Please check the examination details below before entering your candidate information

Candidate surname	Other names

Centre Number | Candidate Number

Pearson Edexcel Level 3 GCE

Time 1 hour 30 minutes

Paper reference **9FM0/01**

Further Mathematics

Advanced
PAPER 1: Core Pure Mathematics 1

You must have:
Mathematical Formulae and Statistical Tables (Green), calculator

Total Marks

Candidates may use any calculator permitted by Pearson regulations. Calculators must not have the facility for algebraic manipulation, differentiation and integration, or have retrievable mathematical formulae stored in them.

Instructions

- Use **black** ink or ball-point pen.
- If pencil is used for diagrams/sketches/graphs it must be dark (HB or B).
- **Fill in the boxes** at the top of this page with your name, centre number and candidate number.
- Answer **all** questions and ensure that your answers to parts of questions are clearly labelled.
- Answer the questions in the spaces provided – *there may be more space than you need.*
- You should show sufficient working to make your methods clear. Answers without working may not gain full credit.
- Inexact answers should be given to three significant figures unless otherwise stated.

Information

- A booklet 'Mathematical Formulae and Statistical Tables' is provided.
- There are 10 questions in this question paper. The total mark for this paper is 75.
- The marks for **each** question are shown in brackets – *use this as a guide as to how much time to spend on each question.*

Advice

- Read each question carefully before you start to answer it.
- Try to answer every question.
- Check your answers if you have time at the end.

Turn over

1. $f(z) = z^3 + az + 52$ where a is a real constant

Given that $2 - 3i$ is a root of the equation $f(z) = 0$

(a) write down the other complex root.

(1)

(b) Hence

(i) solve completely $f(z) = 0$

(ii) determine the value of a

(4)

(c) Show all the roots of the equation $f(z) = 0$ on a single Argand diagram.

(1)

a) $2 + 3i$ (1)

b) (i) let $\alpha = 2+3i$ $\quad \alpha + \beta = 4$

$\beta = 2-3i$ $\quad \alpha\beta = 13$

$f(z) = (z^2 - 4z + 13)(z - \gamma)$ (1)

consider constant: $-13\gamma = 52$

$\gamma = -4$

roots: $z = 2 \pm 3i, -4$ (1)

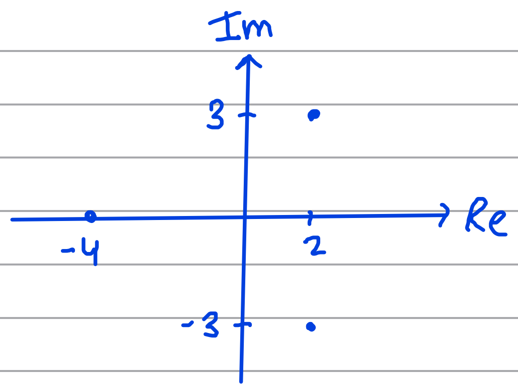
(ii) $f(z) = (z^2 - 4z + 13)(z + 4) = z^3 + az + 52$

consider coefficient of z:

$-4(4) + 13 = a$ (1)

$a = -3$ (1)

c)



(1)

Question 1 continued

(Total for Question 1 is 6 marks)

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2. **In this question you must show all stages of your working.**

Solutions relying entirely on calculator technology are not acceptable.

Determine the values of x for which

$$64\cosh^4 x - 64\cosh^2 x - 9 = 0$$

Give your answers in the form $q\ln 2$ where q is rational and in simplest form.

(4)

hidden quadratic in $\cosh^2 x$:

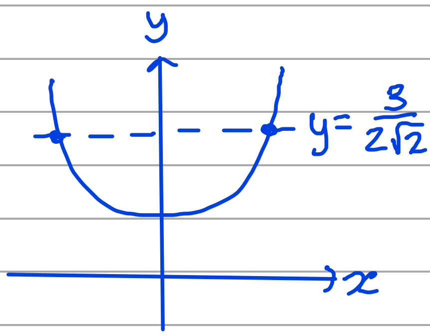
$64\cosh^4 x - 64\cosh^2 x - 9 = 0$

$(8\cosh^2 x - 9)(8\cosh^2 x + 1) = 0$ ①

either $8\cosh^2 x - 9 = 0 \Rightarrow \cosh^2 x = \frac{9}{8}$

or $8\cosh^2 x + 1 = 0 \Rightarrow \cosh^2 x = -\frac{1}{8}$ (no solutions)

① $\cosh^2 x = \frac{9}{8} \Rightarrow \cosh x = \pm\frac{3}{2\sqrt{2}}$



$\cosh x$ is symmetrical in the y axis, so consider $\cosh x = \frac{3}{2\sqrt{2}}$ first only

$x = \text{arcosh}\left(\frac{3}{2\sqrt{2}}\right) = \ln\left[\frac{3}{2\sqrt{2}} + \sqrt{\left(\frac{3}{2\sqrt{2}}\right)^2 - 1}\right]$ ①

$x = \ln\sqrt{2}$

then, $x = -\ln\sqrt{2}$ is also a solution.

$x = \pm\ln\sqrt{2} = \pm\frac{1}{2}\ln 2$ ①

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Question 2 continued

(Total for Question 2 is 4 marks)

3. (a) Determine the general solution of the differential equation

$$\cos x \frac{dy}{dx} + y \sin x = e^{2x} \cos^2 x$$

giving your answer in the form $y = f(x)$

(3)

Given that $y = 3$ when $x = 0$

(b) determine the smallest positive value of x for which $y = 0$

(3)

a) $\cos x \frac{dy}{dx} + y\sin x = e^{2x}\cos^2 x$

$\frac{dy}{dx} + y\tan x = e^{2x}\cos x$

Integration factor (IF) $= e^{\int \tan x \, dx} = e^{\ln(\sec x)} = \sec x$

$\sec x \frac{dy}{dx} + y \tan x \sec x = e^{2x} \cos x \sec x$

$\frac{d}{dx}(y \sec x) = e^{2x}$

$y\sec x = \int e^{2x} \, dx$ (1)

$y \sec x = \frac{1}{2}e^{2x} + c$ (1)

$y = \cos x\left(\frac{1}{2}e^{2x} + c\right)$ (1)

b) $y = 3$ when $x = 0$:

$3 = \cos 0\left(\frac{1}{2}e^0 + c\right) \Rightarrow 3 = \frac{1}{2} + c \Rightarrow c = \frac{5}{2}$ (1)

$y = \cos x\left(\frac{1}{2}e^{2x} + \frac{5}{2}\right) = 0$

$e^{2x} > 0$ for all x, so $\frac{1}{2}e^{2x} + \frac{5}{2} \neq 0$

(1) $\cos x = 0$

$x = \pi/2$ (1)

Question 3 continued

(Total for Question 3 is 6 marks)

4. (a) Use the method of differences to prove that for $n > 2$

$$\sum_{r=2}^{n} \ln\left(\frac{r+1}{r-1}\right) \equiv \ln\left(\frac{n(n+1)}{2}\right)$$

(4)

(b) Hence find the exact value of

$$\sum_{r=51}^{100} \ln\left(\frac{r+1}{r-1}\right)^{35}$$

Give your answer in the form $a\ln\left(\frac{b}{c}\right)$ where a, b and c are integers to be determined.

(3)

$\ln\left(\frac{r+1}{r-1}\right) = \ln(r+1) - \ln(r-1)$ (1)

when $r=2$: $\ln(3) - \ln(1)$
$r=3$: $\ln(4) - \ln(2)$
$r=4$: $\ln(5) - \ln(3)$
$r=5$: $\ln(6) - \ln(4)$
⋮
$r=n-2$: $\ln(n-1) - \ln(n-3)$
$r=n-1$: $\ln(n) - \ln(n-2)$
$r=n$: $\ln(n+1) - \ln(n-1)$ (1)

adding all together: $\sum_{r=2}^{n} \ln\left(\frac{r+1}{r-1}\right) = -\ln 1 - \ln 2 + \ln(n) + \ln(n+1)$ (1)

$= \ln\left(\frac{n(n+1)}{2}\right)$ (1)

b) $\sum_{r=51}^{100} \ln\left(\frac{r+1}{r-1}\right)^{35} = 35\sum_{r=51}^{100} \ln\left(\frac{r+1}{r-1}\right)$ (1)

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Question 4 continued

$$= 35\left(\sum_{r=1}^{100} \ln\left(\frac{r+1}{r-1}\right) - \sum_{r=1}^{50} \ln\left(\frac{r+1}{r-1}\right)\right) \quad (1)$$

$$= 35\left[\ln\left(\frac{100 \times 101}{2}\right) \div \ln\left(\frac{50 \times 51}{2}\right)\right] \quad \leftarrow \text{using part (a)}$$

$$= 35\ln\left(\frac{202}{51}\right) \quad (1)$$

Question 4 continued

Question 4 continued

(Total for Question 4 is 7 marks)

5.

$$\mathbf{M} = \begin{pmatrix} a & 2 & -3 \\ 2 & 3 & 0 \\ 4 & a & 2 \end{pmatrix} \qquad \text{where } a \text{ is a constant}$$

(a) Show that **M** is non-singular for all values of a.

(2)

(b) Determine, in terms of a, $\mathbf{M}^{-1}$

(4)

a) consider $\det M = a\begin{vmatrix} 3 & 0 \\ a & 2 \end{vmatrix} - 2\begin{vmatrix} 2 & 0 \\ 4 & 2 \end{vmatrix} - 3\begin{vmatrix} 2 & 3 \\ 4 & a \end{vmatrix}$

$= a(6) - 2(4) - 3(2a-12)$ (1)

$= 28$

$28 \neq 0$, so M is non-singular for all a. (1)

b) matrix of minors:

$$\begin{pmatrix} \begin{vmatrix} 3 & 0 \\ a & 2 \end{vmatrix} & \begin{vmatrix} 2 & 0 \\ 4 & 2 \end{vmatrix} & \begin{vmatrix} 2 & 3 \\ 4 & a \end{vmatrix} \\ \begin{vmatrix} 2 & -3 \\ a & 2 \end{vmatrix} & \begin{vmatrix} a & -3 \\ 4 & 2 \end{vmatrix} & \begin{vmatrix} a & 2 \\ 4 & a \end{vmatrix} \\ \begin{vmatrix} 2 & -3 \\ 3 & 0 \end{vmatrix} & \begin{vmatrix} a & -3 \\ 2 & 0 \end{vmatrix} & \begin{vmatrix} a & 2 \\ 2 & 3 \end{vmatrix} \end{pmatrix} = \begin{pmatrix} 6 & 4 & 2a-12 \\ 4+3a & 2a+12 & a^2-8 \\ 9 & 6 & 3a-4 \end{pmatrix}$$ (1)

matrix of cofactors:

$$\begin{pmatrix} 6 & -4 & 2a-12 \\ -4-3a & 2a+12 & 8-a^2 \\ 9 & -6 & 3a-4 \end{pmatrix}$$

transpose:

$$\begin{pmatrix} 6 & -4-3a & 9 \\ -4 & 2a+12 & -6 \\ 2a-12 & 8-a^2 & 3a-4 \end{pmatrix}$$ (1)

$$M^{-1} = \frac{1}{28}\begin{pmatrix} 6 & -4-3a & 9 \\ -4 & 2a+12 & -6 \\ 2a-12 & 8-a^2 & 3a-4 \end{pmatrix}$$ (2)

Question 5 continued

(Total for Question 5 is 6 marks)

6. (a) Express as partial fractions

$$\frac{2x^2 + 3x + 6}{(x+1)(x^2+4)}$$

(3)

(b) Hence, show that

$$\int_0^2 \frac{2x^2 + 3x + 6}{(x+1)(x^2+4)}\,dx = \ln\left(a\sqrt{2}\right) + b\pi$$

where a and b are constants to be determined.

(4)

a) $\frac{2x^2+3x+6}{(x+1)(x^2+4)} = \frac{A}{x+1} + \frac{Bx+C}{x^2+4}$

$2x^2+3x+6 = A(x^2+4) + (Bx+C)(x+1)$ ①

$= Ax^2 + 4A + Bx^2 + Bx + Cx + C$

$= (A+B)x^2 + (B+C)x + (4A+C)$

comparing coefficients:

$A+B=2$ ①

$B+C=3$ ②

$4A+C=6$ ③

② − ①: $B+C-A-B = 3-2$

$C = A+1$

sub into ③: $4A+A+1=6$

$5A=5$

$A=1$ ①

①: $1+B=2$, $B=1$ ②: $1+C=3$, $C=2$

$\frac{2x^2+3x+6}{(x+1)(x^2+4)} = \frac{1}{x+1} + \frac{x+2}{x^2+4}$ ①

b) $\int_0^2 \left(\frac{1}{x+1} + \frac{x+2}{x^2+4}\right)dx = \int_0^2 \left(\frac{1}{x+1} + \frac{x}{x^2+4} + \frac{2}{x^2+4}\right)dx$

$= \left[\ln(x+1) + \frac{1}{2}\ln(x^2+4) + \frac{2}{2}\arctan\left(\frac{x}{2}\right)\right]_0^2$ ②

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Question 6 continued

$= \left[\ln 3 + \frac{1}{2}\ln 8 + \arctan 1\right] - \left[\ln 1 + \frac{1}{2}\ln 4 + \arctan 0\right]$ (1)

$= \ln 3 + \ln\sqrt{8} + \frac{\pi}{4} - 0 - \ln 2 - 0$

$= \ln\left(\frac{3\sqrt{8}}{2}\right) + \frac{\pi}{4}$

$= \ln(3\sqrt{2}) + \frac{\pi}{4}$ (1)

Question 6 continued

Question 6 continued

(Total for Question 6 is 7 marks)

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7. Given that $z = a + bi$ is a complex number where a and b are real constants,

(a) show that zz^* is a real number.

(2)

Given that

- $zz^* = 18$
- $\dfrac{z}{z^*} = \dfrac{7}{9} + \dfrac{4\sqrt{2}}{9}\text{i}$

(b) determine the possible complex numbers z

(5)

a) $z = a + bi, \quad a, b \in \mathbb{R}$

$z^* = a - bi$

$zz^* = (a+bi)(a-bi) = a^2 - b^2i^2 = a^2 + b^2$ (1)

which is a real number. (1)

b) $\dfrac{z}{z^*} = \dfrac{a+bi}{a-bi} = \dfrac{(a+bi)(a+bi)}{(a-bi)(a+bi)} = \dfrac{a^2-b^2+2abi}{a^2+b^2}$ (1)

$= \dfrac{a^2-b^2}{a^2+b^2} + \dfrac{2ab}{a^2+b^2}i = \dfrac{7}{9} + \dfrac{4\sqrt{2}}{9}i$

given $zz^* = 18 \Rightarrow a^2 + b^2 = 18$ (1) (1)

comparing real parts: $\dfrac{a^2-b^2}{18} = \dfrac{7}{9} \Rightarrow a^2 - b^2 = 14$ (2) (1)

comparing imaginary parts: $\dfrac{2ab}{18} = \dfrac{4\sqrt{2}}{9} \Rightarrow ab = 4\sqrt{2}$ (3)

(1) + (2): $2a^2 = 18 + 14$

$a^2 = 16$

$a = \pm 4$ (1)

if $a = 4$, from (3): $b = \sqrt{2}$ if $a = -4$, from (3): $b = -2$

so $z = \pm(4 + \sqrt{2}i)$ (1)

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Question 7 continued

Question 7 continued

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Question 7 continued

(Total for Question 7 is 7 marks)

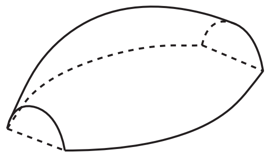
8. (a) Given

$$z^n + \frac{1}{z^n} = 2\cos n\theta \qquad n \in \mathbb{N}$$

show that

$$32\cos^6\theta \equiv \cos 6\theta + 6\cos 4\theta + 15\cos 2\theta + 10$$

(5)



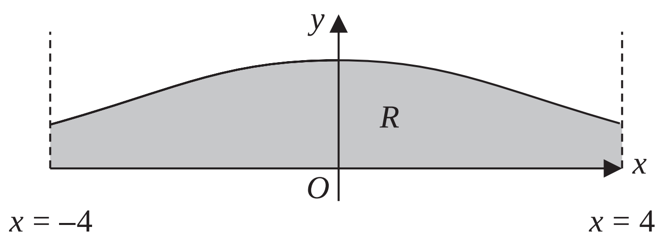


Figure 1 **Figure 2**

Figure 1 shows a solid paperweight with a flat base.

Figure 2 shows the curve with equation

$$y = H\cos^3\left(\frac{x}{4}\right) \qquad -4 \leqslant x \leqslant 4$$

where H is a positive constant and x is in radians.

The region R, shown shaded in Figure 2, is bounded by the curve, the line with equation $x = -4$, the line with equation $x = 4$ and the x-axis.

The paperweight is modelled by the solid of revolution formed when R is rotated **180°** about the x-axis.

Given that the maximum height of the paperweight is 2 cm,

(b) write down the value of H.

(1)

(c) Using algebraic integration and the result in part (a), determine, in cm^3, the volume of the paperweight, according to the model. Give your answer to 2 decimal places.

[*Solutions based entirely on calculator technology are not acceptable.*]

(5)

(d) State a limitation of the model.

(1)

a) $z + \frac{1}{z} = 2\cos\theta$

$\left(z + \frac{1}{z}\right)^6 = (2\cos\theta)^6 = 64\cos^6\theta$ (1)

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Question 8 continued

from binomial expansion:

$$\left(z+\frac{1}{z}\right)^6 = z^6 + 6(z^5)\left(\frac{1}{z}\right) + 15(z^4)\left(\frac{1}{z}\right)^2 + 20(z^3)\left(\frac{1}{z}\right)^3$$

$$+ 15(z^2)\left(\frac{1}{z}\right)^3 + 6(z)\left(\frac{1}{z}\right)^4 + \left(\frac{1}{z}\right)^5 \quad (1)$$

$$= z^6 + 6z^4 + 15z^2 + 20 + \frac{15}{z^2} + \frac{6}{z^4} + \frac{1}{z^6}$$

$$= \left[z^6 + \frac{1}{z^6}\right] + 6\left[z^4 + \frac{1}{z^4}\right] + 15\left[z^2 + \frac{1}{z^2}\right] + 20 \quad (1)$$

$$64\cos^6\theta = 2\cos 6\theta + 6(2\cos 4\theta) + 15(2\cos 2\theta) + 20$$

$$64\cos^6\theta = 2\cos 6\theta + 12\cos 4\theta + 30\cos 2\theta + 20 \quad (1)$$

$$32\cos^6\theta = \cos 6\theta + 6\cos 4\theta + 15\cos 2\theta + 10$$ as required (1)

b) $y = H\cos^3\left(\frac{x}{4}\right)$ has max $y = 2$

$\cos^3\left(\frac{x}{4}\right) \leq 1$ so $H = 2$ (1)

c) $$\text{Volume} = \frac{\pi}{2}\int_{-4}^{4}\left(2\cos^3\left(\frac{x}{4}\right)\right)^2 dx \quad (1)$$

$$= 2\pi\int_{-4}^{4}\cos^6\left(\frac{x}{4}\right) dx$$

since the curve is symmetrical in the y-axis

$$= 4\pi\int_{0}^{4}\cos^6\left(\frac{x}{4}\right) dx$$

Question 8 continued

$$= \frac{4\pi}{32}\int_0^4 \left(\cos\left(\frac{6x}{4}\right) + 6\cos\left(\frac{4x}{4}\right) + 15\cos\left(\frac{2x}{4}\right) + 10\right) dx \quad (1)$$

$$= \frac{\pi}{8}\left[\frac{2}{3}\sin\left(\frac{3x}{2}\right) + 6\sin x + 30\sin\left(\frac{x}{2}\right) + 10x\right]_0^4 \quad (1)$$

$$= \frac{\pi}{8}\left[\frac{2}{3}\sin\left(\frac{3}{2}\times 4\right) + 6\sin 4 + 30\sin\left(\frac{4}{2}\right) + 10(4) - 0\right] \quad (1)$$

$= 24.56$ (2dp) cm^3 (1)

d) the paperweight may not be perfectly smooth (1)

Question 8 continued

(Total for Question 8 is 12 marks)

9. (i) (a) Explain why $\int_0^\infty \cosh x \, dx$ is an improper integral.

(1)

(b) Show that $\int_0^\infty \cosh x \, dx$ is divergent.

(3)

(ii) $$4\sinh x = p\cosh x \qquad \text{where } p \text{ is a real constant}$$

Given that this equation has real solutions, determine the range of possible values for p

(2)

(i) a) the upper limit is infinite ①

b) $\int_0^\infty \cosh x \, dx = \lim_{t\to\infty} \int_0^t \cosh x \, dx$ ①

$= \lim_{t\to\infty} \left[\sinh x\right]_0^t = \lim_{t\to\infty} \left[\sinh t - \sinh 0\right]$

$= \lim_{t\to\infty} \left[\frac{1}{2}(e^t - e^{-t})\right]$ ①

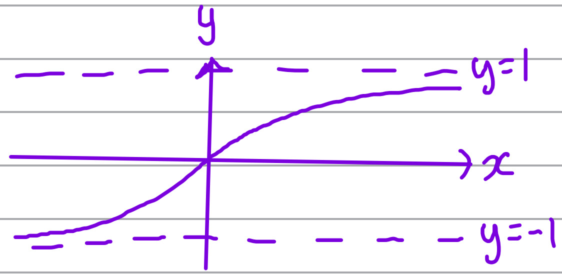
when $t\to\infty$, $e^t \to \infty$ and $e^{-t} \to 0$ so the integral is divergent ①

(ii) $4\sinh x = p\cosh x$ has real solutions

$4\tanh x = p$

$\tanh x = \frac{p}{4}$ ①

$-1 < \tanh x < 1 \Rightarrow -1 < \frac{p}{4} < 1 \Rightarrow -4 < p < 4$ ①



graph of $\tanh x$

Question 9 continued

(Total for Question 9 is 6 marks)

10.

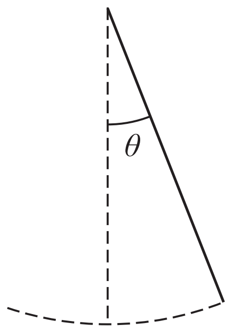


Figure 3

The motion of a pendulum, shown in Figure 3, is modelled by the differential equation

$$\frac{d^2\theta}{dt^2} + 9\theta = \frac{1}{2}\cos 3t$$

where θ is the angle, in radians, that the pendulum makes with the downward vertical, t seconds after it begins to move.

(a) (i) Show that a particular solution of the differential equation is

$$\theta = \frac{1}{12}t\sin 3t$$

(4)

(ii) Hence, find the general solution of the differential equation.

(4)

Initially, the pendulum

- makes an angle of $\frac{\pi}{3}$ radians with the downward vertical
- is at rest

Given that, 10 seconds after it begins to move, the pendulum makes an angle of α radians with the downward vertical,

(b) determine, according to the model, the value of α to 3 significant figures.

(4)

Given that the true value of α is 0.62

(c) evaluate the model.

(1)

The differential equation

$$\frac{d^2\theta}{dt^2} + 9\theta = \frac{1}{2}\cos 3t$$

models the motion of the pendulum as moving with forced harmonic motion.

(d) Refine the differential equation so that the motion of the pendulum is simple harmonic motion.

(1)

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Question 10 continued

a) (i) Method: take $\theta = \frac{1}{12} t\sin 3t$ and show that it satisfies the differential equation

$$\frac{d\theta}{dt} = \frac{1}{12}\sin 3t + \frac{3}{12} t\cos 3t$$

$$= \frac{1}{12}\sin 3t + \frac{1}{4} t\cos 3t \quad (1)$$

$$\frac{d^2\theta}{dt^2} = \frac{3}{12}\cos 3t + \frac{1}{4}\cos 3t - \frac{3}{4} t\sin 3t$$

$$= \frac{1}{2}\cos 3t - \frac{3}{4} t\sin 3t \quad (1)$$

$$\frac{d^2\theta}{dt^2} + 9\theta = \frac{1}{2}\cos 3t - \frac{3}{4} t\sin 3t + \frac{9}{12} t\sin 3t = \frac{1}{2}\cos 3t \quad (1) ✓$$

so PI is $\theta = \frac{1}{12} t\sin 3t$ (1)

(ii) auxillary equation: $m^2 + 9 = 0$

$m = \pm 3i$ (1)

complementary function: $\theta = e^{0}(A\cos 3t + B\sin 3t)$ (1)

general solution: $\theta = A\cos 3t + B\sin 3t + \frac{1}{12} t\sin 3t$ (1) (1)

b) when $t = 0$, $\theta = \frac{\pi}{3}$: $\frac{\pi}{3} = A\cos 0 + B\sin 0 + 0$

$A = \frac{\pi}{3}$ (1)

Question 10 continued

when $t=0$, $\frac{d\theta}{dt}=0$

$$\frac{d\theta}{dt} = \pi\sin 3t + 3B\cos 3t + \frac{1}{12}\sin 3t + \frac{1}{4}t\cos 3t$$

$$0 = \pi\sin 0 + 3B\cos 0 + \frac{1}{12}\sin 0 + 0 \quad (1)$$

$$3B = 0$$

$$B = 0$$

when $t=10$, $\theta = \alpha$

$$\alpha = \frac{\pi}{3}\cos(3\times 10) + \frac{1}{12}(10)\sin(3\times 10) \quad (1)$$

$$\alpha = -0.662 \quad (1)$$

so the pendulum makes an angle of 0.662 radians with the downwards vertical

c) 0.662 is close to 0.62 so the model is good. (1)

d) $\frac{d^2\theta}{dt^2} + 9\theta = 0$ (1)

Question 10 continued

Question 10 continued

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(Total for Question 10 is 14 marks)

TOTAL FOR PAPER IS 75 MARKS