

Cambridge  
International  
**A Level**

**Cambridge Assessment International Education**  
Cambridge International Advanced Level

CANDIDATE  
NAME

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**MATHEMATICS**

**9709/32**

Paper 3 Pure Mathematics 3 (P3)

**May/June 2019**

**1 hour 45 minutes**

Candidates answer on the Question Paper.

Additional Materials: List of Formulae (MF9)

**READ THESE INSTRUCTIONS FIRST**

Write your centre number, candidate number and name in the spaces at the top of this page.

Write in dark blue or black pen.

You may use an HB pencil for any diagrams or graphs.

Do not use staples, paper clips, glue or correction fluid.

**DO NOT WRITE IN ANY BARCODES.**

Answer **all** the questions in the space provided. If additional space is required, you should use the lined page at the end of this booklet. The question number(s) must be clearly shown.

Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.

The use of an electronic calculator is expected, where appropriate.

You are reminded of the need for clear presentation in your answers.

At the end of the examination, fasten all your work securely together.

The number of marks is given in brackets [ ] at the end of each question or part question.

The total number of marks for this paper is 75.

This document consists of **18** printed pages and **2** blank pages.







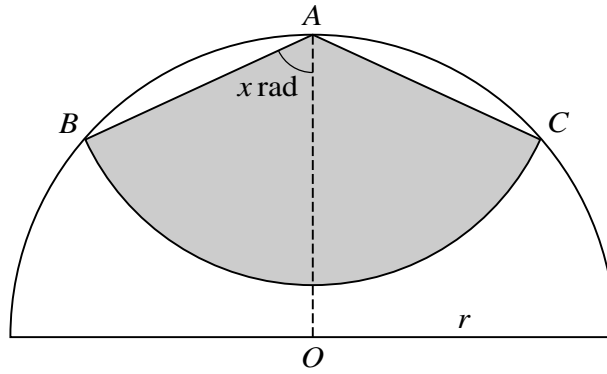








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In the diagram,  $A$  is the mid-point of the semicircle with centre  $O$  and radius  $r$ . A circular arc with centre  $A$  meets the semicircle at  $B$  and  $C$ . The angle  $OAB$  is equal to  $x$  radians. The area of the shaded region bounded by  $AB$ ,  $AC$  and the arc with centre  $A$  is equal to half the area of the semicircle.

(i) Use triangle  $OAB$  to show that  $AB = 2r \cos x$ . [1]

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(ii) Hence show that  $x = \cos^{-1} \sqrt{\left(\frac{\pi}{16x}\right)}$ . [2]

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(iii) Verify by calculation that  $x$  lies between 1 and 1.5.

[2]

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(iv) Use an iterative formula based on the equation in part (ii) to determine  $x$  correct to 3 decimal places. Give the result of each iteration to 5 decimal places.

[3]

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(ii) Explain why  $x$  can only take values that are less than 1. [1]

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- (ii) The plane  $m$  is perpendicular to  $AB$  and passes through the mid-point of  $AB$ . The plane  $m$  intersects the line  $l$  at the point  $P$ . Find the equation of  $m$  and the position vector of  $P$ . [5]

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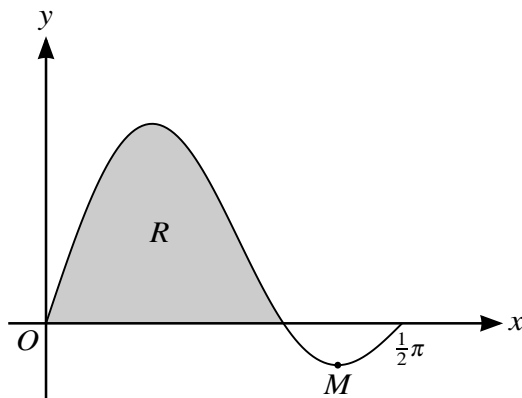
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The diagram shows the curve  $y = \sin 3x \cos x$  for  $0 \leq x \leq \frac{1}{2}\pi$  and its minimum point  $M$ . The shaded region  $R$  is bounded by the curve and the  $x$ -axis.

(i) By expanding  $\sin(3x + x)$  and  $\sin(3x - x)$  show that

$$\sin 3x \cos x = \frac{1}{2}(\sin 4x + \sin 2x). \quad [3]$$

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(ii) Using the result of part (i) and showing all necessary working, find the exact area of the region  $R$ . [4]

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