Please check the examination details below before entering your candidate information

Candidate surname	Other names

Centre Number | Candidate Number

Pearson Edexcel Level 3 GCE

Thursday 25 May 2023

Afternoon
(Time: 1 hour 30 minutes)

Paper reference **9FM0/01**

Further Mathematics

Advanced
PAPER 1: Core Pure Mathematics 1

You must have:
Mathematical Formulae and Statistical Tables (Green), calculator

Total Marks

Candidates may use any calculator permitted by Pearson regulations. Calculators must not have the facility for algebraic manipulation, differentiation and integration, or have retrievable mathematical formulae stored in them.

Instructions

- Use **black** ink or ball-point pen.
- If pencil is used for diagrams/sketches/graphs it must be dark (HB or B).
- **Fill in the boxes** at the top of this page with your name, centre number and candidate number.
- Answer **all** questions and ensure that your answers to parts of questions are clearly labelled.
- Answer the questions in the spaces provided – *there may be more space than you need.*
- You should show sufficient working to make your methods clear. Answers without working may not gain full credit.
- Inexact answers should be given to three significant figures unless otherwise stated.

Information

- A booklet 'Mathematical Formulae and Statistical Tables' is provided.
- There are 8 questions in this question paper. The total mark for this paper is 75.
- The marks for **each** question are shown in brackets – *use this as a guide as to how much time to spend on each question.*

Advice

- Read each question carefully before you start to answer it.
- Try to answer every question.
- Check your answers if you have time at the end.

Turn over

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Pearson

1. The cubic equation

$$x^3 - 7x^2 - 12x + 6 = 0$$

has roots α, β and γ.

Without solving the equation, determine a cubic equation whose roots are $(\alpha + 2)$, $(\beta + 2)$ and $(\gamma + 2)$, giving your answer in the form $w^3 + pw^2 + qw + r = 0$, where p, q and r are integers to be found.

(5)

let $w = x + 2 \Rightarrow x = w - 2$ 1

$(w-2)^3 - 7(w-2)^2 - 12(w-2) + 6 = 0$ 1

$(w^3 - 6w^2 + 12w - 8) - 7(w^2 - 4w + 4) - 12(w-2) + 6 = 0$

$w^3 - 6w^2 + 12w - 8 - 7w^2 + 28w - 28 - 12w + 24 + 6 = 0$ 2

$w^3 - 13w^2 + 28w - 6 = 0$ 1

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Question 1 continued

(Total for Question 1 is 5 marks)

2. (a) Write $x^2 + 4x - 5$ in the form $(x + p)^2 + q$ where p and q are integers.

(1)

(b) Hence use a standard integral from the formula book to find

$$\int \frac{1}{\sqrt{x^2 + 4x - 5}}\,\mathrm{d}x$$

(2)

(c) Determine the mean value of the function

$$\mathrm{f}(x) = \frac{1}{\sqrt{x^2 + 4x - 5}} \qquad 3 \leqslant x \leqslant 13$$

giving your answer in the form $A \ln B$ where A and B are constants in simplest form.

(3)

a) $x^2 + 4x - 5 = (x+2)^2 - 4 - 5 = (x+2)^2 - 9$ 1

b) $\displaystyle\int \frac{1}{\sqrt{(x+2)^2 - 9}}\,dx$

from booklet:

$\displaystyle\int \frac{1}{x^2 - a^2}\,dx = \operatorname{arcosh}\left(\frac{x}{a}\right)$

set "x" $= x+2$ and "a" $= 3$

$= \operatorname{arcosh}\left(\dfrac{x+2}{3}\right)$ 1 1

c) mean value $= \dfrac{1}{13-3}\displaystyle\int_3^{13} \frac{1}{\sqrt{x^2+4x+5}}\,dx$ 1

$= \dfrac{1}{10}\left[\operatorname{arcosh}\left(\dfrac{x+2}{3}\right)\right]_3^{13}$

$= \dfrac{1}{10}\left[\operatorname{arcosh}\left(\dfrac{15}{3}\right) - \operatorname{arcosh}\left(\dfrac{5}{3}\right)\right]$ 1

$= \dfrac{1}{10}\ln\left(\dfrac{5+2\sqrt{6}}{3}\right)$ 1

Question 2 continued

(Total for Question 2 is 6 marks)

3. **In this question you must show all stages of your working.**

Solutions relying on calculator technology are not acceptable.

$$z_1 = -4 + 4\text{i}$$

(a) Express z_1 in the form $r(\cos\theta + \text{i}\sin\theta)$, where $r \in \mathbb{R}$, $r > 0$ and $0 \leqslant \theta < 2\pi$

(2)

$$z_2 = 3\left(\cos\frac{17\pi}{12} + \text{i}\sin\frac{17\pi}{12}\right)$$

(b) Determine in the form $a + \text{i}b$, where a and b are exact real numbers,

(i) $\dfrac{z_1}{z_2}$

(2)

(ii) $(z_2)^4$

(2)

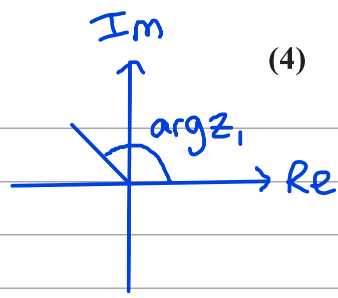
(c) Show on a single Argand diagram

(i) the complex numbers z_1, z_2 and $\dfrac{z_1}{z_2}$

(ii) the region defined by $\{z \in \mathbb{C} : |z - z_1| < |z - z_2|\}$

(4)

a) $z_1 = -4 + 4i \qquad |z_1| = \sqrt{(-4)^2 + 4^2} = 4\sqrt{2}$



$\arg z_1 = \pi - \tan^{-1}\left(\frac{4}{4}\right) = \frac{3\pi}{4}$ 1

$z_1 = 4\sqrt{2}\left(\cos\frac{3\pi}{4} + i\sin\frac{3\pi}{4}\right)$ 1

b) (i) $\frac{z_1}{z_2}$: divide moduli, subtract arguments

$= \frac{4\sqrt{2}}{3}\left(\cos\left(\frac{3\pi}{4} - \frac{17\pi}{12}\right) + i\sin\left(\frac{3\pi}{4} - \frac{17\pi}{12}\right)\right)$ 1

$= -\frac{2\sqrt{2}}{3} - \frac{2\sqrt{6}}{3}i$ 1

(ii) $(z_2)^4$: raise modulus to power of 4, multiply argument by 4

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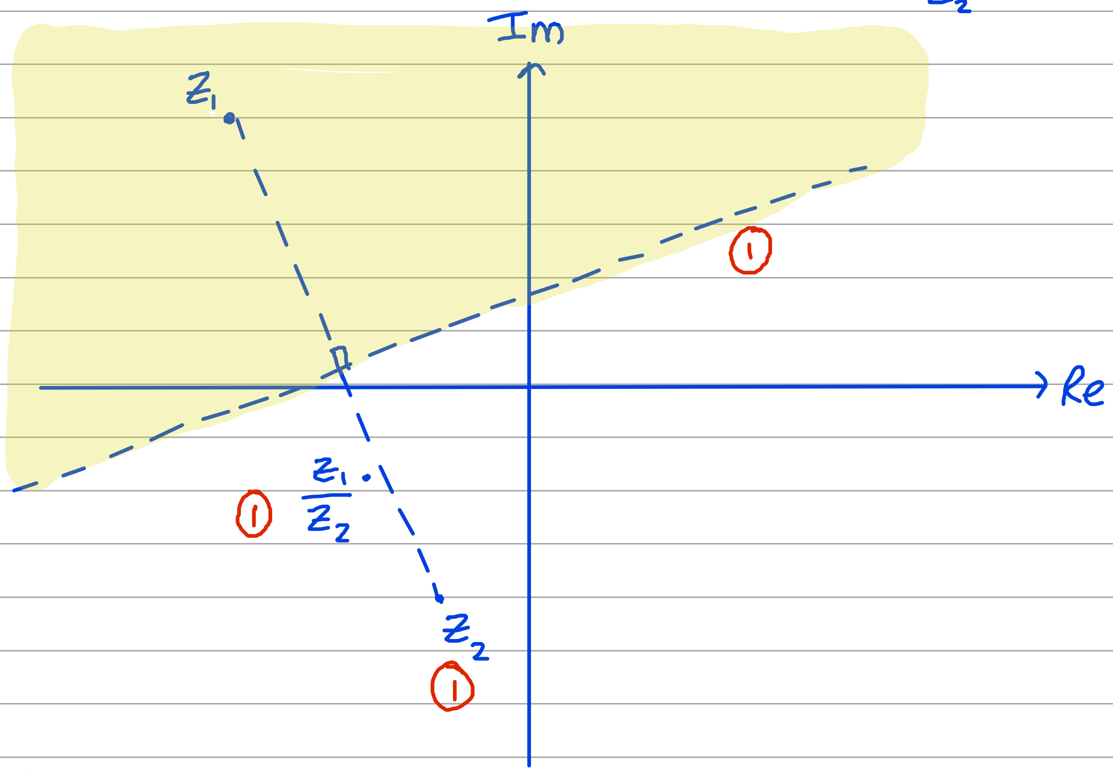
Question 3 continued

$$(z_2)^4 = 3^4\left(\cos\left(4 \times \frac{17\pi}{12}\right) + i\sin\left(4 \times \frac{17\pi}{12}\right)\right) \; 1$$

$$= \frac{81}{2} - \frac{81\sqrt{3}}{2}i \; 1$$

c) (i) $z_1 = -4 + 4i$ $z_2 = -0.78 - 2.90i$ $\frac{z_1}{z_2} = -0.94 - 1.63i$



(ii) perpendicular bisector of z_1 and z_2, with shaded region being closer to z_1

Question 3 continued

Question 3 continued

(Total for Question 3 is 10 marks)

4. Prove by induction that for $n \in \mathbb{N}$

$$\begin{pmatrix} 1 & -2 \\ 0 & 1 \end{pmatrix}^n = \begin{pmatrix} 1 & -2n \\ 0 & 1 \end{pmatrix}$$

(5)

Base case $n=1$:

$$\begin{pmatrix} 1 & -2 \\ 0 & 1 \end{pmatrix}^1 = \begin{pmatrix} 1 & -2\times 1 \\ 0 & 1 \end{pmatrix}$$ so true when $n=1$ (1)

Assume true for $n=k$:

$$\begin{pmatrix} 1 & -2 \\ 0 & 1 \end{pmatrix}^k = \begin{pmatrix} 1 & -2k \\ 0 & 1 \end{pmatrix}$$

Show true for $n=k+1$:

$$\begin{pmatrix} 1 & -2 \\ 0 & 1 \end{pmatrix}^{k+1} = \begin{pmatrix} 1 & -2 \\ 0 & 1 \end{pmatrix}^k \begin{pmatrix} 1 & -2 \\ 0 & 1 \end{pmatrix}$$ (1)

$$= \begin{pmatrix} 1 & -2k \\ 0 & 1 \end{pmatrix}\begin{pmatrix} 1 & -2 \\ 0 & 1 \end{pmatrix}$$

$$= \begin{pmatrix} 1 & -2-2k \\ 0 & 1 \end{pmatrix}$$ (1)

$$= \begin{pmatrix} 1 & -2(k+1) \\ 0 & 1 \end{pmatrix}$$ (1)

Hence it is true for $n=k+1$. As it is true when $n=1$ and have shown if true for $n=k$ then true for $n=k+1$, so it is true for all positive integers n. (1)

Question 4 continued

(Total for Question 4 is 5 marks)

5. The line l_1 has equation $\dfrac{x+5}{1} = \dfrac{y+4}{-3} = \dfrac{z-3}{5}$

The plane Π_1 has equation $2x + 3y - 2z = 6$

(a) Find the point of intersection of l_1 and Π_1

(2)

The line l_2 is the reflection of the line l_1 in the plane Π_1

(b) Show that a vector equation for the line l_2 is

$$\mathbf{r} = \begin{pmatrix} -7 \\ 2 \\ -7 \end{pmatrix} + \mu \begin{pmatrix} 10 \\ 6 \\ 2 \end{pmatrix}$$

where μ is a scalar parameter.

(5)

The plane Π_2 contains the line l_1 and the line l_2

(c) Determine a vector equation for the line of intersection of Π_1 and Π_2

(2)

The plane Π_3 has equation $\mathbf{r}.\begin{pmatrix} 1 \\ 1 \\ a \end{pmatrix} = b$ where a and b are constants.

Given that the planes Π_1, Π_2 and Π_3 form a sheaf,

(d) determine the value of a and the value of b.

(3)

a) vector equation of l_1: $\mathbf{r} = \begin{pmatrix} -5 \\ -4 \\ 3 \end{pmatrix} + \lambda \begin{pmatrix} 1 \\ -3 \\ 5 \end{pmatrix} = \begin{pmatrix} -5+\lambda \\ -4-3\lambda \\ 3+5\lambda \end{pmatrix}$

sub into Π: $\begin{pmatrix} -5+\lambda \\ -4-3\lambda \\ 3+5\lambda \end{pmatrix} \cdot \begin{pmatrix} 2 \\ 3 \\ -2 \end{pmatrix} = 6$

$2(-5+\lambda) + 3(-4-3\lambda) - 2(3+5\lambda) = 6$ ①

$-17\lambda - 28 = 6$

$\lambda = -2$

sub into l_1: intersection $= \begin{pmatrix} -5-2 \\ -4-3(-2) \\ 3+5(-2) \end{pmatrix}$ coordinates: $(-7, 2, -7)$ ①

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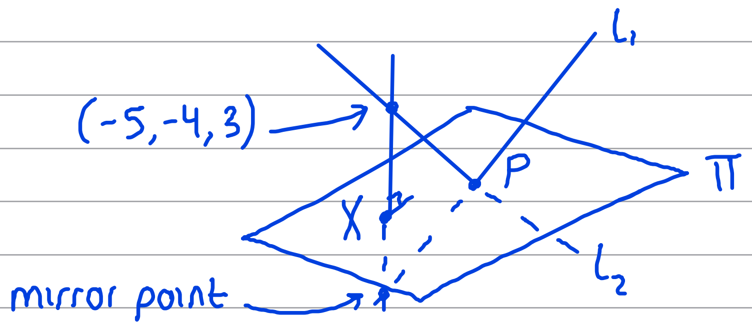
Question 5 continued

c) Method: find two points on l_2. $P(-7, 2, -7)$ will be on l_2 as it is in Π.



To find the second point, take a line which is perpendicular to Π and passes through a point on l_1:

$$r = \begin{pmatrix} -5 \\ -4 \\ 3 \end{pmatrix} + t \begin{pmatrix} 2 \\ 3 \\ -2 \end{pmatrix} = \begin{pmatrix} -5+2t \\ -4+3t \\ 3-2t \end{pmatrix} \quad (1)$$

find where it intersects Π:

$$\begin{pmatrix} -5+2t \\ -4+3t \\ 3-2t \end{pmatrix} \cdot \begin{pmatrix} 2 \\ 3 \\ -2 \end{pmatrix} = 6$$

$$\Rightarrow 2(-5+2t) + 3(-4+3t) - 2(3-2t) = 6 \quad (1)$$

$$t = 2$$

if $\begin{pmatrix} -5 \\ -4 \\ 3 \end{pmatrix} + 2\begin{pmatrix} 2 \\ 3 \\ -2 \end{pmatrix}$ reaches Π, then $\begin{pmatrix} -5 \\ -4 \\ 3 \end{pmatrix} + 4\begin{pmatrix} 2 \\ 3 \\ -2 \end{pmatrix}$ reaches l_2.

$$\text{mirror point} = \begin{pmatrix} -5+8 \\ -4+12 \\ 3-8 \end{pmatrix} \overset{(1)}{=} \begin{pmatrix} 3 \\ 8 \\ -5 \end{pmatrix} \ (1)$$

Question 5 continued

$$l_2: \underline{r} = \begin{pmatrix} -7 \\ 2 \\ -7 \end{pmatrix} + \mu \begin{pmatrix} 3-(-7) \\ 8-2 \\ -5-(-7) \end{pmatrix} \quad 1$$

$$\underline{r} = \begin{pmatrix} -7 \\ 2 \\ -7 \end{pmatrix} + \mu \begin{pmatrix} 10 \\ 6 \\ 2 \end{pmatrix} \quad 1$$

c) line of intersection of Π_1 and Π_2 will cross through X and P.

$$P = \begin{pmatrix} -7 \\ 2 \\ -7 \end{pmatrix} \qquad X = \begin{pmatrix} -5 \\ -4 \\ 3 \end{pmatrix} + 2\begin{pmatrix} 2 \\ 3 \\ -2 \end{pmatrix} = \begin{pmatrix} -1 \\ 2 \\ -1 \end{pmatrix}$$

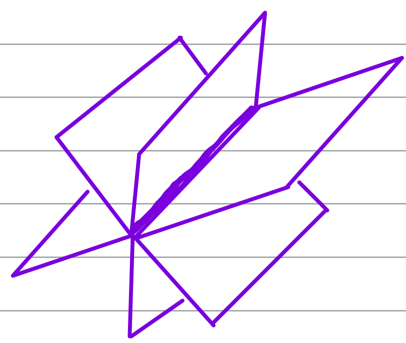
$\therefore$ equation of line: $\underline{r} = \begin{pmatrix} -7 \\ 2 \\ -7 \end{pmatrix} + s \begin{pmatrix} -1-(-7) \\ 2-2 \\ -1-(-7) \end{pmatrix}$ 1

$$\underline{r} = \begin{pmatrix} -7 \\ 2 \\ -7 \end{pmatrix} + s \begin{pmatrix} 6 \\ 0 \\ 6 \end{pmatrix} \quad 1$$

d) since planes form a sheaf, the line from c) will also be inside Π_3.

$\therefore \begin{pmatrix} 6 \\ 0 \\ 6 \end{pmatrix}$ is perpendicular to $\underline{n}_3 = \begin{pmatrix} 1 \\ 1 \\ a \end{pmatrix}$

$$\begin{pmatrix} 6 \\ 0 \\ 6 \end{pmatrix} \cdot \begin{pmatrix} 1 \\ 1 \\ a \end{pmatrix} = 0 \Rightarrow a = -1 \quad 1 \quad 1$$



sheaf.

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Question 5 continued

$\begin{pmatrix} -7 \\ 2 \\ -7 \end{pmatrix}$ is on the line going throug Π_3, so

$$\begin{pmatrix} -7 \\ 2 \\ -7 \end{pmatrix} \cdot \begin{pmatrix} 1 \\ 1 \\ -1 \end{pmatrix} = 6$$

$$6 = -7 + 2 + 7$$
$$= 2 \quad 1$$

(Total for Question 5 is 12 marks)

6. Water is flowing into and out of a large tank.

Initially the tank contains 10 litres of water.

The rate of flow of the water is modelled so that

- there are V litres of water in the tank at time t minutes after the water begins to flow
- water enters the tank at a rate of $\left(3 - \dfrac{4}{1 + e^{0.8t}}\right)$ litres per minute
- water leaves the tank at a rate proportional to the volume of water remaining in the tank

Given that when $t = 0$ the volume of water in the tank is decreasing at a rate of 3 litres per minute, use the model to

(a) show that the volume of water in the tank at time t satisfies

$$\frac{dV}{dt} = 3 - \frac{4}{1 + e^{0.8t}} - 0.4V$$

(3)

(b) Determine $\dfrac{d}{dt}\left(\arctan e^{0.4t}\right)$

(2)

Hence, by solving the differential equation from part (a),

(c) determine an equation for the volume of water in the tank at time t.

Give your answer in simplest form as $V = f(t)$

(6)

After 10 minutes, the volume of water in the tank was 8 litres.

(d) Evaluate the model in light of this information.

(1)

a) $\dfrac{dV}{dt} = 3 - \dfrac{4}{1+e^{0.8t}} + kV$ 1

sub in $t=0$, $V=10$, $\dfrac{dV}{dt} = -3$:

$-3 = 3 - \dfrac{4}{1+1} + 10k$ 1

$10k = -4$

$k = -0.4$

$\dfrac{dV}{dt} = 3 - \dfrac{4}{1+e^{0.8t}} - 0.4V$ 1

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Question 6 continued

b) $\frac{d}{dt}\arctan(e^{0.4t}) = \frac{1}{1+(e^{0.4t})^2} \times 0.4e^{0.4t}$ (1)

$= \frac{0.4e^{0.4t}}{1+e^{0.8t}}$ (1)

c) $\frac{dV}{dt} + 0.4V = 3 - \frac{4}{1+e^{0.8t}}$

Integration factor $= e^{\int 0.4\,dt} = e^{0.4t}$ (1)

$e^{0.4t}\frac{dV}{dt} + 0.4e^{0.4t}V = 3e^{0.4t} - \frac{4e^{0.4t}}{1+e^{0.8t}}$

$\frac{d}{dt}(Ve^{0.4t}) = 3e^{0.4t} - \frac{4e^{0.4t}}{1+e^{0.8t}}$

$Ve^{0.4t} = \int\left(3e^{0.4t} - \frac{4e^{0.4t}}{1+e^{0.8t}}\right)dt$ (1)

$Ve^{0.4t} = \frac{3}{0.4}e^{0.4t} - 10\arctan(e^{0.4t}) + c$ (1) (1)

sub in $t=0$, $V=10$:

$10e^0 = 7.5e^0 - 10\arctan(e^0) + c$

$10 = 7.5 - 2.5\pi + c$

$c = 2.5(1+\pi)$ (1)

$V = 7.5 - 10e^{-0.4t}\arctan(e^{0.4t}) + 2.5(\pi+1)e^{-0.4t}$ (1)

d) using model, when $t=10$, $V = 7.4$ so the model is not very accurate (1)

Question 6 continued

Question 6 continued

(Total for Question 6 is 12 marks)

7. **In this question you must show all stages of your working.**

Solutions relying on calculator technology are not acceptable.

(a) Explain why, for $n \in \mathbb{N}$

$$\sum_{r=1}^{2n} (-1)^r \mathrm{f}(r) = \sum_{r=1}^{n} \left(\mathrm{f}(2r) - \mathrm{f}(2r-1)\right)$$

for any function $\mathrm{f}(r)$.

(2)

(b) Use the standard summation formulae to show that, for $n \in \mathbb{N}$

$$\sum_{r=1}^{2n} r\left((-1)^r + 2r\right)^2 = n(2n+1)(8n^2 + 4n + 5)$$

(6)

(c) Hence evaluate

$$\sum_{r=14}^{50} r\left((-1)^r + 2r\right)^2$$

(4)

$2n-1$ is odd, so $(-1)^{2n-1} = -1$

a) $\sum_{r=1}^{2n} (-1)^r f(r) = -f(1) + f(2) - f(3) + f(4) - \cdots - f(2n-1) + f(2n)$

$= f(2) + f(4) + \cdots + f(2n) - [f(1) + f(3) + \cdots + f(2n-1)]$

$= \sum_{r=1}^{n} \left(f(2r) - f(2r-1)\right)$ (1) (1)

b) $\sum_{r=1}^{2n} r\left((-1)^r + 2r\right)^2 = \sum_{r=1}^{2n} r\left((-1)^{2r} + 4r(-1)^r + 4r^2\right)$ (1)

$(-1)^{2r} = 1$ for all r.

$= \sum_{r=1}^{2n} \left(r + 4r^2(-1)^r + 4r^3\right)$

$= \frac{1}{2}(2n)(2n+1) + 4\sum_{r=1}^{2n} (-1)^r r^2 + \frac{1}{4}(2n)^2(2n+1)^2$ (1)

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Question 7 continued

let $f(r) = r^2$. $\sum_{r=1}^{2n} (-1)^r r^2 = \sum_{r=1}^{2n} \left((2r)^2 - (2r-1)^2 \right)$ 1

$(2r)^2 - (2r-1)^2 = (2r + 2r - 1)(2r - (2r-1))$
$= 4r - 1$

$\sum_{r=1}^{2n} (4r-1) = \frac{4n}{2}(n+1) - n$

1

$\therefore \sum_{r=1}^{2n} r\left((-1)^r + 2r\right)^2 = \frac{1}{2}(2n)(2n+1) + 4\left[\frac{4n}{2}(n+1) - n\right] + \frac{1}{4}(2n)^2(2n+1)^2$

$= n(2n+1) + 4n(2n+1) + 4n^2(2n+1)^2$ 1

$= n(2n+1)\left[1 + 4 + 4n(2n+1)\right]$

$= n(2n+1)(8n^2 + 4n + 5)$ as required 1

c) $\sum_{r=14}^{50} r\left((-1)^r + 2r\right)^2 = \sum_{r=1}^{50} r\left((-1)^r + 2r\right)^2 - \sum_{r=1}^{13} r\left((-1)^r + 2r\right)^2$ 1

13 is odd, so cannot be used as 2n.

$= \sum_{r=1}^{50} r\left((-1)^r + 2r\right)^2 - \left[\sum_{r=1}^{12} r\left((-1)^r + 2r\right)^2 + \sum_{r=13}^{13} r\left((-1)^r + 2r\right)^2\right]$ 1

split the summation

50 is 2n here, so n is actually 25. Don't trip up!

$= (25)(51)(5105) - 6(13)(317) - 13\left((-1)^{13} + 2(13)\right)^2$ 1

$= 6508875 - 24726 - 8125$

$= 6476024$ 1

Question 7 continued

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Question 7 continued

(Total for Question 7 is 12 marks)

8. A colony of small mammals is being studied.
In the study, the mammals are divided into 3 categories

N (newborns)	0 to less than 1 month old
J (juveniles)	1 to 3 months old
B (breeders)	over 3 months old

(a) State one limitation of the model regarding the division into these categories.
(1)

A model for the population of the colony is given by the matrix equation

$$\begin{pmatrix} N_{n+1} \\ J_{n+1} \\ B_{n+1} \end{pmatrix} = \begin{pmatrix} 0 & 0 & 2 \\ a & b & 0 \\ 0 & 0.48 & 0.96 \end{pmatrix} \begin{pmatrix} N_n \\ J_n \\ B_n \end{pmatrix}$$

where a and b are constants, and N_n, J_n and B_n are the respective numbers of the mammals in each category n months after the start of the study.

At the start of the study the colony has breeders only, with no newborns or juveniles.

According to the model, after 2 months the number of newborns is 48 and the number of juveniles is 40

(b) (i) Determine the number of mammals in the colony at the start of the study.

(ii) Show that $a = 0.8$
(4)

(c) Determine, in terms of b,

$$\begin{pmatrix} 0 & 0 & 2 \\ 0.8 & b & 0 \\ 0 & 0.48 & 0.96 \end{pmatrix}^{-1}$$

(3)

Given that the model predicts approximately 1015 mammals **in total** at the start of a particular month, and approximately 596 **newborns**, 464 **juveniles** and 437 **breeders** at the start of the next month,

(d) determine the value of b, giving your answer to 2 decimal places.
(3)

It is decided to monitor the number of **newborn** males and females as a part of the study. Assuming that 42% of newborns are male,

(e) refine the matrix equation for the model to reflect this information, giving a reason for your answer.
(*There is no need to estimate any unknown values for the refined model, but any known values should be made clear.*)
(2)

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Question 8 continued

a) the mammals will stop breeding past a certain age (1)

b) (i) when $n=2$, $N_2=48$ and $J_2=40$
when $n=0$, $N_0=0$ and $J_0=0$. Let $B_0=k$

$$\begin{pmatrix} N_{n+1} \\ J_{n+1} \\ B_{n+1} \end{pmatrix} = \begin{pmatrix} 0 & 0 & 2 \\ a & b & 0 \\ 0 & 0.48 & 0.96 \end{pmatrix}\begin{pmatrix} N_n \\ J_n \\ B_n \end{pmatrix}$$

$$\begin{pmatrix} 0 & 0 & 2 \\ a & b & 0 \\ 0 & 0.48 & 0.96 \end{pmatrix}\begin{pmatrix} 0 \\ 0 \\ k \end{pmatrix} = \begin{pmatrix} 2k \\ 0 \\ 0.96k \end{pmatrix} \quad n=1$$

$$\begin{pmatrix} 0 & 0 & 2 \\ a & b & 0 \\ 0 & 0.48 & 0.96 \end{pmatrix}\begin{pmatrix} 2k \\ 0 \\ 0.96k \end{pmatrix} = \begin{pmatrix} 2\times 0.96k \\ 2ak \\ 0.96^2 k \end{pmatrix} \quad n=2 \quad (1)$$

comparing first row: $2\times 0.96k = 48$ (1)
$k = 25$

$\therefore$ there are 25 mammals at the start of the study (1)

(ii) comparing second row $2a(25)=60$
$a = 0.8$ (1)

c) $\det\begin{pmatrix} 0 & 0 & 2 \\ 0.8 & b & 0 \\ 0 & 0.48 & 0.96 \end{pmatrix} = 0 - 0 + 2\begin{vmatrix} 0.8 & b \\ 0 & 0.48 \end{vmatrix}$

$= 0.768$ (1)

matrix of minors:

$$\begin{pmatrix} \begin{vmatrix} b & 0 \\ 0.48 & 0.96 \end{vmatrix} & \begin{vmatrix} 0.8 & 0 \\ 0 & 0.96 \end{vmatrix} & \begin{vmatrix} 0.8 & b \\ 0 & 0.48 \end{vmatrix} \\ \begin{vmatrix} 0 & 2 \\ 0.48 & 0.96 \end{vmatrix} & \begin{vmatrix} 0 & 2 \\ 0 & 0.96 \end{vmatrix} & \begin{vmatrix} 0 & 0 \\ 0 & 0.48 \end{vmatrix} \\ \begin{vmatrix} 0 & 2 \\ b & 0 \end{vmatrix} & \begin{vmatrix} 0 & 2 \\ 0.8 & 0 \end{vmatrix} & \begin{vmatrix} 0 & 0 \\ 0.8 & b \end{vmatrix} \end{pmatrix} = \begin{pmatrix} 0.96b & 0.768 & 0.384 \\ -0.96 & 0 & 0 \\ -2b & -1.6 & 0 \end{pmatrix}$$

Question 8 continued

matrix of cofactors: $\begin{pmatrix} 0.966 & -0.768 & 0.384 \\ 0.96 & 0 & 0 \\ -26 & 1.6 & 0 \end{pmatrix}$

transpose: $\begin{pmatrix} 0.966 & 0.96 & -26 \\ -0.768 & 0 & 1.6 \\ 0.384 & 0 & 0 \end{pmatrix}$ (1)

$\therefore \begin{pmatrix} 0 & 0 & 2 \\ 0.8 & b & 0 \\ 0 & 0.48 & 0.96 \end{pmatrix}^{-1} = \frac{1}{0.768}\begin{pmatrix} 0.966 & 0.96 & -26 \\ -0.768 & 0 & 1.6 \\ 0.384 & 0 & 0 \end{pmatrix}$ (1)

d) $\begin{pmatrix} 596 \\ 464 \\ 437 \end{pmatrix} = \begin{pmatrix} 0 & 0 & 2 \\ 0.8 & b & 0 \\ 0 & 0.48 & 0.96 \end{pmatrix}\begin{pmatrix} x \\ y \\ z \end{pmatrix}$ $x+y+z=1015$

$\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \frac{1}{0.768}\begin{pmatrix} 0.966 & 0.96 & -26 \\ -0.768 & 0 & 1.6 \\ 0.384 & 0 & 0 \end{pmatrix}\begin{pmatrix} 596 \\ 464 \\ 437 \end{pmatrix}$ (1)

$x+y+z = 1.256 \times 596 + 1.25 \times 464 - \frac{125}{48} b \times 437 - 596 + \frac{25}{12} \times 437 + 298$

$1015 = 7456 + 580 - 11386 - 596 + 910.4 + 298$ (1)

$b = 0.4513...$

$b = 0.45$ (2sf) (1)

e) separate N_n into NM_n and NF_n

$\begin{pmatrix} NM_{n+1} \\ NF_{n+1} \\ J_{n+1} \\ B_{n+1} \end{pmatrix} = \begin{pmatrix} 0 & 0 & 0 & A \\ 0 & 0 & 0 & B \\ ? & ? & 0.45 & 0 \\ 0 & 0 & 0.48 & 0.96 \end{pmatrix}\begin{pmatrix} NM_n \\ NF_n \\ J_n \\ B_n \end{pmatrix}$

Question 8 continued

? ?: we dont know what percentage of male and female newborns become juveniles each month. We only know that 80% of the <u>total</u> newborn population become juveniles each month.

A: in the original model, every breeder had two newborns each month. Now 42% are male, so $0.42 \times 2 = 0.84$

B: in the original model, every breeder had two newborns each month. Now 58% are male, so $0.58 \times 2 = 1.16$

$$\begin{pmatrix} NM_{n+1} \\ NF_{n+1} \\ J_{n+1} \\ B_{n+1} \end{pmatrix} = \begin{pmatrix} 0 & 0 & 0 & 0.84 \\ 0 & 0 & 0 & 1.16 \\ ? & ? & 0.45 & 0 \\ 0 & 0 & 0.48 & 0.96 \end{pmatrix} \begin{pmatrix} NM_n \\ NF_n \\ J_n \\ B_n \end{pmatrix}$$ 2

Question 8 continued

(Total for Question 8 is 13 marks)

TOTAL FOR PAPER IS 75 MARKS