**UNIVERSITY OF CAMBRIDGE INTERNATIONAL EXAMINATIONS**

GCE Advanced Subsidiary and Advanced Level

# MARK SCHEME for the November 2004 question paper

## 9709 MATHEMATICS
## 8719 HIGHER MATHEMATICS

**9709/03, 8719/03 Paper 3, maximum raw mark 75**

This mark scheme is published as an aid to teachers and students, to indicate the requirements of the examination. It shows the basis on which Examiners were initially instructed to award marks. It does not indicate the details of the discussions that took place at an Examiners' meeting before marking began. Any substantial changes to the mark scheme that arose from these discussions will be recorded in the published *Report on the Examination*.

All Examiners are instructed that alternative correct answers and unexpected approaches in candidates' scripts must be given marks that fairly reflect the relevant knowledge and skills demonstrated.

Mark schemes must be read in conjunction with the question papers and the *Report on the Examination*.

- CIE will not enter into discussion or correspondence in connection with these mark schemes.

CIE is publishing the mark schemes for the November 2004 question papers for most IGCSE and GCE Advanced Level syllabuses.

**Grade thresholds** taken for Syllabus 9709/8719 (Mathematics and Higher Mathematics)) in the November 2004 examination.

| | maximum mark available | minimum mark required for grade: | | |
|---|---|---|---|---|
| | | A | B | E |
| Component 3 | 75 | 59 | 53 | 30 |

The thresholds (minimum marks) for Grades C and D are normally set by dividing the mark range between the B and the E thresholds into three. For example, if the difference between the B and the E threshold is 24 marks, the C threshold is set 8 marks below the B threshold and the D threshold is set another 8 marks down. If dividing the interval by three results in a fraction of a mark, then the threshold is normally rounded down.

**Mark Scheme Notes**

Marks are of the following three types:

M Method mark, awarded for a valid method applied to the problem. Method marks are not lost for numerical errors, algebraic slips or errors in units. However, it is not usually sufficient for a candidate just to indicate an intention of using some method or just to quote a formula; the formula or idea must be applied to the specific problem in hand, e.g. by substituting the relevant quantities into the formula. Correct application of a formula without the formula being quoted obviously earns the M mark and in some cases an M mark can be implied from a correct answer.

A Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. Accuracy marks cannot be given unless the associated method mark is earned (or implied).

B Mark for a correct result or statement independent of method marks.

- When a part of a question has two or more "method" steps, the M marks are generally independent unless the scheme specifically says otherwise; and similarly when there are several B marks allocated. The notation DM or DB (or dep*) is used to indicate that a particular M or B mark is dependent on an earlier M or B (asterisked) mark in the scheme. When two or more steps are run together by the candidate, the earlier marks are implied and full credit is given.

- The symbol √ implies that the A or B mark indicated is allowed for work correctly following on from previously incorrect results. Otherwise, A or B marks are given for correct work only. A and B marks are not given for fortuitously "correct" answers or results obtained from incorrect working.

- Note: B2 or A2 means that the candidate can earn 2 or 0.
  2/1/0 means that the candidate can earn anything from 0 to 2.

  The marks indicated in the scheme may not be subdivided. If there is genuine doubt whether a candidate has earned a mark, allow the candidate the benefit of the doubt. Unless otherwise indicated, marks once gained cannot subsequently be lost, e.g. wrong working following a correct form of answer is ignored.

- Wrong or missing units in an answer should not lead to the loss of a mark unless the scheme specifically indicates otherwise.

- For a numerical answer, allow the A or B mark if a value is obtained which is correct to 3 s.f., or which would be correct to 3 s.f. if rounded (1 d.p. in the case of an angle). As stated above, an A or B mark is not given if a correct numerical answer arises fortuitously from incorrect working. For Mechanics questions, allow A or B marks for correct answers which arise from taking *g* equal to 9.8 or 9.81 instead of 10.

The following abbreviations may be used in a mark scheme or used on the scripts:

| | |
|---|---|
| AEF | Any Equivalent Form (of answer is equally acceptable) |
| AG | Answer Given on the question paper (so extra checking is needed to ensure that the detailed working leading to the result is valid) |
| BOD | Benefit of Doubt (allowed when the validity of a solution may not be absolutely clear) |
| CAO | Correct Answer Only (emphasising that no "follow through" from a previous error is allowed) |
| CWO | Correct Working Only – often written by a ‘fortuitous' answer |
| ISW | Ignore Subsequent Working |
| MR | Misread |
| PA | Premature Approximation (resulting in basically correct work that is insufficiently accurate) |
| SOS | See Other Solution (the candidate makes a better attempt at the same question) |
| SR | Special Ruling (detailing the mark to be given for a specific wrong solution, or a case where some standard marking practice is to be varied in the light of a particular circumstance) |

**Penalties**

| | |
|---|---|
| MR -1 | A penalty of MR -1 is deducted from A or B marks when the data of a question or part question are genuinely misread and the object and difficulty of the question remain unaltered. In this case all A and B marks then become "follow through √" marks. MR is not applied when the candidate misreads his own figures – this is regarded as an error in accuracy. An MR-2 penalty may be applied in particular cases if agreed at the coordination meeting. |
| PA -1 | This is deducted from A or B marks in the case of premature approximation. The PA -1 penalty is usually discussed at the meeting. |

**November 2004**

GCE AS AND A LEVEL

| |
|---|
| MARK SCHEME |
| MAXIMUM MARK: 75 |
| **SYLLABUS/COMPONENT: 9709/03, 8719/03**<br><br>**MATHEMATICS AND HIGHER MATHEMATICS**<br>**PAPER 3** |

UNIVERSITY of CAMBRIDGE
International Examinations

| | | | | |
|---|---|---|---|---|
| 1 | *EITHER*: Obtain correct unsimplified version of the $x$ or $x^2$ term in the expansion of $(2+x)^{-3}$ or $\left(1+\frac{1}{2}x\right)^{-3}$ | M1 | |
| | State correct first term $\frac{1}{8}$ | B1 | |
| | Obtain next two terms $-\frac{3}{16}x+\frac{3}{16}x^2$ | A1 + A1 | |
| | [The M mark is not earned by versions with unexpanded binomial coefficients such as $\begin{pmatrix}-3\\1\end{pmatrix}$.] | | |
| | [Accept exact decimal equivalents of fractions.] | | |
| | [SR: Answers given as $\frac{1}{8}\left(1-\frac{3}{2}x+\frac{3}{2}x^2\right)$ can earn M1B1A1.] | | |
| | [SR: Solutions involving $k\left(1+\frac{1}{2}x\right)^{-3}$, where $k$ = 2, 8 or $\frac{1}{2}$, can earn M1 and A1√ for correctly simplifying both the terms in $x$ and $x^2$.] | | |
| | *OR*: Differentiate expression and evaluate f(0) and f′(0), where $f'(x) = k(2 + x)^{-4}$ | M1 | |
| | State correct first term $\frac{1}{8}$ | B1 | |
| | Obtain next two terms $-\frac{3}{16}x+\frac{3}{16}x^2$ | A1 + A1 | 4 |
| | [Accept exact decimal equivalents of fractions.] | | |
| 2 | Use law for subtraction or addition of logarithms, or the equivalent in exponentials | M1 | |
| | Use ln e = 1 or e = exp(1) | M1 | |
| | Obtain a correct equation free of logarithms e.g. $\frac{1+x}{x}=e$ or $1 + x = ex$ | A1 | |
| | Obtain answer $x$ = 0.58 (allow 0.582 or answer rounding to it) | A1 | 4 |
| 3 (i) | Substitute 2 for $x$ and equate to zero, or divide by $x$ – 2 and equate remainder to zero | M1 | |
| | Obtain answer $a = -3$ | A1 | 2 |
| (ii) | Attempt to find quadratic factor by division or inspection | M1 | |
| | State quadratic factor $2x^2 + x + 2$ | A1 | 2 |
| | [The M1 is earned if division reaches a partial quotient of $2x^2 + kx$, or if inspection has an unknown factor of $2x^2 + bx + c$ and an equation in $b$ and/or $c$, or if two coefficients with the correct moduli are stated without working.] | | |
| (iii) | State answer $x > 2$ (and nothing else) | B1* | |
| | Make a correct justification e.g. $2x^2 + x + 2$ (has no zeros) and is always positive | B1(dep*) | 2 |
| | [SR: The answer $x \geq 2$ gets B0, but in this case allow the second B mark if the remaining work is correct.] | | |

| | | | | |
|---|---|---|---|---|
| **4 (i)** | *EITHER*: Use $\tan(A \pm B)$ formula correctly to obtain an equation in $\tan x$ | M1 | |
| | State or imply the equation $\dfrac{1+\tan x}{1-\tan x} = \dfrac{2(1-\tan x)}{1+\tan x}$ or equivalent | A1 | |
| | Transform to an expanded horizontal quadratic equation in $\tan x$ | M1 | |
| | Obtain given answer correctly | A1 | |
| | *OR*: Use $\sin(A \pm B)$ and $\cos(A \pm B)$ formulae correctly to obtain an equation in $\sin x$ and $\cos x$ | M1 | |
| | Using values of sin 45° and cos 45°, or their equality, obtain an expanded horizontal equation in $\sin x$ and $\cos x$ | A1 | |
| | Transform to a quadratic equation in $\tan x$ | M1 | |
| | Obtain given answer correctly | A1 | **4** |
| **(ii)** | Solve the given quadratic and calculate an angle in degrees or radians | M1 | |
| | Obtain one answer e.g. 80.3° | A1 | |
| | Obtain second answer 9.7° and no others in the range<br>[Ignore answers outside the given range.] | A1 | **3** |
| **5 (i)** | Obtain area of *ONB* in terms of $r$ and $\alpha$ e.g. $\frac{1}{2}r^2 \cos\alpha \sin\alpha$ | B1 | |
| | Equate area of triangle in terms of $r$ and $\alpha$ to $\frac{1}{2}\left(\frac{1}{2}r^2\alpha\right)$ or equivalent | M1 | |
| | Obtain given form, $\sin 2\alpha = \alpha$, correctly<br>[Allow use of *OA* and/or *OB* for $r$.] | A1 | **3** |
| **(ii)** | Make recognisable sketch in one diagram over the given range of two suitable graphs, e.g. $y = \sin 2x$ and $y = x$ | B1 | |
| | State or imply link between intersections and roots and justify the given answer<br>[Allow a single graph and its intersection with $y = 0$ to earn full marks.] | B1 | **2** |
| **(iii)** | Use the iterative formula correctly at least once | M1 | |
| | Obtain final answer 0.95 | A1 | |
| | Show sufficient iterations to justify its accuracy to 2d.p., or show there is a sign change in (0.945, 0.955)<br>[SR: Allow the M mark if calculations are attempted in degree mode.] | A1 | **3** |
| **6 (i)** | State $u - v$ is $-3 + \mathrm{i}$ | B1 | |
| | *EITHER*: Carry out multiplication of numerator and denominator of $u/v$ by $4 - 2\mathrm{i}$, or equivalent | M1 | |
| | Obtain answer $\frac{1}{2} + \frac{1}{2}\mathrm{i}$, or any equivalent | A1 | |
| | *OR*: Obtain two equations in $x$ and $y$, and solve for $x$ or for $y$ | M1 | |
| | Obtain answer $\frac{1}{2} + \frac{1}{2}\mathrm{i}$, or any equivalent | A1 | **3** |
| **(ii)** | State argument is $\frac{1}{4}\pi$ (or 0.785 radians or 45°) | A1√ | **1** |
| **(iii)** | State that *OC* and *BA* are equal (in length) | B1 | |
| | State that *OC* and *BA* are parallel or have the same direction | B1 | **2** |

| | | | | |
|---|---|---|---|---|
| **(iv)** | *EITHER*: | Use fact that angle $AOB$ = arg $u$ – arg $v$ = arg($u/v$) | M1 | |
| | | Obtain given answer (or 45°) | A1 | |
| | *OR*: | Obtain tan $AOB$ from gradients of $OA$ and $OB$ and the tan($A \pm B$) formula | M1 | |
| | | Obtain given answer (or 45°) | A1 | |
| | *OR*: | Obtain cos $AOB$ by using the cosine rule or a scalar product | M1 | |
| | | Obtain given answer (or 45°) | A1 | |
| | *OR*: | Prove angle $OAB$ = 90° and $OA$ = $AB$ | M1 | |
| | | Derive the given answer (or 45°) | A1 | **2** |
| | [SR: Obtaining a value for angle $AOB$ by calculating $\arctan(3) - \arctan\left(\frac{1}{2}\right)$ earns a maximum of B1.] | | | |
| **7 (i)** | Use product or quotient rule | | M1* | |
| | Obtain first derivative $2xe^{-\frac{1}{2}x} - \frac{1}{2}x^2e^{-\frac{1}{2}x}$ or equivalent | | A1 | |
| | Equate derivative to zero and solve for non-zero $x$ | | M1(dep*) | |
| | Obtain answer $x$ = 4 | | A1 | **4** |
| **(ii)** | Integrate by parts once, obtaining $kx^2e^{-\frac{1}{2}x} + l\int xe^{-\frac{1}{2}x}\,dx$, where $kl \neq 0$ | | M1 | |
| | Obtain integral $-2x^2e^{-\frac{1}{2}x} + 4\int xe^{-\frac{1}{2}x}\,dx$, or any unsimplified equivalent | | A1 | |
| | Complete the integration, obtaining $-2(x^2 + 4x + 8)e^{-\frac{1}{2}x}$ or equivalent | | A1 | |
| | Having integrated by parts twice, use limits $x$ = 0 and $x$ = 1 in the complete integral | | M1 | |
| | Obtain simplified answer $16 - 26e^{-\frac{1}{2}}$ or equivalent | | A1 | **5** |
| **8 (a)(i)** | State answer $\frac{A}{x+4} + \frac{Bx+C}{x^2+3}$ | | B1 | **1** |
| **(ii)** | State answer $\frac{A}{x-2} + \frac{Bx+C}{(x+2)^2}$ or $\frac{A}{x-2} + \frac{B}{x+2} + \frac{C}{(x+2)^2}$ | | B2 | **2** |
| | [Award B1 if the $B$ term is omitted or for the form $\frac{A}{x-2} + \frac{B}{x+2} + \frac{Cx+D}{(x+2)^2}$.] | | | |
| **(b)** | Stating or implying $f(x) \equiv \frac{A}{x+1} + \frac{B}{x-2}$, use a relevant method to determine $A$ or $B$ | | M1 | |
| | Obtain $A$ = 1 and $B$ = 2 | | A1 | |
| | [SR: If $A$ = 1 and $B$ = 2 stated without working, award B1 + B1.] | | | |
| | Integrate and obtain terms ln ($x$ + 1) + 2 ln ($x$ – 2) | | A1√ + A1√ | |
| | Use correct limits correctly in the complete integral | | M1 | |
| | Obtain given answer ln 5 following full and exact working | | A1 | **6** |

| | | | | |
|---|---|---|---|---|
| **9 (i)** | *EITHER*: | Express general point of $l$ or $m$ in component form e.g. $(2 + s, -1 + s, 4 - s)$ or $(-2 - 2t, 2 + t, 1 + t)$ | B1 | |
| | | Equate at least two pairs of components and solve for $s$ or for $t$ | M1 | |
| | | Obtain correct answer for $s$ or $t$ (possible answers are $\frac{2}{3}$, 10, or 3 for $s$ and $-\frac{7}{3}$, $-7$, or 0 for $t$) | A1 | |
| | | Verify that all three component equations are not satisfied | A1 | |
| | *OR*: | State a Cartesian equation for $l$ or for $m$, e.g. $\frac{x-2}{1} = \frac{y-(-1)}{1} = \frac{z-4}{-1}$ for $l$ | B1 | |
| | | Solve a pair of equations for a pair of values, e.g. $x$ and $y$ | M1 | |
| | | Obtain a pair of correct answers, e.g. $x = \frac{8}{3}$ and $y = -\frac{1}{3}$ | A1 | |
| | | Find corresponding remaining values, e.g. of $z$, and show lines do not intersect | A1 | |
| | *OR*: | Form a relevant triple scalar product, e.g. $(4\mathbf{i} - 3\mathbf{j} + 3\mathbf{k}).((\mathbf{i} + \mathbf{j} - \mathbf{k})\times(-2\mathbf{i} + \mathbf{j} + \mathbf{k}))$ | B1 | |
| | | Attempt to use correct method of evaluation | M1 | |
| | | Obtain at least two correct simplified terms of the three terms of the complete expansion of the triple product or of the corresponding determinant | A1 | |
| | | Obtain correct non-zero value, e.g.14, and state that the lines cannot intersect | A1 | **4** |
| **(ii)** | *EITHER*: | Express $\overrightarrow{PQ}$ or $\left(\overrightarrow{QP}\right)$ in terms of $s$ in any correct form e.g. $-s\mathbf{i} + (1 - s)\mathbf{j} + (-5 + s)\mathbf{k}$ | B1 | |
| | | Equate its scalar product with a direction vector for $l$ to zero, obtaining a linear equation in $s$ | M1 | |
| | | Solve for $s$ | M1 | |
| | | Obtain $s = 2$ and $\overrightarrow{OP}$ is $4\mathbf{i} + \mathbf{j} + 2\mathbf{k}$ | A1 | |
| | *OR*: | Take a point $A$ on $l$, e.g. $(2, -1, 4)$, and use scalar product to calculate $AP$, the length of the projection of $AQ$ onto $l$ | M1 | |
| | | Obtain answer $AP = 2\sqrt{3}$, or equivalent | A1 | |
| | | Carry out method for finding $\overrightarrow{OP}$ | M1 | |
| | | Obtain answer $4\mathbf{i} + \mathbf{j} + 2\mathbf{k}$ | A1 | **4** |
| **(iii)** | Show that $Q$ is the point on $m$ with parameter $t = -2$, or that $(2, 0, -1)$ satisfies the Cartesian equation of $m$ | | B1 | |
| | Show that $PQ$ is perpendicular to $m$ e.g. by verifying fully that $(-2\mathbf{i} - \mathbf{j} - 3\mathbf{k}).(-2\mathbf{i} + \mathbf{j} + \mathbf{k}) = 0$ | | B1 | **2** |

| | | | |
|---|---|---|---|
| **10 (i)** | State or imply $\frac{dV}{dt} = 1000\frac{dh}{dt}$ | B1 | |
| | State or imply $\frac{dV}{dt} = 30 - k\sqrt{h}$ or $\frac{dh}{dt} = 0.03 - m\sqrt{h}$ | B1 | |
| | Show that $k$ = 10 or $m$ = 0.01 and justify the given equation<br>[Allow the first B1 for the statement that 0.03 = 30/1000.] | B1 | **3** |
| **(ii)** | Separate variables and attempt integration of $\frac{x-3}{x}$ with respect to $x$ | M1* | |
| | Obtain $x - 3 \ln x$, or equivalent | A1 | |
| | Obtain $0.005t$, or equivalent | A1 | |
| | Use $x = 3$, $t = 0$ in the evaluation of a constant or as limits in an answer involving ln $x$ and $kt$ | M1(dep*) | |
| | Obtain answer in any correct form e.g. $t = 200(x - 3 - 3 \ln x + 3 \ln 3)$<br>[To qualify for the first M mark, an attempt to solve the earlier differential equation in $h$ and $t$ must involve correct separation of variables, the use of a substitution such as $\sqrt{h} = u$, and an attempt to integrate the resulting function of $u$.] | A1 | **5** |
| **(iii)** | Substitute $x$ = 1 and calculate $t$ | M1 | |
| | Obtain answer $t$ = 259 correctly | A1 | **2** |