Cambridge International A Level

Cambridge Assessment International Education
Cambridge International Advanced Level

MATHEMATICS **9709/33**

Paper 3 **May/June 2018**

MARK SCHEME

Maximum Mark: 75

Published

This mark scheme is published as an aid to teachers and candidates, to indicate the requirements of the examination. It shows the basis on which Examiners were instructed to award marks. It does not indicate the details of the discussions that took place at an Examiners' meeting before marking began, which would have considered the acceptability of alternative answers.

Mark schemes should be read in conjunction with the question paper and the Principal Examiner Report for Teachers.

Cambridge International will not enter into discussions about these mark schemes.

Cambridge International is publishing the mark schemes for the May/June 2018 series for most Cambridge IGCSE™, Cambridge International A and AS Level and Cambridge Pre-U components, and some Cambridge O Level components.

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Generic Marking Principles

These general marking principles must be applied by all examiners when marking candidate answers. They should be applied alongside the specific content of the mark scheme or generic level descriptors for a question. Each question paper and mark scheme will also comply with these marking principles.

GENERIC MARKING PRINCIPLE 1: Marks must be awarded in line with: • the specific content of the mark scheme or the generic level descriptors for the question • the specific skills defined in the mark scheme or in the generic level descriptors for the question • the standard of response required by a candidate as exemplified by the standardisation scripts.
GENERIC MARKING PRINCIPLE 2: Marks awarded are always **whole marks** (not half marks, or other fractions).
GENERIC MARKING PRINCIPLE 3: Marks must be awarded **positively**: • marks are awarded for correct/valid answers, as defined in the mark scheme. However, credit is given for valid answers which go beyond the scope of the syllabus and mark scheme, referring to your Team Leader as appropriate • marks are awarded when candidates clearly demonstrate what they know and can do • marks are not deducted for errors • marks are not deducted for omissions • answers should only be judged on the quality of spelling, punctuation and grammar when these features are specifically assessed by the question as indicated by the mark scheme. The meaning, however, should be unambiguous.
GENERIC MARKING PRINCIPLE 4: Rules must be applied consistently e.g. in situations where candidates have not followed instructions or in the application of generic level descriptors.
GENERIC MARKING PRINCIPLE 5: Marks should be awarded using the full range of marks defined in the mark scheme for the question (however; the use of the full mark range may be limited according to the quality of the candidate responses seen).
GENERIC MARKING PRINCIPLE 6: Marks awarded are based solely on the requirements as defined in the mark scheme. Marks should not be awarded with grade thresholds or grade descriptors in mind.

Mark Scheme Notes

Marks are of the following three types:

M Method mark, awarded for a valid method applied to the problem. Method marks are not lost for numerical errors, algebraic slips or errors in units. However, it is not usually sufficient for a candidate just to indicate an intention of using some method or just to quote a formula; the formula or idea must be applied to the specific problem in hand, e.g. by substituting the relevant quantities into the formula. Correct application of a formula without the formula being quoted obviously earns the M mark and in some cases an M mark can be implied from a correct answer.

A Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. Accuracy marks cannot be given unless the associated method mark is earned (or implied).

B Mark for a correct result or statement independent of method marks.

- When a part of a question has two or more 'method' steps, the M marks are generally independent unless the scheme specifically says otherwise; and similarly when there are several B marks allocated. The notation DM or DB (or dep*) is used to indicate that a particular M or B mark is dependent on an earlier M or B (asterisked) mark in the scheme. When two or more steps are run together by the candidate, the earlier marks are implied and full credit is given.

- The symbol FT implies that the A or B mark indicated is allowed for work correctly following on from previously incorrect results. Otherwise, A or B marks are given for correct work only. A and B marks are not given for fortuitously 'correct' answers or results obtained from incorrect working.

  - Note: B2 or A2 means that the candidate can earn 2 or 0.
    B2/1/0 means that the candidate can earn anything from 0 to 2.

  The marks indicated in the scheme may not be subdivided. If there is genuine doubt whether a candidate has earned a mark, allow the candidate the benefit of the doubt. Unless otherwise indicated, marks once gained cannot subsequently be lost, e.g. wrong working following a correct form of answer is ignored.

- Wrong or missing units in an answer should not lead to the loss of a mark unless the scheme specifically indicates otherwise.

- For a numerical answer, allow the A or B mark if a value is obtained which is correct to 3 s.f., or which would be correct to 3 s.f. if rounded (1 d.p. in the case of an angle). As stated above, an A or B mark is not given if a correct numerical answer arises fortuitously from incorrect working. For Mechanics questions, allow A or B marks for correct answers which arise from taking g equal to 9.8 or 9.81 instead of 10.

The following abbreviations may be used in a mark scheme or used on the scripts:

AEF/OE	Any Equivalent Form (of answer is equally acceptable) / Or Equivalent
AG	Answer Given on the question paper (so extra checking is needed to ensure that the detailed working leading to the result is valid)
CAO	Correct Answer Only (emphasising that no 'follow through' from a previous error is allowed)
CWO	Correct Working Only – often written by a 'fortuitous' answer
ISW	Ignore Subsequent Working
SOI	Seen or implied
SR	Special Ruling (detailing the mark to be given for a specific wrong solution, or a case where some standard marking practice is to be varied in the light of a particular circumstance)

<u>Penalties</u>

MR –1	A penalty of MR –1 is deducted from A or B marks when the data of a question or part question are genuinely misread and the object and difficulty of the question remain unaltered. In this case all A and B marks then become 'follow through' marks. MR is not applied when the candidate misreads his own figures – this is regarded as an error in accuracy. An MR –2 penalty may be applied in particular cases if agreed at the coordination meeting.
PA –1	This is deducted from A or B marks in the case of premature approximation. The PA –1 penalty is usually discussed at the meeting.

Question	Answer	Marks
1	Obtain a correct unsimplified version of the x or x^2 term of the expansion of $(4-3x)^{-\frac{1}{2}}$ or $\left(1-\frac{3}{4}x\right)^{-\frac{1}{2}}$	**M1**
	State correct first term 2	**B1**
	Obtain the next two terms $\frac{3}{4}x+\frac{27}{64}x^2$	**A1 + A1**
	Total:	**4**

Question	Answer	Marks
2	State or imply $u^2 = u + 5$, or equivalent in 5^x	**B1**
	Solve for u, or 5^x	**M1**
	Obtain root $\frac{1}{2}(1+\sqrt{21})$, or decimal in [2.79, 2.80]	**A1**
	Use correct method for finding x from a positive root	**M1**
	Obtain answer x = 0.638 and no other answer	**A1**
	Total:	**5**

Question	Answer	Marks
3	Integrate by parts and reach $ax\sin 3x + b\int \sin 3x\,dx$	**M1***
	Obtain $\frac{1}{3}x\sin 3x - \frac{1}{3}\int \sin 3x\,dx$, or equivalent	**A1**
	Complete the integration and obtain $\frac{1}{3}x\sin 3x + \frac{1}{9}\cos 3x$, or equivalent	**A1**
	Substitute limits correctly having integrated twice and obtained $ax \sin 3x + b \cos 3x$	**M1(dep*)**
	Obtain answer $\frac{1}{18}(\pi - 2)$ OE	**A1**
	Total:	**5**

Question	Answer	Marks
4(i)	Use the quotient or product rule	**M1**
	Obtain correct derivative in any form	**A1**
	Equate derivative to zero and obtain the given equation	**A1**
	Total:	**3**
4(ii)	Sketch a relevant graph, e.g. $y = \ln x$	**B1**
	Sketch a second relevant graph, e.g. $y = 1 + \dfrac{3}{x}$, and justify the given statement	**B1**
	Total:	**2**
4(iii)	Use iterative formula $x_{n+1} = \dfrac{3 + x}{\ln x_n}$ correctly at least once	**M1**
	Obtain final answer 4.97	**A1**
	Show sufficient iterations to 4 d.p.to justify 4.97 to 2 d.p. or show there is a sign change in the interval (4.965, 4.975)	**A1**
	Total:	**3**

Question	Answer	Marks
5(i)	Attempt cubic expansion and equate to 1	**M1**
	Obtain a correct equation	**A1**
	Use Pythagoras and double angle formula in the expansion	**M1**
	Obtain the given result correctly	**A1**
	Total:	**4**
5(ii)	Use the identity and carry out a method for finding a root	**M1**
	Obtain answer 20.9°	**A1**
	Obtain a second answer, e.g. 69.1°	**A1FT**
	Obtain the remaining answers, e.g. 110.9° and 159.1°, and no others in the given interval	**A1FT**
	Total:	**4**

Question	Answer	Marks
6(i)	Carry out relevant method to find A and B such that $\frac{1}{4-y^2} \equiv \frac{A}{2+y} + \frac{B}{2-y}$	M1
	Obtain $A = B = \frac{1}{4}$	A1
	Total:	**2**
6(ii)	Separate variables correctly and integrate at least one side to obtain one of the terms $a \ln x$, $b \ln (2 + y)$ or $c \ln (2 - y)$	M1
	Obtain term $\ln x$	B1
	Integrate and obtain terms $\frac{1}{4}\ln(2+y) - \frac{1}{4}\ln(2-y)$	A1FT
	Use $x = 1$ and $y = 1$ to evaluate a constant, or as limits, in a solution containing at least two terms of the form $a \ln x$, $b \ln (2 + y)$ and $c \ln (2 - y)$	M1
	Obtain a correct solution in any form, e.g. $\ln x = \frac{1}{4}\ln(2+y) - \frac{1}{4}\ln(2-y) - \frac{1}{4}\ln 3$	A1
	Rearrange as $\frac{2(3x^4-1)}{(3x^4+1)}$, or equivalent	A1
	Total:	**6**

Question	Answer	Marks
7(i)	State answer $R = \sqrt{5}$	B1
	Use trig formulae to find $\tan \alpha$	M1
	Obtain $\tan\alpha = 2$	A1
	Total:	**3**
7(ii)	State that the integrand is $3\sec^2(\theta - \alpha)$	B1FT
	State correct indefinite integral $3\tan(\theta - \alpha)$	B1FT
	Substitute limits correctly	M1
	Use $\tan(A \pm B)$ formula	M1
	Obtain the given exact answer correctly	A1
	Total:	**5**

Question	Answer	Marks
8(i)	State or imply $3y^2 \frac{dy}{dx}$ as derivative of y^3	**B1**
	State or imply $3y^2 + 6xy \frac{dy}{dx}$ as derivative of $3xy^2$	**B1**
	Equate derivative of LHS to zero and solve for $\frac{dy}{dx}$	**M1**
	Obtain the given answer	**A1**
	Total:	**4**
8(ii)	Equate denominator to zero and solve for *y*	**M1***
	Obtain $y = 0$ and $x = a$	**A1**
	Obtain $y = \alpha x$ and substitute in curve equation to find *x* or to find *y*	**M1(dep*)**
	Obtain $x = -a$	**A1**
	Obtain $y = 2a$	**A1**
	Total:	**5**

Question	Answer	Marks
9(a)	Substitute and obtain a correct equation in *x* and *y*	**B1**
	Use $i^2 = -1$ and equate real and imaginary parts	**M1**
	Obtain two correct equations in *x* and *y*, e.g. $3x - y = 1$ and $3y - x = 5$	**A1**
	Solve and obtain answer $z = 1 +$ **2(i)**	**A1**
	Total:	**4**
9(b)	Show a circle with radius 3	**B1**
	Show the line $y = 2$ extending in both quadrants	**B1**
	Shade the correct region	**B1**
	Carry out a complete method for finding the greatest value of arg *z*	**M1**
	Obtain answer 2.41	**A1**
	Total:	**5**

Question	Answer	Marks
10(i)	Carry out a correct method for finding a vector equation for *AB*	**M1**
	Obtain $\mathbf{r} = 2\mathbf{i} + \mathbf{j} + 3\mathbf{k} + \lambda(2\mathbf{i} - 2\mathbf{k})$, or equivalent	**A1**
	Equate pair(s) of components *AB* and *l* and solve for λ or μ	**M1(dep*)**
	Obtain correct answer for λ or μ	**A1**
	Verify that all three component equations are not satisfied	**A1**
	Total:	**5**
10(ii)	State or imply a direction vector for *AP* has components $(2 + t, 5 + 2t, -3 - 2t)$	**B1**
	State or imply that cos 120° equals the scalar product of $\overrightarrow{AP}$ and $\overrightarrow{AB}$ divided by the product of their moduli	**M1**
	Carry out the correct processes for finding the scalar product and the product of the moduli in terms of *t*, and obtain an equation in terms of *t*	**M1**
	Obtain the given equation correctly	**A1**
	Solve the quadratic and use a root to find a position vector for *P*	**M1**
	Obtain position vector $2\mathbf{i} + 2\mathbf{j} + 4\mathbf{k}$ from $t = -2$, having rejected the root $t = -\frac{2}{3}$	**A1**
	Total:	**6**