

CAMBRIDGE
INTERNATIONAL EXAMINATIONS

JUNE 2002

**GCE Advanced Level
GCE Advanced Subsidiary Level**

MARK SCHEME

MAXIMUM MARK : 75

SYLLABUS/COMPONENT : 9709 /3, 8719 /3

**MATHEMATICS
(Pure 3)**



UNIVERSITY of CAMBRIDGE
Local Examinations Syndicate

Page 1	Mark Scheme	Syllabus	Paper
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- 1 *EITHER*: Express LHS in terms of $\cos\theta$ and $\sin\theta$ or in terms of $\tan\theta$ M1
 Make sufficient relevant use of double-angle formula(e) M1
 Complete proof of the result A1
OR: Express RHS in terms of $\cos\theta$ and $\sin\theta$ or in terms of $\tan\theta$ M1
 Express RHS as the difference (or sum) of two fractions M1
 Complete proof of the result A1 **3**
- [SR: an attempt ending with $\frac{1 \cdot \tan^2\theta}{\tan\theta} = \cot\theta - \tan\theta$ earns M1 B1 only.]
- 2 *EITHER*: Show correct (unsimplified) version of the x or the x^2 or the x^3 term M1
 Obtain correct first two terms $1 + x$ A1
 Obtain correct quadratic term $2x^2$ A1
 Obtain correct cubic term $\frac{14}{3}x^3$ (allow $\frac{28}{6}$, 4.67, 4.66 for the coefficient) A1
 [The M mark may be implied by correct simplified terms, if no working is shown. It is not earned by unexpanded binomial coefficients involving $-\frac{1}{3}$, e.g. ${}^{-\frac{1}{3}}C_1$ or $\binom{-\frac{1}{3}}{2}$.]
 [An attempt to divide 1 by the expansion of $(1 - 3x)^{\frac{1}{3}}$ earns M1 if the expansion has a correct (unsimplified) x , x^2 , or x^3 term and if the partial quotient contains a term in x . The remaining A marks are awarded as above.]
- OR*: Differentiate and calculate $f(0)$, $f'(0)$, where $f(x) = k(1 - 3x)^{-\frac{1}{3}}$ M1
 Obtain correct first two terms $1 + x$ A1
 Obtain correct quadratic term $2x^2$ A1
 Obtain correct cubic term $\frac{14}{3}x^3$ (allow $\frac{28}{6}$, 4.67, 4.66 for the coefficient) A1 **4**
- 3 Attempt to find a and/or quadratic factor by division or by inspection M1
 Obtain partial quotient or factor $x^2 - x$ A1
 State answer $a = 6$ B1
 State or imply the other factor is $x^2 - x + 3$ A1 **4**
- [The M1 is earned if division has produced a partial quotient $x^2 + bx$, or if inspection has an unknown factor $x^2 + bx + c$ and has reached an equation in b and/or c .]
 [SR: a correct division with unresolved constant remainder can earn M1A1B0A1.]
 [NB: successive division by a pair of incorrect linear factors, e.g. $x - 1$ and $x + 2$ or $x + 1$ and $x + 2$, can earn M1A0 or M1A1 (if their product is of the form $x^2 + x + k$).]

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- 4 (i) Use the formula correctly at least once M1
 State $\alpha = 1.26$ as final answer A1
 Show sufficient iterations to justify $\alpha = 1.26$ to 2d.p., or show there is a sign change in the interval (1.255, 1.265) A1 3
- (ii) State any suitable equation in one unknown e.g. $x = \frac{2}{3} \left(x + \frac{1}{x^2} \right)$ B1
 State exact value of α (or x) is $\sqrt[3]{2}$ or $2^{\frac{1}{3}}$ B1 2
- 5 Obtain derivative $\pm 2\sin x + k \cos 2x$ or $\pm 2\sin x + k(\cos^2 x \pm \sin^2 x)$ M1
 Equate derivative to zero and use trig formula to obtain an equation involving only one trig function M1
 Obtain a correct equation of this type e.g. $2\sin^2 x + \sin x - 1 = 0$ or $\cos 2x = \cos \left(\frac{1}{2} \pi - x \right)$ A1
 Obtain value $x = \frac{1}{6} \pi$ (allow 0.524 radians or 30°) A1
 Show by any method that the corresponding point is a maximum point A1
 Obtain second value $x = \frac{5}{6} \pi$ (allow 2.62 radians or 150°), and no others in range A1 ✓
 Determine that it corresponds to a minimum point A1 7
- 6 (i) State or imply $f(x) = \frac{A}{(3x+1)} + \frac{B}{(x+1)^2} + \frac{C}{(x+1)}$ B1
 State or obtain $A = -3$ B1
 State or obtain $B = 2$ B1
 Use any relevant method to find C M1
 Obtain $C = 1$ A1 5
 [Special case: allow the form $\frac{A}{(3x+1)} + \frac{Dx+E}{(x+1)^2}$ and apply the above scheme ($A = -3, D = 1, E = 3$).]
 {SR: if $f(x)$ is given an incomplete form of partial fractions, give B1 for a form equivalent to the omission of C , or E , or B in the above, and M1 for finding one coefficient.}
- (ii) Integrate and obtain terms $-\ln(3x+1) - \frac{2}{(x+1)} + \ln(x+1)$ B1 + B1 + B1 ✓
 Use limits correctly M1
 Obtain the given answer correctly A1 5

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- 7 (i) State that $\frac{dm}{dt} = k(50 - m)^2$ B1
Justify $k = 0.002$ B1 2
- (ii) Separate variables and attempt to integrate $\frac{1}{(50 - m)^2}$ M1
Obtain $\pm \frac{1}{(50 - m)}$ and $0.002t$, or equivalent A1
Evaluate a constant or use limits $t = 0, m = 0$ M1
Obtain any correct form of solution e.g. $\frac{1}{(50 - m)} = 0.002t + \frac{1}{50}$ A1
Obtain given answer correctly A1 5
- (iii) Obtain answer $m = 25$ when $t = 10$ B1
Obtain answer $t = 90$ when $m = 45$ B1 2
- (iv) State that m approaches 50 B1 1
- 8 (i) State or imply a simplified direction vector of l is $3\mathbf{i} - \mathbf{j} + 2\mathbf{k}$, or equivalent B1
State equation of l is $\mathbf{r} = \mathbf{i} + \mathbf{k} + \lambda(3\mathbf{i} - \mathbf{j} + 2\mathbf{k})$, or $\frac{x-1}{3} = \frac{y}{-1} = \frac{z-1}{2}$, or equivalent B1 ✓
Substitute in equation of p and solve for λ , or one of x, y , or z M1
Obtain point of intersection $-2\mathbf{i} + \mathbf{j} - \mathbf{k}$ A1 4
[Any notation is acceptable.]
- (ii) State or imply a normal vector of p is $\mathbf{i} + 3\mathbf{j} - 2\mathbf{k}$ B1
EITHER: Use scalar product to obtain $a + 3b - 2c = 0$ M1
Use points on l to obtain two equations in a, b, c e.g. $a + c = 1, 4a - b + 3c = 1$ B1 ✓
Solve simultaneous equations, obtaining one unknown M1
Obtain one correct unknown e.g. $a = -\frac{2}{3}$ A1
Obtain the other unknowns e.g. $b = \frac{4}{3}, c = \frac{5}{3}$ A1
- OR:* Use scalar product to obtain $a + 3b - 2c = 0$ M1
Use scalar product to obtain $3a - b + 2c = 0$ B1 ✓
Solve simultaneous equations to obtain one ratio e.g. $a : b$ M1
Obtain $a : b : c = 2 : -4 : -5$, or equivalent A1
Obtain $a = -\frac{2}{3}, b = \frac{4}{3}, c = \frac{5}{3}$ A1
- [NB: candidates may transfer from the *EITHER* to *OR* scheme by subtracting the two "point" equations, or transfer from *OR* to *EITHER* by finding one of the "point" equations.]
- OR:* Calculate the vector product $(3\mathbf{i} - \mathbf{j} + 2\mathbf{k}) \times (\mathbf{i} + 3\mathbf{j} - 2\mathbf{k})$ M1
Obtain answer $-4\mathbf{i} + 8\mathbf{j} + 10\mathbf{k}$, or equivalent A1 ✓
Substitute in $-4x + 8y + 10z = d$ to find d , or equivalent M1
Obtain $d = 6$, or equivalent A1
Obtain $a = -\frac{2}{3}, b = \frac{4}{3}, c = \frac{5}{3}$ A1
- OR:* State or imply a correct equation of the plane e.g. $\mathbf{r} = \lambda(3\mathbf{i} - \mathbf{j} + 2\mathbf{k}) + \mu(\mathbf{i} + 3\mathbf{j} - 2\mathbf{k}) + \mathbf{i} + \mathbf{k}$ M1
State 3 equations in x, y, z, λ , and μ , e.g. $x = 3\lambda + \mu + 1, y = -\lambda + 3\mu, z = 2\lambda - 2\mu + 1$ A1 ✓
Eliminate λ and μ M1
Obtain equation $-4x + 8y + 10z = 6$, or equivalent A1
Obtain $a = -\frac{2}{3}, b = \frac{4}{3}, c = \frac{5}{3}$ A1 6
- [SR: condone the use of $x\mathbf{i} + y\mathbf{j} + z\mathbf{k}$ for $a\mathbf{i} + b\mathbf{j} + c\mathbf{k}$ in the *EITHER* scheme and the first *OR* scheme.]

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- 9 (i) State or imply that $r = 2$ B1
 State or imply that $\theta = \frac{1}{3}\pi$ (allow 1.05 radians or 60°) B1
 Obtain modulus 4, and argument $\frac{2}{3}\pi$ of u^2 (allow 2^2 ; 2.09 or 2.10 radians or 120°) B1 + B1✓
 Obtain modulus 8 and argument π of u^3 (allow 2^3 ; 3.14 or 3.15 radians or 180°) B1✓ 5
 [Follow through on wrong r and θ]
 [SR: if u^2 and u^3 are only given in polar form, give B1✓ for u^2 and B1✓ for u^3 .]
 (ii) EITHER: Deduce that $u^2 - 2u + 4 = 0$ from $u^3 + 8 = 0$
 OR: Verify that $u^2 - 2u + 4 = 0$ by calculation B1
 State that the other root is $1 - i\sqrt{3}$, or equivalent B1 2
 [NB: stating that the roots are $1 \pm i\sqrt{3}$ is sufficient for both B marks.]
 (iii) Show both points correctly on an Argand diagram B1
 Show the correct relevant circle B1
 Show line (segment) correctly B1
 Shade the correct region B1 4
 [SR: allow work on separate diagrams to be eligible for the first three B marks.]
- 10 (i) State at any stage that the x -coordinate of A is equal to 1, or that A is the point (1,0) B1 1
 (ii) State $f'(x) = 2 \frac{\ln x}{x}$, or equivalent B1
 Use product or quotient rule for the next differentiation M1
 Obtain $2 \cdot \frac{1}{x} \cdot \frac{1}{x} + 2 \ln x \cdot \left(\frac{-1}{x^2}\right)$, or any equivalent correct unsimplified form A1
 Verify that $f''(e) = 0$ A1 4
 (iii) State or imply area is $\int_1^e (\ln x)^2 dx$ B1
 Use $\frac{dx}{du} = e^u$, or equivalent, in substituting for x throughout M1
 Obtain given answer correctly (allow change of limits to be done mentally) A1 3
 (iv) Attempt the first integration by parts, going the correct way M1
 Obtain $(u^2 - 2u \pm 2)e^u$, or equivalent, after two applications of the rule A1
 Obtain exact answer in terms of e , in any correct form, e.g. $(e - 2e + 2e) - 2$, or $e - 2$ A1 3
- [The substitution in (iii) may be done in reverse i.e. starting with the u integral and obtaining the x integral. The M1A1 scheme applies, but only an explicit statement will earn the B1.]
 [The M1A1A1 in (iv) applies to those working in terms of x and obtaining $x((\ln x)^2 - 2 \ln x \pm 2)$, or equivalent.]