0.04\left(\frac{\cos 4\theta+1}{2}\right)$	M1	2.1
	$\frac{1}{2}\int (0.4+0.2\cos 2\theta)^2 \mathrm{d}\theta = \frac{1}{2}\left[0.18\theta + 0.08\sin 2\theta + 0.005\sin 4\theta (+c)\right]$ $= 0.09\theta + 0.04\sin 2\theta + 0.0025\sin 4\theta (+c)$ o.e.	A1ft	1.1b
	Area enclosed by curve = $\left[0.09\theta + 0.04\sin 2\theta + 0.0025\sin 4\theta\right]_0^{2\pi}$ or Area enclosed by curve = $2\left[0.09\theta + 0.04\sin 2\theta + 0.0025\sin 4\theta\right]_0^{\pi}$ or Area enclosed by curve = $4\left[0.09\theta + 0.04\sin 2\theta + 0.0025\sin 4\theta\right]_0^{\pi/2}$	dM1	3.1a
	$= \frac{9}{50}\pi$ or $0.18\pi (= 0.5654...)$	A1	1.1b

	Area of wood $=1.2\times0.6-0.18\pi$	M1	1.1b
	$=$ awrt 0.155 (m^2)	A1	1.1b
		(8)	
(10 marks)			

Notes

(a)
M1: Interprets the information from the model and realises that the maximum value of r gives half the length of the table top (or equivalent) and solves to find a value for a. Use $\theta=0$ and $r=0.6$ or $\theta=\pi$ and $r=-0.6$ to find a value for a.
Using $\theta=2\pi$ is M0
A1*: Correct value for a.

<u>Alternative</u>
M1: Uses $a = 0.2$ and $\theta=0$ to find a value for r
A1: Finds $r = 0.6$ and concludes that $a = 0.2$

(b)
B1: 1.2×0.6 or 0.72
M1: A correct strategy identified for finding an area enclosed by the polar curve using a correct formula with r substituted. Attempt at area $=\frac{1}{2}\int (0.4+0.2\cos 2\theta)^2\,\mathrm{d}\theta=...$

Look for $=\lambda\times\frac{1}{2}\int (0.4+0.2\cos 2\theta)^2\,\mathrm{d}\theta=...$

If the $\frac{1}{2}$ is not explicitly seen then look at the limits and it must be either

$$=\int_0^{\pi}(0.4+0.2\cos 2\theta)^2\,\mathrm{d}\theta=... \text{ or } =2\int_0^{\frac{\pi}{2}}(0.4+0.2\cos 2\theta)^2\,\mathrm{d}\theta=...$$

Condone missing $\mathrm{d}\theta$

M1: Squares to achieve three terms and uses $\cos^2 2\theta=\frac{\pm1\pm\cos 4\theta}{2}$ to obtain an expression in an integrable form.
A1ft: Correct follow through integration as long as the previous two method marks have been awarded.
dM1: Dependent of first method mark. Finds the required area enclosed by the curve using the correct limits.
There are only three cases either $\frac{1}{2}\int_0^{2\pi}(0.4+0.2\cos 2\theta)^2\,\mathrm{d}\theta$ or $\int_0^{\pi}(0.4+0.2\cos 2\theta)^2\,\mathrm{d}\theta$ or

$2\int_0^{\frac{\pi}{2}}(0.4+0.2\cos 2\theta)^2\,\mathrm{d}\theta$

The use of the limit 0 can be implied if it gives 0 but the use of 0 must been seen or implied if it does not result in 0 (just writing 0 is insufficient)

A1: Correct area of the glass following fully correct working. **Do not award for the correct answer following incorrect working.**
M1: Subtracts their area of the glass from their area of the rectangle, as long as it does not give a negative area
A1: awrt 0.155 or awrt $0.155\,\text{m}^2$ (If the units are stated they must be correct)

Note: Using a calculator to find the area scores a maximum of B1M0M0A0M0A0M1A1

Question	Scheme	Marks	AOs
4	$\frac{1}{(r+1)(r+2)(r+3)} \equiv \frac{A}{r+1}+\frac{B}{r+2}+\frac{C}{r+3} \Rightarrow A=..., B=..., C=...$ $\left(\text{NB } A=\frac{1}{2} \quad B=-1 \quad C=\frac{1}{2}\right)$	M1	3.1a
	$r=0$ $\quad \frac{1}{2}\left[\frac{1}{1}-\frac{2}{2}+\frac{1}{3}\right]$ or $\frac{1}{2\ 1}-\frac{1}{2}+\frac{1}{2\ 3}$ or $\frac{1}{2}-\frac{1}{2}+\frac{1}{6}$ $r=1$ $\quad \frac{1}{2}\left[\frac{1}{2}-\frac{2}{3}+\frac{1}{4}\right]$ or $\frac{1}{2\ 2}-\frac{1}{3}+\frac{1}{2\ 4}$ or $\frac{1}{4}-\frac{1}{3}+\frac{1}{8}$ $r=n-1$ $\quad \frac{1}{2}\left[\frac{1}{n}-\frac{2}{n+1}+\frac{1}{n+2}\right]$ or $\frac{1}{2\ n}-\frac{1}{n+1}+\frac{1}{2\ n+2}$ or $\frac{1}{2n}-\frac{1}{n+1}+\frac{1}{2n+4}$ $r=n$ $\quad \frac{1}{2}\left[\frac{1}{n+1}-\frac{2}{n+2}+\frac{1}{n+3}\right]$ or $\frac{1}{2\ n+1}-\frac{1}{n+2}+\frac{1}{2\ n+3}$ or $\frac{1}{2n+2}-\frac{1}{n+2}+\frac{1}{2n+6}$	M1	2.1
	$\frac{1}{2}-\frac{1}{2}+\frac{1}{4}+\frac{1}{2(n+2)}-\frac{1}{n+2}+\frac{1}{2(n+3)}$ or $\frac{1}{4}-\frac{1}{2(n+2)}+\frac{1}{2(n+3)}$	A1	1.1b
	$=\frac{n^2+5n+6+2n+6-4n-12+2n+4}{4(n+2)(n+3)}$	M1	1.1b
	$=\frac{(n+1)(n+4)}{4(n+2)(n+3)}$	A1	2.2a
		(5)	
		(5 marks)	

Notes
M1: A complete strategy to find A, B and C e.g. partial fractions. Allow slip when finding the constant but must be the correct form of partial fractions and correct identity. M1: Starts the process of differences to identify the relevant fractions at the start and end. Must have attempted a minimum of $r=0,\quad r=1,\quad \ldots\quad r=n-1$ and $r=n$ Follow through on their values of A, B and C. Look for $r=0\to \frac{A}{1}-\frac{B}{2}+\frac{C}{3}$ $\qquad r=1\to \frac{A}{2}-\frac{B}{3}+\frac{C}{4}$ $r=n-1\to \frac{A}{n}-\frac{B}{n+1}+\frac{C}{n+2}$ $\qquad r=n\to \frac{A}{n+1}-\frac{B}{n+2}+\frac{C}{n+3}$ A1: Correct fractions from the beginning and end that do not cancel stated. M1 Combines all 'their' fractions (at least two algebraic fractions) over their correct common denominator, does not need to be the lowest common denominator (allow a slip in the numerator). A1: Correct answer. **Note**: if they start with $r=1$ the maximum they can score is M1M0A0M1A0 **Note:** Proof by induction gains no marks

Question	Scheme	Marks	AOs
5(a)	The tank initially contains 100L. 3 L are entering every minute and 2 L are leaving every minute so overall 1 L increase in volume each minute so the tank contains 100 + t litres after t minutes	M1	3.3
	2 L leave the tank each minute and if there are Sg of salt in the tank, the concentration will be $\frac{S}{100+t}\,g/L$ so salt leaves the tank at a rate of $2\times\frac{S}{100+t}\,g$ per minute	M1	3.3
	Salt enters the tank at a rate of $3\times 1g$ per minute	B1	2.2a
	$\therefore \frac{dS}{dt}=3-\frac{2S}{100+t}$ * cso	A1*	1.1b
		(4)	
(b)	$\frac{dS}{dt}+\frac{2S}{100+t}=3$		
	$I=e^{\int\frac{2}{100+t}dt}=(100+t)^2\Rightarrow S(100+t)^2=\int 3(100+t)^2 dt$	M1	3.1b
	$S(100+t)^2=(100+t)^3\,(+c)$ OR $S(100+t)^2=30\,000t+300t^2+t^3\,(+c)$	A1	1.1b
	$t=0,\ S=0\Rightarrow c=-10^6$	M1	3.4
	$t=10\Rightarrow S=100+10-\frac{10^6}{(100+10)^2}$	dM1	1.1b

	OR $S(100+10)^2=(100+10)^3(+c) \Rightarrow S=...$		
	$=\text{awrt } 27 \text{ (g) or } \frac{3310}{121} \text{ (g)}$	A1	2.2b
		(5)	
(c)	Concentration is $\left(100+t-\frac{10^6}{(100+t)^2}\right)\div(100+t)=0.9$ OR $S=0.9\ 100+t \Rightarrow 0.9\ 100+t\ =\ 100+t\ -\frac{10^6}{100+10\ ^2}$ OR $S=0.9\ 100+t \Rightarrow 0.9\ 100+t\ ^3=\ 100+t\ ^3-10^6$	M1	3.4
	$(100+t)^3=10^7 \Rightarrow t=...$ OR $t^3+300t^2+30\,000t-9\,000\,000=0 \Rightarrow t=...$	dM1	1.1b
	$t=\text{awrt } 115 \text{ (minutes)}$	A1	2.2b
		(3)	
(d)	E.g. • It is unlikely that mixing is instantaneous • The model will only be valid when the tank is not full • When the valve is closed, the model is not valid • It is unlikely that the concentration of salt water entering the tank remains exactly the same	B1	3.5a
		(1)	
			(13 marks)

Notes

(a)

M1: A suitable explanation for the "100 + t" e.g. as a minimum (v) = 100 + 3t – 2t = 100 + t

M1: A suitable explanation for the $\frac{2S}{100+t}$

There need to be some explanation (words) for this part of the formula.

e.g. the concentration of (salt) $=\frac{S}{100+t}$ therefore (salt) out $=2\times\frac{S}{100+t}=\frac{2S}{100+t}$

e.g. salt out = $\frac{2S}{\text{volume of water}}=\frac{2S}{100+t}$

Note: M0 for $2\times\frac{S}{100+t}=\frac{2S}{100+t}$ only with no explanation

B1: Correct interpretation for the "3" e.g. salt in = 3 or $\frac{dS}{dt}$ in = 3

<u>**Note:**</u> Salt **water** in = 3 is **B0**

A1*: Puts all the components together to form the given differential equation cso (b) M1: Uses the model to find the integrating factor and attempts the solution of the differential equation. Look for $I.F. = e^{\int \frac{2}{100+t} dt} \Rightarrow S \times \text{'their } I.F.\text{'} = \int 3 \times \text{'their } I.F.\text{' d}t$ A1: Correct solution condone missing + c **For the next three mark there must be a constant of integration** M1: Interprets the initial conditions, $t = 0 \;\; S = 0,$ and uses in their equation to find the constant of integration. dM1: Dependent on having a constant of integration. Uses their solution to the problem to find the amount of salt after 10 minutes. A1: Awrt 27 or $\frac{3310}{121}$. (If the units are stated they must be correct) **Note:** If achieves $S(100+t)^2 = 30\,000t + 300t^2 + t^3 + c$ the constant of integration $c = 0$ and the correct amount of salt can be achieved. If there is no + c the maximum they can score is M1A1M0M0A0
Notes continued
(c) **Note:** Look out for setting $S = 0.9$ in this part, which scores no marks. M1: Uses their solution to the model and divides by 100 + t as an interpretation of the concentration and sets = 0.9. Alternatively recognises that the amount of salt = 0.9(100 + t) and substitutes for S in their solution to the model. dM1: Dependent on previous method mark. Solves their equation to obtain a value for t. May use a calculator. A1: Awrt 115 (If the units are stated they must be correct) or 1hr 45 mins with units (d) B1: Evaluates the model by making a suitable comment – see scheme for examples.

Question	Scheme	Marks	AOs
6	**Way 1** f(k +1) – f(k)		
	When $n = 1$, $3^{2n+4} - 2^{2n} = 729 - 4 = 725$ ($725 = 145 \times 5$) so the statement is true for $n = 1$	B1	2.2a
	Assume true for $n = k$ so $3^{2k+4} - 2^{2k}$ is divisible by 5	M1	2.4
	$\text{f}(k+1) - \text{f}(k) = 3^{2k+6} - 2^{2k+2} - 3^{2k+4} + 2^{2k}$	M1	2.1
	either $8\text{f}(k) + 5 \times 2^{2k}$ or $3\text{f}(k) + 5 \times 3^{2k+4}$	A1	1.1b
	$\text{f}(k+1) = 9\text{f}(k) + 5 \times 2^{2k}$ or $\text{f}(k+1) = 4\text{f}(k) + 5 \times 3^{2k+4}$ o.e.	A1	1.1b
	If true for $n = k$ then it is true for	A1	2.4

	<u>$n = k + 1$</u> and as it is <u>true for $n = 1$</u>, the statement is <u>true for all (positive integers) n</u>. (Allow 'for all values')		
		(6)	
	<u>Way 2</u> f$(k+1)$		
	When $n = 1$, $3^{2n+4} - 2^{2n} = 729 - 4 = 725$ ($725 = 145\times 5$)so the statement is true for $n = 1$	B1	2.2a
	Assume true for $n = k$ so $3^{2k+4} - 2^{2k}$ is divisible by 5	M1	2.4
	$\mathrm{f}\left(k+1\right) = 3^{2(k+1)+4} - 2^{2(k+1)} \left(= 3^{2k+6} - 2^{2k+2}\right)$	M1	2.1
	$\mathrm{f}\ \ k+1\ \ = 9\mathrm{f}\ \ k\ \ + 5\times 2^{2k}$ or $\mathrm{f}\ \ k+1\ \ = 4\mathrm{f}\ \ k\ \ + 5\times 3^{2k+4}$ o.e.	A1 A1	1.1b 1.1b
	<u>If true for $n = k$ then it is true for</u> <u>$n = k + 1$</u> and as it is <u>true for $n = 1$</u>, the statement is <u>true for all (positive integers) n</u>. (Allow 'for all values')	A1	2.4
		(6)	
	<u>Way 3</u> f$(k) = 5M$		
	When $n = 1$, $3^{2n+4} - 2^{2n} = 729 - 4 = 725$ ($725 = 145\times 5$) so the statement is true for $n = 1$	B1	2.2a
	Assume true for $n = k$ so $3^{2k+4} - 2^{2k} = 5M$	M1	2.4
	$\mathrm{f}\left(k+1\right) = 3^{2(k+1)+4} - 2^{2(k+1)} \left(= 3^{2k+6} - 2^{2k+2}\right)$	M1	2.1
	$\left(\mathrm{f}\left(k+1\right) = 3^2\times 3^{2k+4} - 2^2\times 2^{2k} = 3^2\times\left(5M + 2^{2k+2}\right) - 2^2\times 2^{2k}\right)$ $\mathrm{f}\ \ k+1\ \ = 45M + 5\times 2^{2k}$ o.e. OR $\left(\mathrm{f}\left(k+1\right) = 3^2\times 3^{2k+4} - 2^2\times 2^{2k} = 3^2\times 3^{2k+4} - 2^2\times\left(3^{2k+4} - 5M\right)\right)$ $\mathrm{f}\ \ k+1\ \ = 5\times 3^{2k+4} + 20M$ o.e.	A1 A1	1.1b 1.1b
	<u>If true for $n = k$ then it is true for</u> <u>$n = k + 1$</u> and as it is <u>true for $n = 1$</u>, the statement is <u>true for all (positive integers) n</u>. (Allow 'for all values')	A1	2.4
		(6)	
	<u>Way 4</u> f$(k+1)$ + f(k)		
	When $n = 1$, $3^{2n+4} - 2^{2n} = 729 - 4 = 725$ ($725 = 145\times 5$)so the statement is true for $n = 1$	B1	2.2a
	Assume true for $n = k$ so $3^{2k+4} - 2^{2k}$ is divisible by 5	M1	2.4
	$\mathrm{f}\left(k+1\right) + \mathrm{f}\left(k\right) = 3^{2k+6} - 2^{2k+2} + 3^{2k+4} - 2^{2k}$	M1	2.1
	$\mathrm{f}\left(k+1\right) + \mathrm{f}\left(k\right) = 3^2\times 3^{2k+4} - 2^2\times 2^{2k} + 3^{2k+4} - 2^{2k}$	A1	1.1b

	leading to $10\times3^{2k+4}-5\times2^{2k}$		
	$f(k+1)=5\left[2\times3^{2k+4}-2^{2k}\right]-f(k)$ o.e.	A1	1.1b
	If true for $n = k$ then it is true for $n = k + 1$ and as it is true for $n = 1$, the statement is true for all (positive integers) n. (Allow 'for all values')	A1	2.4
		(6)	
	Way 5 f(k +1) – 'M'f(k) (Selecting a value of M that will lead to multiples of 5)		
	When n = 1, $3^{2n+4}-2^{2n}=729-4=725$ ($725=145\times5$)so the statement is true for n = 1	B1	2.2a
	Assume true for $n = k$ so $3^{2k+4}-2^{2k}$ is divisible by 5	M1	2.4
	$f(k+1)-'M'f(k)=3^{2k+6}-2^{2k+2}-'M'\times3^{2k+4}+'M'\times2^{2k}$	M1	2.1
	$f(k+1)-'M'f(k)=(9-'M')\times3^{2k+4}-(4-'M')\times2^{2k}$	A1	1.1b
	$f(k+1)=(9-'M')\times3^{2k+4}-(4-'M')\times2^{2k}+'M'f(k)$ o.e.	A1	1.1b
	If true for $n = k$ then it is true for $n = k + 1$ and as it is true for $n = 1$, the statement is true for all (positive integers) n. (Allow 'for all values')	A1	2.4
		(6)	

(6 marks)

Notes

Way 1 f(k +1) – f(k)
B1: Shows the statement is true for n = 1. Needs to show $f(1)=725$ and conclusion true for n = 1, this statement can be recovered in their conclusion if says e.g. true for n = 1
M1: Makes an assumption statement that assumes the result is true for $n = k$. Assume (true for) $n=k$ is sufficient. This mark may be recovered in their conclusion if they say e.g. if true for $n=k$ then …etc
M1: Attempts f(k +1) – f(k) or equivalent work
A1: Achieves a correct simplified expression for f(k +1) – f(k)
A1: Achieves a correct expression for f(k +1) in terms of f(k)
A1: Correct complete conclusion. This mark is dependent on **all** previous marks. It is gained by conveying the ideas of **all** underlined points either at the end of their solution **or** as a narrative in their solution.

Way 2 f(k +1)
B1: Shows the statement is true for n = 1. Needs to show $f(1)=725$ and conclusion true for n = 1, this statement can be recovered in their conclusion if says e.g. true for n = 1.
M1: Makes an assumption statement that assumes the result is true for $n = k$. Assume (true for) $n=k$ is sufficient. This mark may be recovered in their conclusion if they say e.g. if true for $n=k$ then …etc
M1: Attempts f(k +1)
A1: Correctly achieves either $9f(k)$ **or** 5×2^{2k} or either $4f(k)$ **or** $5\times3^{2k+4}$

A1: Achieves a correct expression for f(k +1) in terms of f(k)
A1: Correct complete conclusion. This mark is dependent on **all** previous marks. It is gained by conveying the ideas of **all** underlined points either at the end of their solution **or** as a narrative in their solution.

Way 3 f(k) = 5M
B1: Shows the statement is true for n = 1. Needs to show $f(1) = 725$ and conclusion true for n = 1, this statement can be recovered in their conclusion if says e.g. true for n = 1.
M1: Makes an assumption statement that assumes the result is true for n = k. Assume (true for) $n = k$ is sufficient. This mark may be recovered in their conclusion if they say e.g. if true for $n = k$ then …etc
M1: Attempts f(k +1)
A1: Correctly achieves either $45M$ **or** 5×2^{2k} or either $20M$ **or** $5 \times 3^{2k+4}$
A1: Achieves a correct expression for f(k +1) in terms of M and 2^{2k} or M and 3^{2k+4}
A1: Correct complete conclusion. This mark is dependent on **all** previous marks. It is gained by conveying the ideas of **all** underlined points either at the end of their solution **or** as a narrative in their solution.

Way 4 f(k +1) + f(k)
B1: Shows the statement is true for n = 1. Needs to show $f(1) = 725$ and conclusion true for n = 1, this statement can be recovered in their conclusion if says e.g. true for n = 1
M1: Makes an assumption statement that assumes the result is true for n = k. Assume (true for) $n = k$ is sufficient. This mark may be recovered in their conclusion if they say e.g. if true for $n = k$ then …etc
M1: Attempts f(k +1) + f(k) or equivalent work
A1: Achieves a correct simplified expression for f(k +1) + f(k)
A1: Achieves a correct expression for $f(k+1) = 5\left[2 \times 3^{2k+4} - 2^{2k}\right] - f(k)$
A1: Correct complete conclusion. This mark is dependent on **all** previous marks. It is gained by conveying the ideas of **all** underlined points either at the end of their solution **or** as a narrative in their solution.

Notes continued

Way 5 f(k +1) – Mf(k) (Selects a suitable value for M which leads to divisibility of 5)
B1: Shows the statement is true for n = 1. Needs to show $f(1) = 725$ and conclusion true for n = 1, this statement can be recovered in their conclusion if says e.g. true for n = 1
M1: Makes an assumption statement that assumes the result is true for n = k. Assume (true for) $n = k$ is sufficient. This mark may be recovered in their conclusion if they say e.g. if true for $n = k$ then …etc
M1: Attempts f(k +1) – Mf(k) or equivalent work
A1: Achieves a correct simplified expression, $f(k+1) - \text{'M'}f(k)$ which is divisible by 5

$f(k+1) - \text{'M'}f(k) = (9 - \text{'M'}) \times 3^{2k+4} - (4 - \text{'M'}) \times 2^{2k}$

A1: Achieves a correct expression for $f(k+1) = (9 - \text{'M'}) \times 3^{2k+4} - (4 - \text{'M'}) \times 2^{2k} + \text{'M'}f(k)$ which is divisible by 5
A1: Correct complete conclusion. This mark is dependent on **all** previous marks. It is gained by conveying the ideas of **all** underlined points either at the end of their solution **or** as a narrative in their solution.

Question	Scheme	Marks	AOs
7(a) **Way 1**	$1+2\lambda=1+t$ $-1-\lambda=-t$ $4+3\lambda=3+2t$ $\Rightarrow t=\ldots \text{or } \lambda=\ldots$	M1	3.1a
	Checks the third equation with $t=2$ and $\lambda=1$ Or shows that the coordinate (3, -2, 7) lies on both lines	A1	1.1b
	As the lines intersect at a point the lines lie in the same plane.	A1	2.4
		(3)	
(a) **Way 2**	$1=1+2\lambda+t$ $-1=\quad -\lambda-t$ $4=3+3\lambda+2t$ $\Rightarrow t=\ldots \text{or } \lambda=\ldots$ $1=1+2\lambda+t$ $0=-1-\lambda-t$ $3=4+3\lambda+2t$ $\Rightarrow t=\ldots \text{or } \lambda=\ldots$	M1	3.1a
	Checks the third equation with $t=2$ and $\lambda=-1$ Checks the third equation with $t=-2$ and $\lambda=1$	A1	1.1b
	Second coordinates lie on the plane; therefore, the lines lie on the same plane	A1	2.4
		(3)	
(a) **Way 3**	$x=1+t, \quad y=-t, \quad z=3+2t$ $\dfrac{1+t-1}{2}=\dfrac{-t+1}{-1}=\dfrac{3+2t-4}{3}$ Solves a pair of equations $t=\ldots$	M1	3.1a
	Solve two pairs of equations to find $t=2$	A1	1.1b
	As the lines intersect at a point the lines lie in the same plane.	A1	2.4
		(3)	
(a) **Way 4 (Using Further Pure 2 knowledge)**	$\begin{pmatrix}2\\-1\\3\end{pmatrix}.\begin{pmatrix}x\\y\\z\end{pmatrix}\Rightarrow 2x-y+3z=0$ and $\begin{pmatrix}1\\-1\\2\end{pmatrix}.\begin{pmatrix}x\\y\\z\end{pmatrix}\Rightarrow x-y+2z=0$ attempts to solve the equations to find a normal vector OR attempts the cross product $\begin{pmatrix}2\\-1\\3\end{pmatrix}\times\begin{pmatrix}1\\-1\\2\end{pmatrix}=\ldots$ **AND** either finds the equation of one plane **OR** finds dot product between the normal and one coordinate	M1	3.1a

	$\mathbf{r}.\begin{pmatrix}1\\-1\\-1\end{pmatrix}=\begin{pmatrix}1\\-1\\4\end{pmatrix}.\begin{pmatrix}1\\-1\\-1\end{pmatrix}=...$ or $\mathbf{r}.\begin{pmatrix}1\\-1\\-1\end{pmatrix}=\begin{pmatrix}1\\0\\3\end{pmatrix}.\begin{pmatrix}1\\-1\\-1\end{pmatrix}=...$ **OR** $\begin{pmatrix}1\\-1\\4\end{pmatrix}.\begin{pmatrix}1\\-1\\-1\end{pmatrix}=...$ or $\begin{pmatrix}1\\0\\3\end{pmatrix}.\begin{pmatrix}1\\-1\\-1\end{pmatrix}=...$		
	Achieves the correct planes containing **each** line $\mathbf{r}.\begin{pmatrix}1\\-1\\-1\end{pmatrix}=-2$ or $x-y-z=-2$ o.e. **OR** Shows that $\begin{pmatrix}1\\-1\\4\end{pmatrix}.\begin{pmatrix}1\\-1\\-1\end{pmatrix}=-2$ **and** $\begin{pmatrix}1\\0\\3\end{pmatrix}.\begin{pmatrix}1\\-1\\-1\end{pmatrix}=-2$ o.e.	A1	1.1b
	Both planes are the same, therefore the lines lie in the same plane.	A1	2.4
		(3)	
(b)	e.g. $\mathbf{r}=\begin{pmatrix}1\\0\\3\end{pmatrix}+p\begin{pmatrix}2\\-1\\3\end{pmatrix}+q\begin{pmatrix}1\\-1\\2\end{pmatrix}$ or $\mathbf{r}=\begin{pmatrix}1\\-1\\4\end{pmatrix}+p\begin{pmatrix}2\\-1\\3\end{pmatrix}+q\begin{pmatrix}1\\-1\\2\end{pmatrix}$ or $\mathbf{r}=\begin{pmatrix}3\\-2\\7\end{pmatrix}+p\begin{pmatrix}2\\-1\\3\end{pmatrix}+q\begin{pmatrix}1\\-1\\2\end{pmatrix}$ or $\mathbf{r}=\begin{pmatrix}3\\-2\\7\end{pmatrix}+p\begin{pmatrix}0\\-1\\1\end{pmatrix}+q\begin{pmatrix}1\\-1\\2\end{pmatrix}$ or $\mathbf{r}.k\begin{pmatrix}1\\-1\\-1\end{pmatrix}=-2k$	B1	2.5
		(1)	
(c) **Way 1**	$\begin{pmatrix}2\\-1\\3\end{pmatrix}\bullet\begin{pmatrix}1\\-1\\2\end{pmatrix}=2+1+6$	M1	1.1b
	$\sqrt{2^2+(-1)^2+3^2}\sqrt{1^2+(-1)^2+2^2}\cos\theta=9$ $\Rightarrow\cos\theta=\dfrac{9}{\sqrt{2^2+(-1)^2+3^2}\sqrt{1^2+(-1)^2+2^2}}$	dM1	2.1
	$\theta=11$ cao	A1	1.1b
		(3)	

Way 2 (Using Further Pure 2 knowledge)	$\begin{pmatrix}2\\-1\\3\end{pmatrix}\times\begin{pmatrix}1\\-1\\2\end{pmatrix}=\begin{pmatrix}1\\-1\\-1\end{pmatrix}$	M1	1.1b
	$\sqrt{2^2+(-1)^2+3^2}\sqrt{1^2+(-1)^2+2^2}\sin\theta=\sqrt{1^2+(-1)^2+(-1)^2}$ $\Rightarrow \sin\theta=\dfrac{\sqrt{1^2+(-1)^2+(-1)^2}}{\sqrt{2^2+(-1)^2+3^2}\sqrt{1^2+(-1)^2+2^2}}$	dM1	2.1
	$\theta=11$ cao	A1	1.1b
		(3)	
			(7 marks)

Notes

(a)

Allow using $\begin{pmatrix}1\\3\\0\end{pmatrix}$ instead of $\begin{pmatrix}1\\0\\3\end{pmatrix}$ for the method mark.

Way 1

M1: Starts by attempting to find where the two lines intersect. They must set up a parametric equation for line 1 (allow sign slips and as long as the intention is clear), forms simultaneous equations by equating coordinates and attempts to solve to find a value for $t=\ldots$ or $\lambda=\ldots$.

A1: Shows that there is a unique solution by checking the third equation or shows that the coordinate (3, -2, 7) lies on both lines.

A1: Achieves the correct values $t=2$ and $\lambda=1$, checks the third equation and concludes that either

- a common point,
- the lines intersect
- the equations are consistent

therefore, the lines lie in the same plane

Way 2

M1: Finds the vector equation of the plane with the both direction vectors and one coordinate (allow a sign slip), sets equal to the other coordinate, forms simultaneous equations and attempts to solve to find a value for $t=\ldots$ or $\lambda=\ldots$.

A1: Shows that the other coordinate lies on the plane by checking the third equation.

A1: Achieves the correct values $t=-2$ and $\lambda=1$ or $t=2$ and $\lambda=-1$ and concludes that the second coordinate lie on the plane; therefore, the lines lie on the same plane

Way 3

M1: Substitutes line 2 into line 1 and solves a pair of equations to find a value for *t*. Allow slip with the position of 0 and sign slips as long as the intention is clear.

A1: Solve two pairs of equations to achieve $t = 2$ for each.
A1: Achieves the correct value $t = 2$ and concludes that either

- a common point,
- the lines intersect
- the equations are consistent

therefore, the lines lie in the same plane

Way 4 (Using Further Pure 2 knowledge)
M1: A complete method to finds a vector which is normal to both lines and attempts to finds the equation of the plane containing one line.
A1: Achieves the correct equation for the plane containing each line.
A1: Conclusion, planes are the same, therefore the lines lie in the same plane.

(b) **This may be seen in part (a)**
B1: Correct **vector** equation allow any letter for the scalers.

Must start with **r** = … and uses two out of the following direction vectors $\pm\begin{pmatrix}1\\-1\\2\end{pmatrix}$, $\pm\begin{pmatrix}2\\-1\\3\end{pmatrix}$ or $\pm\begin{pmatrix}0\\-1\\1\end{pmatrix}$ and one of the following position vectors $\begin{pmatrix}1\\0\\3\end{pmatrix}$, $\begin{pmatrix}1\\-1\\4\end{pmatrix}$ or $\begin{pmatrix}3\\-2\\7\end{pmatrix}$

(c)
Way 1
M1: Calculates the scalar product between the direction vectors, allow one slip, if the intention is clear
dM1: Dependent on the previous method mark. Applies the scalar product formula with their scalar product to find a value for $\cos\theta$
A1: Correct answer only
Way 2 (Using Further Pure 2 knowledge)
M1: Calculates the vector product between the direction vectors, allow one slip, if the intention is clear
dM1: Dependent on the previous method mark. Applies the vector product formula with their vector product to find a value for $\sin\theta$
A1: Correct answer only

Question	Scheme	Marks	AOs
8(a)	$\frac{d^2w}{dt^2}=\frac{5}{2}\left(\frac{dw}{dt}-\frac{ds}{dt}\right)$ or $\frac{ds}{dt}=\frac{dw}{dt}-\frac{2}{5}\frac{d^2w}{dt^2}$ o.e.	B1	1.1b
	$\frac{ds}{dt}=\frac{dw}{dt}-\frac{2}{5}\frac{d^2w}{dt^2}\Rightarrow\frac{dw}{dt}-\frac{2}{5}\frac{d^2w}{dt^2}=\frac{2}{5}w-90e^{-t}$	M1	2.1
	$2\frac{d^2w}{dt^2}-5\frac{dw}{dt}+2w=450e^{-t}$ *	A1*	1.1b
		(3)	
(b)	$2m^2-5m+2=0\Rightarrow m=...$	M1	3.4
	$m=2,\frac{1}{2}$	A1	1.1b
	$(w)=Ae^{\alpha t}+Be^{\beta t}$	M1	3.4
	$(w)=Ae^{0.5t}+Be^{2t}$	A1	1.1b
	PI: Try $w=ke^{-t}\Rightarrow\frac{dw}{dt}=-ke^{-t}\Rightarrow\frac{d^2w}{dt^2}=ke^{-t}$ $2ke^{-t}+5ke^{-t}+2ke^{-t}=450e^{-t}\Rightarrow k=...$	M1	3.4
	$w=$ 'their C.F.' $+\ 50e^{-t}$ $\left(w=Ae^{0.5t}+Be^{2t}+50e^{-t}\right)$	A1ft	1.1b
		(6)	
(c)	$s=w-\frac{2}{5}\frac{dw}{dt}=Ae^{0.5t}+Be^{2t}+50e^{-t}-\frac{2}{5}\left(\frac{A}{2}e^{0.5t}+2Be^{2t}-50e^{-t}\right)$	M1	3.4
	$s=\frac{4A}{5}e^{0.5t}+\frac{B}{5}e^{2t}+70e^{-t}$	A1	1.1b
		(2)	
(d)	$65=A+B+50,\ 85=\frac{4A}{5}+\frac{B}{5}+70\Rightarrow A=...,B=...$ $(\text{NB } A=20\ \ B=-5)$	M1	3.3
	$w=0\Rightarrow 20e^{0.5t}-5e^{2t}+50e^{-t}=0$	dM1	1.1b
	$e^{3t}-4e^{1.5t}-10(=0)$ or a multiple	A1	3.1a
	$e^{1.5t}=\frac{4\pm\sqrt{4^2-4\times(1)(-10)}}{2}$	M1	1.1b
	$1.5t=\ln\left(\frac{4+\sqrt{56}}{2}\right)$	M1	2.3
	$T=\frac{2}{3}\ln\left(\frac{4+\sqrt{56}}{2}\right)=$ awrt 1.165	A1	3.2a
		(6)	

(e)	E.g. • Either population becomes negative which is not possible • When the white-clawed crayfish have died out, the model will not be valid	B1	3.5b
		(1)	
			(18 marks)

Notes

(a)
B1: Differentiates the first equation with respect to t correctly.
M1: Substitutes $\frac{\mathrm{d}s}{\mathrm{d}t}$ into their derivative.
A1*: Achieves the printed answer with no errors.

(b) **<u>Note:</u> All the mark except the final A1 are available if they use other variables.**
M1: Uses the model to form and solve the Auxiliary Equation.
A1: Correct roots of the AE.
M1: Uses the model to form the Complementary Function for their roots (they may be complex roots)
A1: Correct CF
M1: Chooses the correct form of the PI according to the model and uses a complete method to find the PI. Uses $w = k\mathrm{e}^{-t}$ finds both $\frac{\mathrm{d}w}{\mathrm{d}t}$ and $\frac{\mathrm{d}^2 w}{\mathrm{d}t^2}$ substitutes into the differential equation and find the value of k.
A1ft: Dependent on all three of the previous method marks. Following through on their CF only to give $w = \text{'their CF'} + 50\mathrm{e}^{-t}$

(c)
M1: Substitutes into the first equation the answer for part (b) in place of w and the derivative of their (b) in place of $\frac{\mathrm{d}w}{\mathrm{d}t}$. If they rearrange to make S the subject first and make a slip but still substitutes for w and $\frac{\mathrm{d}w}{\mathrm{d}t}$ allow this mark.
A1: Correct simplified equation.

(d)
M1: Uses the initial conditions $t = 0$, $w = 65$ and $s = 85$ to form simulations equations and solves to find the values of their constants
dM1: Dependent on the previous method mark. Sets $w = 0$
A1: Processes the indices correctly to obtain a 3-term quadratic equation in terms of $\mathrm{e}^{1.5t}$. It does not need to all be on one side and condone missing = 0.
M1: Solves their three-term quadratic (3TQ) to reach $\mathrm{e}^{pt} = q$
M1: Correct use of logarithms to reach $pt = \ln q$ where $q > 0$ and rejects the other solution
A1: awrt 1.165

Note: the final 3 marks only can be implied by a correct answer following the correct 3-term quadratic equation in terms of $e^{1.5t}$

(e)
B1: Suggests a suitable limitation of the model, not valid when negative population
Any mention of other factors such as does not take into account e.g. other predictors, fishing, disease, lack of food etc is B0

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