Cambridge International AS & A Level

Cambridge Assessment International Education
Cambridge International Advanced Subsidiary and Advanced Level

CANDIDATE NAME		
CENTRE NUMBER		CANDIDATE NUMBER

9810374883

MATHEMATICS **9709/12**

Paper 1 Pure Mathematics 1 **(P1)** **October/November 2019**

1 hour 45 minutes

Candidates answer on the Question Paper.

Additional Materials: List of Formulae (MF9)

READ THESE INSTRUCTIONS FIRST

Write your centre number, candidate number and name in the spaces at the top of this page.
Write in dark blue or black pen.
You may use an HB pencil for any diagrams or graphs.
Do not use staples, paper clips, glue or correction fluid.
DO **NOT** WRITE IN ANY BARCODES.

Answer **all** the questions in the space provided. If additional space is required, you should use the lined page at the end of this booklet. The question number(s) must be clearly shown.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
The use of an electronic calculator is expected, where appropriate.
You are reminded of the need for clear presentation in your answers.

At the end of the examination, fasten all your work securely together.
The number of marks is given in brackets [ ] at the end of each question or part question.
The total number of marks for this paper is 75.

This document consists of **19** printed pages and **1** blank page.

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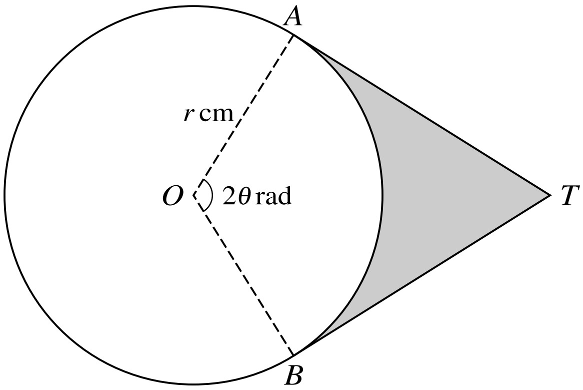
1 The coefficient of x^2 in the expansion of $(4 + ax)\left(1 + \frac{x}{2}\right)^6$ is 3. Find the value of the constant a. [4]

2 The point M is the mid-point of the line joining the points $(3, 7)$ and $(-1, 1)$. Find the equation of the line through M which is parallel to the line $\frac{x}{3} + \frac{y}{2} = 1$. [4]

3 A curve is such that $\frac{dy}{dx} = \frac{k}{\sqrt{x}}$, where k is a constant. The points $P(1, -1)$ and $Q(4, 4)$ lie on the curve. Find the equation of the curve. [4]

4



The diagram shows a circle with centre O and radius r cm. Points A and B lie on the circle and angle $AOB = 2\theta$ radians. The tangents to the circle at A and B meet at T.

(i) Express the perimeter of the shaded region in terms of r and θ. [3]

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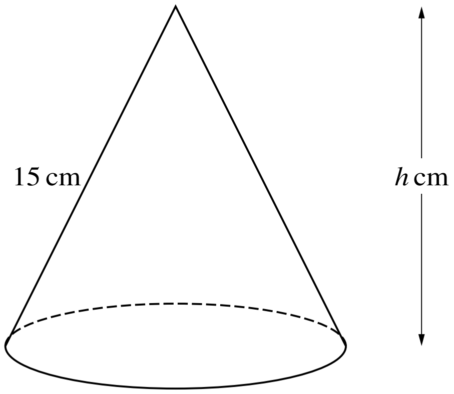
(ii) In the case where $r = 5$ and $\theta = 1.2$, find the area of the shaded region. [4]

5



The diagram shows a solid cone which has a slant height of 15 cm and a vertical height of h cm.

(i) Show that the volume, V cm^3, of the cone is given by $V = \frac{1}{3}\pi(225h - h^3)$. [2]

[The volume of a cone of radius r and vertical height h is $\frac{1}{3}\pi r^2 h$.]

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(ii) Given that h can vary, find the value of h for which V has a stationary value. Determine, showing all necessary working, the nature of this stationary value. [5]

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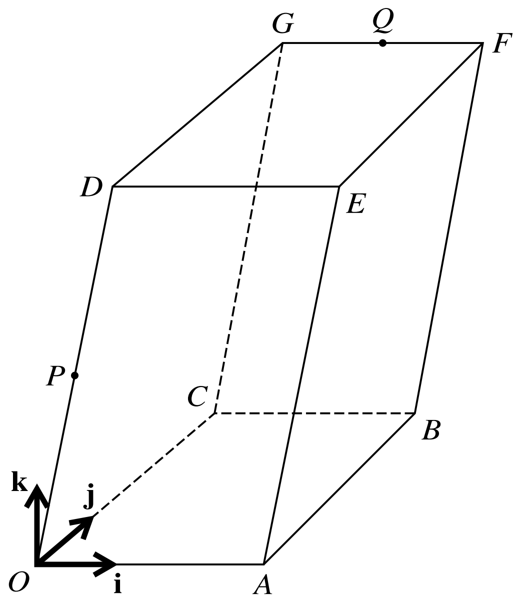
6 **(a)** Given that $x > 0$, find the two smallest values of x, in radians, for which $3\tan(2x + 1) = 1$. Show all necessary working. [4]

(b) The function $f : x \mapsto 3\cos^2 x - 2\sin^2 x$ is defined for $0 \leqslant x \leqslant \pi$.

(i) Express $f(x)$ in the form $a\cos^2 x + b$, where a and b are constants. [1]

(ii) Find the range of f. [2]

7



The diagram shows a three-dimensional shape $OABCDEFG$. The base $OABC$ and the upper surface $DEFG$ are identical horizontal rectangles. The parallelograms $OAED$ and $CBFG$ both lie in vertical planes. Points P and Q are the mid-points of OD and GF respectively. Unit vectors **i** and **j** are parallel to $\overrightarrow{OA}$ and $\overrightarrow{OC}$ respectively and the unit vector **k** is vertically upwards. The position vectors of A, C and D are given by $\overrightarrow{OA} = 6\mathbf{i}$, $\overrightarrow{OC} = 8\mathbf{j}$ and $\overrightarrow{OD} = 2\mathbf{i} + 10\mathbf{k}$.

(i) Express each of the vectors $\overrightarrow{PB}$ and $\overrightarrow{PQ}$ in terms of **i**, **j** and **k**. [4]

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(ii) Determine whether P is nearer to Q or to B. [2]

(iii) Use a scalar product to find angle BPQ. [3]

8 **(a)** Over a 21-day period an athlete prepares for a marathon by increasing the distance she runs each day by 1.2 km. On the first day she runs 13 km.

(i) Find the distance she runs on the last day of the 21-day period. [1]

(ii) Find the total distance she runs in the 21-day period. [2]

(b) The first, second and third terms of a geometric progression are x, $x - 3$ and $x - 5$ respectively.

(i) Find the value of x. [2]

(ii) Find the fourth term of the progression. [2]

(iii) Find the sum to infinity of the progression. [2]

9 Functions f and g are defined by

$$f(x) = 2x^2 + 8x + 1 \quad \text{for } x \in \mathbb{R},$$
$$g(x) = 2x - k \quad \text{for } x \in \mathbb{R},$$

where k is a constant.

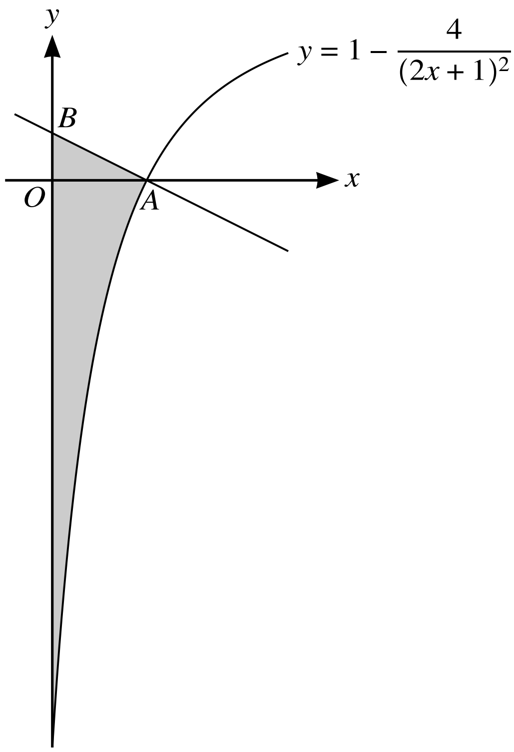
(i) Find the value of k for which the line $y = g(x)$ is a tangent to the curve $y = f(x)$. [3]

(ii) In the case where $k = -9$, find the set of values of x for which $f(x) < g(x)$. [3]

(iii) In the case where $k = -1$, find $g^{-1}f(x)$ and solve the equation $g^{-1}f(x) = 0$. [3]

(iv) Express $f(x)$ in the form $2(x + a)^2 + b$, where a and b are constants, and hence state the least value of $f(x)$. [3]

10



The diagram shows part of the curve $y = 1 - \dfrac{4}{(2x+1)^2}$. The curve intersects the x-axis at A. The normal to the curve at A intersects the y-axis at B.

(i) Obtain expressions for $\dfrac{dy}{dx}$ and $\displaystyle\int y\,dx$. [4]

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(ii) Find the coordinates of B. [4]

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(iii) Find, showing all necessary working, the area of the shaded region. [4]

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Additional Page

If you use the following lined page to complete the answer(s) to any question(s), the question number(s) must be clearly shown.