UNIVERSITY OF CAMBRIDGE INTERNATIONAL EXAMINATIONS

GCE Advanced Subsidiary Level and GCE Advanced Level

MARK SCHEME for the October/November 2009 question paper

for the guidance of teachers

9709 MATHEMATICS

9709/21 Paper 21, maximum raw mark 50

This mark scheme is published as an aid to teachers and candidates, to indicate the requirements of the examination. It shows the basis on which Examiners were instructed to award marks. It does not indicate the details of the discussions that took place at an Examiners' meeting before marking began, which would have considered the acceptability of alternative answers.

Mark schemes must be read in conjunction with the question papers and the report on the examination.

- CIE will not enter into discussions or correspondence in connection with these mark schemes.

CIE is publishing the mark schemes for the October/November 2009 question papers for most IGCSE, GCE Advanced Level and Advanced Subsidiary Level syllabuses and some Ordinary Level syllabuses.

Mark Scheme Notes

Marks are of the following three types:

M Method mark, awarded for a valid method applied to the problem. Method marks are not lost for numerical errors, algebraic slips or errors in units. However, it is not usually sufficient for a candidate just to indicate an intention of using some method or just to quote a formula; the formula or idea must be applied to the specific problem in hand, e.g. by substituting the relevant quantities into the formula. Correct application of a formula without the formula being quoted obviously earns the M mark and in some cases an M mark can be implied from a correct answer.

A Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. Accuracy marks cannot be given unless the associated method mark is earned (or implied).

B Mark for a correct result or statement independent of method marks.

- When a part of a question has two or more "method" steps, the M marks are generally independent unless the scheme specifically says otherwise; and similarly when there are several B marks allocated. The notation DM or DB (or dep*) is used to indicate that a particular M or B mark is dependent on an earlier M or B (asterisked) mark in the scheme. When two or more steps are run together by the candidate, the earlier marks are implied and full credit is given.

- The symbol $\sqrt{}$ implies that the A or B mark indicated is allowed for work correctly following on from previously incorrect results. Otherwise, A or B marks are given for correct work only. A and B marks are not given for fortuitously "correct" answers or results obtained from incorrect working.

- Note: B2 or A2 means that the candidate can earn 2 or 0.
  B2/1/0 means that the candidate can earn anything from 0 to 2.

  The marks indicated in the scheme may not be subdivided. If there is genuine doubt whether a candidate has earned a mark, allow the candidate the benefit of the doubt. Unless otherwise indicated, marks once gained cannot subsequently be lost, e.g. wrong working following a correct form of answer is ignored.

- Wrong or missing units in an answer should not lead to the loss of a mark unless the scheme specifically indicates otherwise.

- For a numerical answer, allow the A or B mark if a value is obtained which is correct to 3 s.f., or which would be correct to 3 s.f. if rounded (1 d.p. in the case of an angle). As stated above, an A or B mark is not given if a correct numerical answer arises fortuitously from incorrect working. For Mechanics questions, allow A or B marks for correct answers which arise from taking *g* equal to 9.8 or 9.81 instead of 10.

The following abbreviations may be used in a mark scheme or used on the scripts:

AEF Any Equivalent Form (of answer is equally acceptable)

AG Answer Given on the question paper (so extra checking is needed to ensure that the detailed working leading to the result is valid)

BOD Benefit of Doubt (allowed when the validity of a solution may not be absolutely clear)

CAO Correct Answer Only (emphasising that no "follow through" from a previous error is allowed)

CWO Correct Working Only - often written by a 'fortuitous' answer

ISW Ignore Subsequent Working

MR Misread

PA Premature Approximation (resulting in basically correct work that is insufficiently accurate)

SOS See Other Solution (the candidate makes a better attempt at the same question)

SR Special Ruling (detailing the mark to be given for a specific wrong solution, or a case where some standard marking practice is to be varied in the light of a particular circumstance)

Penalties

MR -1 A penalty of MR -1 is deducted from A or B marks when the data of a question or part question are genuinely misread and the object and difficulty of the question remain unaltered. In this case all A and B marks then become "follow through √" marks. MR is not applied when the candidate misreads his own figures - this is regarded as an error in accuracy. An MR-2 penalty may be applied in particular cases if agreed at the coordination meeting.

PA -1 This is deducted from A or B marks in the case of premature approximation. The PA -1 penalty is usually discussed at the meeting.

1	*EITHER*:	Obtain a non-modular inequality from $(2x + 3)^2 < (x - 3)^2$, or corresponding quadratic equation, or pair of linear equations $2x + 3 = \pm(x - 3)$	M1	
		Make reasonable solution attempt at a 3-term quadratic, or solve two linear equations	M1	
		Obtain critical values $x = -6$ and $x = 0$	A1	
		State answer $-6 < x < 0$	A1	
	OR:	obtain the critical value $x = -6$ from a graphical method or by inspection, or by solving a linear equation or inequality	B1	
		Obtain the critical value $x = 0$ similarly	B2	
		State answer $-6 < x < 0$	B1	[4]

2	Use $\ln x^2 = 2\ln x$	B1	
	Obtain $3 - x^2 = x^2$, or equivalent	B1	
	Solve for x	M1	
	Obtain answer $x - 1.22$, having rejected $x = -1.22$	A1	[4]

3	**(i)**	Substitute $x = -\frac{1}{2}$ and equate to zero		M1	
		Obtain $a = -11$		A1	[2]
	(ii)	*EITHER*:	Attempt division by $2x + 1$ reaching a partial quotient $2x^2 - 5x$	M1	
			Obtain quadratic factor $2x^2 - 5x - 3$	A1	
			Obtain complete factorisation $(2x + 1)^2(x - 3)$	A1 + A1	
		OR:	Obtain factor $(x - 3)$ by inspection or factor theorem	B2	
			Attempt division by $(x - 3)$ reaching a partial quotient $4x^2 + 4x$	M1	
			Obtain complete factorisation $(2x + 1)^2(x - 3)$	A1	[4]

4	**(i)**	Use trig formulae to express equation in terms of $\sin x$ and $\cos x$	M1	
		Use $\cos 60° = \frac{1}{2}$ and $\sin 60° = \frac{\sqrt{3}}{2}$, or equivalent	M1	
		Obtain equation in $\sin x$ and $\cos x$ in any correct form	A1	
		Obtain $\tan x = \sqrt{3}/5$, or 0.3464…, or equivalent	A1	[4]
	(ii)	Obtain answer $x = 19.1°$	B1	
		Obtain answer $x = 199.1°$ and no others in the range [ignore answers outside the given range.]	B1√	[2]

5	**(i)**	Use double angle formulae and obtain $a + b\cos 4x$	M1	
		Obtain answer $\frac{1}{2} + \frac{1}{2}\cos 4x$, or equivalent	A1	[2]
	(ii)	Integrate and obtain $\frac{1}{2}x + \frac{1}{8}\sin 4x$	A1√ + A1√	
		Substitute limits correctly	M1	
		Obtain answer $\frac{1}{16}\pi + \frac{1}{8}$, or exact equivalent	A1	[4]

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6	**(i)**	Use product rule	M1*	
		Obtain derivative in any correct form	A1	
		Equate derivative to zero and solve for x	M1(dep*)	
		Obtain $x = 1/e$, or exact equivalent	A1	
		Obtain $y = -1/e$, or exact equivalent	A1	[5]
	(ii)	Carry out complete method for determining the nature of a stationary point	M1	
		Show that at $x = 1/e$ there is a minimum point, with no errors seen	A1	[2]
7	**(i)**	*EITHER*: Integrate $1 - e^{-x}$ obtaining $x \pm e^{-x}$	M1	
		Obtain indefinite integral $x - e^{-x}$	A1	
		Substitute limits $x = 0$, $x = p$ correctly	M1	
		Obtain answer $p + e^{-p} - 1$, or equivalent	A1	
		OR: Integrate e^{-x} obtaining $\pm e^{-x}$	M1	
		Substitute limits $x = 0$, $x = p$ correctly	M1	
		Obtain area below curve is $1 - e^{-p}$	A1	
		Obtain answer $p + e^{-p} - 1$, or equivalent	A1	[4]
	(ii)	Show that $p + e^{-p} - 1 = 1$ is equivalent to $p = 2 - e^{-p}$ or *vice versa*	B1	[1]
	(iii)	Use the iterative formula correctly at least once	M1	
		Obtain final answer 1.84	A1	
		Show sufficient iterations to justify its accuracy to 2 d.p.	A1	[3]
8	**(i)**	*EITHER*: Substitute $x = 1$ and attempt to solve 3-term quadratic in y	M1	
		Obtain answers (1, 1) and (1, –3)	A1	
		OR: State answers (1, 1) and (1, –3)	B1 + B1	[2]
	(ii)	State $2y\dfrac{dy}{dx}$ as derivative of y^2	B1	
		State $2y + 2x\dfrac{dy}{dx}$ as derivative of $2xy$	B1	
		Substitute for x and y, and solve for $\dfrac{dy}{dx}$	M1	
		Obtain $\dfrac{dy}{dx} = 0$ when $x = 1$ and $y = 1$	A1	
		Obtain $\dfrac{dy}{dx} = -2$ when $x = 1$ and $y = -3$	A1√	
		Form the equation of the tangent at (1, –3)	M1	
		Obtain answer $2x + y + 1 = 0$	A1	[7]