June 2004

GCE A AND AS LEVEL

MARK SCHEME

MAXIMUM MARK: 75

SYLLABUS/COMPONENT: 9709/03, 8719/03

MATHEMATICS AND HIGHER MATHEMATICS
Paper 3 (Pure 3)

UNIVERSITY of CAMBRIDGE
International Examinations

1	Show correct sketch for $0 \le x < \frac{1}{2}\pi$		B1	
	Show correct sketch for $\frac{1}{2}\pi < x < \frac{3}{2}\pi$ or $\frac{3}{2}\pi < x \le 2\pi$		B1	
	Show completely correct sketch		B1	**3**
	[SR: for a graph with y = 0 when x = 0, π, 2π but otherwise of correct shape, award B1.]			

2	*EITHER*:	State or imply non-modular inequality $(2x+1)^2 < x^2$ or corresponding quadratic equation or pair of linear equations $(2x + 1) = \pm x$	B1	
		Expand and make a reasonable solution attempt at a 3-term quadratic, or solve two linear equations	M1	
		Obtain critical values $x = -1$ and $x = -\frac{1}{3}$ only	A1	
		State answer $-1 < x < -\frac{1}{3}$	A1	
	OR:	Obtain the critical value $x = -1$ from a graphical method , or by inspection, or by solving a linear inequality or equation	B1	
		Obtain the critical value $x = -\frac{1}{3}$ (deduct B1 from B3 if extra values are obtained)	B2	
		State answer $-1 < x < -\frac{1}{3}$	B1	**4**
		[Condone $\le$ for <; accept –0.33 for $-\frac{1}{3}$.]		

3	*EITHER*:	State $6y\frac{dy}{dx}$ as the derivative of $3y^2$	B1	
		State $\pm 4x\frac{dy}{dx} \pm 4y$ as the derivative of $-4xy$	B1	
		Equate attempted derivative of LHS to zero and solve for $\frac{dy}{dx}$	M1	
		Obtain answer 2	A1	
		[The M1 is conditional on at least one of the B marks being obtained. Allow any combination of signs for the second B1.]		
	OR:	Obtain a correct expression for y in terms of x	B1	
		Differentiate using chain rule	M1	
		Obtain derivative in any correct form	A1	
		Substitute x = 2 and obtain answer 2 only	A1	**4**
		[The M1 is conditional on a reasonable attempt at solving the quadratic in y being made.]		

4	(**i**)	State or imply $2^{-x} = \frac{1}{y}$	B1	
		Obtain 3-term quadratic e.g. $y^2 - y - 1 = 0$	B1	**2**
	(**ii**)	Solve a 3-term quadratic, obtaining 1 or 2 roots	M1	
		Obtain answer $y = (1 + \sqrt{5})/2$, or equivalent	A1	
		Carry out correct method for solving an equation of the form $2^x = a$, where *a* > 0, reaching a ratio of logarithms	M1	
		Obtain answer *x* = 0.694 only	A1	**4**
5	(**i**)	Make relevant use of formula for sin 2θ or cos 2θ	M1	
		Make relevant use of formula for cos 4θ	M1	
		Complete proof of the given result	A1	**3**
	(**ii**)	Integrate and obtain $\frac{1}{8}(\theta - \frac{1}{4}\sin 4\theta)$ or equivalent	B1	
		Use limits correctly with an integral of the form $a\theta + b\sin 4\theta$, where $ab \neq 0$	M1	
		Obtain answer $\frac{1}{8}(\frac{1}{3}\pi + \frac{\sqrt{3}}{8})$, or exact equivalent	A1	**3**
6		Separate variables and attempt to integrate	M1	
		Obtain terms $\frac{1}{3}\ln(y^3+1)$ and *x*, or equivalent	A1 + A1	
		Evaluate a constant or use limits *x* = 0, *y* = 1 with a solution containing terms $k\ln(y^3+1)$ and *x*, or equivalent	M1	
		Obtain any correct form of solution e.g. $\frac{1}{3}\ln(y^3+1) = x + \frac{1}{3}\ln 2$	A1√	
		Rearrange and obtain $y = (2e^{3x} - 1)^{\frac{1}{3}}$, or equivalent	A1	**6**
		[f.t. is on $k \neq 0$.]		
7	(**i**)	Evaluate cubic when *x* = –1 and *x* = 0	M1	
		Justify given statement correctly	A1	**2**
		[If calculations are not given but justification uses correct statements about signs, award B1.]		
	(**ii**)	State $x = \frac{2x^3 - 1}{3x^2 + 1}$, or equivalent	B1	
		Rearrange this in the form $x^3 + x + 1 = 0$ (or *vice versa*)	B1	**2**

	(iii) Use the iterative formula correctly at least once	M1	
	Obtain final answer –0.68	A1	
	Show sufficient iterations to justify its accuracy to 2d.p., or show there is a sign change in the interval (–0.685, –0.675)	A1	**3**
8	**(i)** *EITHER*: Solve the quadratic and use $\sqrt{-1} = \mathrm{i}$	M1	
	Obtain roots $\frac{1}{2} + \mathrm{i}\frac{\sqrt{3}}{2}$ and $\frac{1}{2} - \mathrm{i}\frac{\sqrt{3}}{2}$ or equivalent	A1	
	OR: Substitute *x* + i*y* and solve for *x* or *y*	M1	
	Obtain correct roots	A1	**2**
	(ii) State that the modulus of each root is equal to 1	B1√	
	State that the arguments are $\frac{1}{3}\pi$ and $-\frac{1}{3}\pi$ respectively	B1√ + B1√	**3**
	[Accept degrees and $\frac{5}{3}\pi$ instead of $-\frac{1}{3}\pi$. Accept a modulus in the form $\sqrt{\frac{p}{q}}$ or $\sqrt{n}$, where *p*, *q*, *n* are integers. An answer which only gives roots in modulus-argument form earns B1 for both the implied moduli and B1 for both the implied arguments.]		
	(iii) *EITHER*: Verify $z^3 = -1$ for each root	B1 + B1	
	OR: State $z^3 + 1 = (z+1)(z^2 - z + 1)$	B1	
	Justify the given statement	B1	
	OR: Obtain $z^3 = z^2 - z$	B1	
	Justify the given statement	B1	**2**
9	**(i)** State or imply $\mathrm{f}(x) \equiv \frac{A}{x-1} + \frac{B}{x-2} + \frac{C}{x+1}$	B1	
	EITHER: Use any relevant method to obtain a constant	M1	
	Obtain one of the values: *A* = –1, *B* = 4 and *C* = –2	A1	
	Obtain the remaining two values	A1	
	OR: Obtain one value by inspection	B1	
	State a second value	B1	
	State the third value	B1	**4**
	[Apply the same scheme to the form $\frac{A}{x-2} + \frac{Bx+C}{x^2-1}$ which has *A* = 4, *B* = –3 and *C* = 1.]		

	(ii)	Use correct method to obtain the first two terms of the expansion of $(x-1)^{-1}$ or $(x-2)^{-1}$ or $(x+1)^{-1}$	M1	
		Obtain any correct unsimplified expansion of the partial fractions up to the terms in x^3 (deduct A1 for each incorrect expansion)	A1√ + A1√ + A1√	
		Obtain the given answer correctly	A1	5

[Binomial coefficients involving –1, e.g. $\binom{-1}{1}$, are not sufficient for the M1 mark. The f.t. is on *A*, *B*, *C*.]

[Apply a similar scheme to the alternative form of fractions in (**i**), awarding M1*A1√A1√ for the expansions, M1(dep*) for multiplying by *Bx* + *C*, and A1 for obtaining the given answer correctly.]

[In the case of an attempt to expand $(x^2+7x-6)(x-1)^{-1}(x-2)^{-1}(x+1)^{-1}$, give M1A1A1A1 for the expansions and A1 for multiplying out and obtaining the given answer correctly.]

[Allow attempts to multiply out $(x-1)(x-2)(x+1)(-3+2x-\frac{3}{2}x^2+\frac{11}{4}x^3)$, giving B1 for reduction to a product of two expressions correct up to their terms in x^3, M1 for attempting to multiply out at least as far as terms in x^2, A1 for a correct expansion up to terms in x^3, and A1 for correctly obtaining the answer x^2+7x-6 and also showing there is no term in x^3.]

[Allow the use of Maclaurin, giving M1A1√ for f(0) = –3 and f ′(0) = 2, A1√ for f ″(0) = –3, A1√ for f ′″(0) = $\frac{33}{2}$, and A1 for obtaining the given answer correctly (f.t. is on *A*, *B*,*C* if used).]

10	(i)	State *x*-coordinate of *A* is 1	B1	1
	(ii)	Use product or quotient rule	M1	
		Obtain derivative in any correct form e.g. $-\frac{2\ln x}{x^3}+\frac{1}{x}.\frac{1}{x^2}$	A1	
		Equate derivative to zero and solve for ln *x*	M1	
		Obtain *x* = $e^{\frac{1}{2}}$ or equivalent (accept 1.65)	A1	
		Obtain *y* = $\frac{1}{2e}$ or exact equivalent not involving ln	A1	5

[SR: if the quotient rule is misused, with a 'reversed' numerator or x^2 instead of x^4 in the denominator, award M0A0 but allow the following M1A1A1.]

	(iii)	Attempt integration by parts, going the correct way	M1	
		Obtain $-\frac{\ln x}{x}+\int\frac{1}{x}.\frac{1}{x}\,dx$ or equivalent	A1	
		Obtain indefinite integral $-\frac{\ln x}{x}-\frac{1}{x}$	A1	
		Use *x*-coordinate of *A* and e as limits, having integrated twice	M1	
		Obtain exact answer $1-\frac{2}{e}$, or equivalent	A1	5

[If *u* = ln *x* is used, apply an analogous scheme to the result of the substitution.]

11 (**i**)	*EITHER*:	Obtain a vector in the plane e.g. $\overrightarrow{PQ} = -3\mathbf{i} + 4\mathbf{j} + \mathbf{k}$	B1	
		Use scalar product to obtain a relevant equation in a, b, c e.g. $-3a + 4b + c = 0$ or $6a - 2b + c = 0$ or $3a + 2b + 2c = 0$	M1	
		State two correct equations in a, b, c	A1	
		Solve simultaneous equations to obtain one ratio e.g. $a : b$	M1	
		Obtain $a : b : c = 2 : 3 : -6$ or equivalent	A1	
		Obtain equation $2x + 3y - 6z = 8$ or equivalent	A1	
		[The second M1 is also given if say c is given an arbitrary value and a or b is found. The following A1 is then given for finding the correct values of a and b.]		
	OR:	Substitute for P, Q, R in equation of plane and state 3 equations in a, b, c, d	B1	
		Eliminate one unknown, e.g. d, entirely	M1	
		Obtain 2 equations in 3 unknowns	A1	
		Solve to obtain one ratio e.g. $a : b$	M1	
		Obtain $a : b : c = 2 : 3 : -6$ or equivalent	A1	
		Obtain equation $2x + 3y - 6z = 8$ or equivalent	A1	
		[The first M1 is also given if say d is given an arbitrary value and two equations in two unknowns, e.g. a and b, are obtained. The following A1 is for two correct equations. Solving to obtain one unknown earns the second M1 and the following A1 is for finding the correct values of a and b.]		
	OR:	Obtain a vector in the plane e.g. $\overrightarrow{QR} = 6\mathbf{i} - 2\mathbf{j} + \mathbf{k}$	B1	
		Find a second vector in the plane and form correctly a 2-parameter equation for the plane	M1	
		Obtain equation in any correct form e.g. $\mathbf{r} = \lambda(-3\mathbf{i} + 4\mathbf{j} + \mathbf{k}) + \mu(6\mathbf{i} - 2\mathbf{j} + \mathbf{k}) + \mathbf{i} - \mathbf{k}$	A1	
		State 3 equations in x, y, z, λ, and μ	A1	
		Eliminate λ and μ	M1	
		Obtain equation $2x + 3y - 6z = 8$ or equivalent	A1	
	OR:	Obtain a vector in the plane e.g. $\overrightarrow{PR} = 3\mathbf{i} + 2\mathbf{j} + 2\mathbf{k}$	B1	
		Obtain a second vector in the plane and calculate the vector product of the two vectors, e.g. $(-3\mathbf{i} + 4\mathbf{j} + \mathbf{k})\times(3\mathbf{i} + 2\mathbf{j} + 2\mathbf{k})$	M1	
		Obtain 2 correct components of the product	A1	
		Obtain correct product e.g. $6\mathbf{i} + 9\mathbf{j} - 18\mathbf{k}$ or equivalent	A1	
		Substitute in $2x + 3y - 6z = d$ and find d or equivalent	M1	
		Obtain equation $2x + 3y - 6z = 8$ or equivalent	A1	6

(**ii**) *EITHER*:

State equation of *SN* is $\mathbf{r} = 3\mathbf{i} + 5\mathbf{j} - 6\mathbf{k} + \lambda(2\mathbf{i} + 3\mathbf{j} - 6\mathbf{k})$ or equivalent	B1√	
Express *x*, *y*, *z* in terms of λ e.g. $(3 + 2\lambda, 5 + 3\lambda, -6 - 6\lambda)$	B1√	
Substitute in the equation of the plane and solve for λ	M1	
Obtain $\overrightarrow{ON} = \mathbf{i} + 2\mathbf{j}$, or equivalent	A1	
Carry out method for finding *SN*	M1	
Show that *SN* = 7 correctly	A1	

OR:

Letting $\overrightarrow{ON} = x\mathbf{i} + y\mathbf{j} + z\mathbf{k}$, obtain two equations in *x*, *y*, *z* by equating scalar product of $\overrightarrow{NS}$ with two of $\overrightarrow{PQ}, \overrightarrow{QR}, \overrightarrow{RP}$ to zero	B1√+ B1√	
Using the plane equation as third equation, solve for *x*, *y*, and *z*	M1	
Obtain $\overrightarrow{ON} = \mathbf{i} + 2\mathbf{j}$, or equivalent	A1	
Carry out method for finding *SN*	M1	
Show that *SN* = 7 correctly	A1	

OR:

Use Cartesian formula or scalar product of $\overrightarrow{PS}$ with a normal vector to find *SN*	M1	
Obtain *SN* = 7	A1	
State a unit normal $\hat{\mathbf{n}}$ to the plane	B1√	
Use $\overrightarrow{ON} = \overrightarrow{OS} \pm 7\hat{\mathbf{n}}$	M1	
Obtain an unsimplified expression e.g. $3\mathbf{i} + 5\mathbf{j} - 6\mathbf{k} \pm 7(\frac{2}{7}\mathbf{i} + \frac{3}{7}\mathbf{j} - \frac{6}{7}\mathbf{k})$	A1√	
Obtain $\overrightarrow{ON} = \mathbf{i} + 2\mathbf{j}$, or equivalent, only	A1	**6**