Cambridge International AS & A Level

Cambridge International Examinations
Cambridge International Advanced Subsidiary and Advanced Level

MATHEMATICS **9709/12**

Paper 1 Pure Mathematics 1 **(P1)** **October/November 2016**

1 hour 45 minutes

Additional Materials: List of Formulae (MF9)

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READ THESE INSTRUCTIONS FIRST

An answer booklet is provided inside this question paper. You should follow the instructions on the front cover of the answer booklet. If you need additional answer paper ask the invigilator for a continuation booklet.

Answer **all** the questions.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
The use of an electronic calculator is expected, where appropriate.
You are reminded of the need for clear presentation in your answers.

At the end of the examination, fasten all your work securely together.
The number of marks is given in brackets [ ] at the end of each question or part question.
The total number of marks for this paper is 75.

This document consists of **3** printed pages, **1** blank page and **1** insert.

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[Turn over

1 A curve is such that $\frac{dy}{dx} = \frac{8}{\sqrt{(4x+1)}}$. The point (2, 5) lies on the curve. Find the equation of the curve. [4]

2 **(i)** Express the equation $\sin 2x + 3\cos 2x = 3(\sin 2x - \cos 2x)$ in the form $\tan 2x = k$, where k is a constant. [2]

(ii) Hence solve the equation for $-90° \leqslant x \leqslant 90°$. [3]

3 A curve has equation $y = 2x^2 - 6x + 5$.

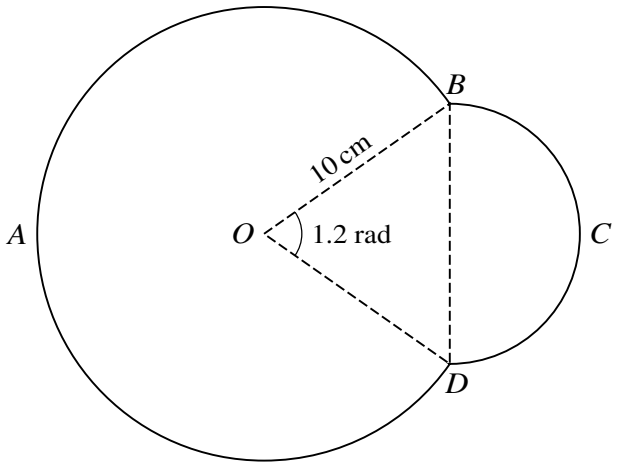
(i) Find the set of values of x for which $y > 13$. [3]

(ii) Find the value of the constant k for which the line $y = 2x + k$ is a tangent to the curve. [3]

4 In the expansion of $(3 - 2x)\left(1 + \frac{x}{2}\right)^n$, the coefficient of x is 7. Find the value of the constant n and hence find the coefficient of x^2. [6]

5 The line $\frac{x}{a} + \frac{y}{b} = 1$, where a and b are positive constants, intersects the x- and y-axes at the points A and B respectively. The mid-point of AB lies on the line $2x + y = 10$ and the distance $AB = 10$. Find the values of a and b. [6]

6



The diagram shows a metal plate $ABCD$ made from two parts. The part BCD is a semicircle. The part DAB is a segment of a circle with centre O and radius 10 cm. Angle BOD is 1.2 radians.

(i) Show that the radius of the semicircle is 5.646 cm, correct to 3 decimal places. [2]

(ii) Find the perimeter of the metal plate. [3]

(iii) Find the area of the metal plate. [3]

7 The equation of a curve is $y = 2 + \dfrac{3}{2x-1}$.

(i) Obtain an expression for $\dfrac{\mathrm{d}y}{\mathrm{d}x}$. [2]

(ii) Explain why the curve has no stationary points. [1]

At the point P on the curve, $x = 2$.

(iii) Show that the normal to the curve at P passes through the origin. [4]

(iv) A point moves along the curve in such a way that its x-coordinate is decreasing at a constant rate of 0.06 units per second. Find the rate of change of the y-coordinate as the point passes through P. [2]

8 **(a)** A cyclist completes a long-distance charity event across Africa. The total distance is 3050 km. He starts the event on May 1st and cycles 200 km on that day. On each subsequent day he reduces the distance cycled by 5 km.

(i) How far will he travel on May 15th? [2]

(ii) On what date will he finish the event? [3]

(b) A geometric progression is such that the third term is 8 times the sixth term, and the sum of the first six terms is $31\frac{1}{2}$. Find

(i) the first term of the progression, [4]

(ii) the sum to infinity of the progression. [1]

9 Relative to an origin O, the position vectors of the points A, B and C are given by

$$\overrightarrow{OA} = \begin{pmatrix} 2 \\ -2 \\ -1 \end{pmatrix}, \quad \overrightarrow{OB} = \begin{pmatrix} -2 \\ 3 \\ 6 \end{pmatrix} \quad \text{and} \quad \overrightarrow{OC} = \begin{pmatrix} 2 \\ 6 \\ 5 \end{pmatrix}.$$

(i) Use a scalar product to find angle AOB. [4]

(ii) Find the vector which is in the same direction as $\overrightarrow{AC}$ and of magnitude 15 units. [3]

(iii) Find the value of the constant p for which $p\overrightarrow{OA} + \overrightarrow{OC}$ is perpendicular to $\overrightarrow{OB}$. [3]

10 A function f is defined by $\mathrm{f} : x \mapsto 5 - 2\sin 2x$ for $0 \leqslant x \leqslant \pi$.

(i) Find the range of f. [2]

(ii) Sketch the graph of $y = \mathrm{f}(x)$. [2]

(iii) Solve the equation $\mathrm{f}(x) = 6$, giving answers in terms of π. [3]

The function g is defined by $\mathrm{g} : x \mapsto 5 - 2\sin 2x$ for $0 \leqslant x \leqslant k$, where k is a constant.

(iv) State the largest value of k for which g has an inverse. [1]

(v) For this value of k, find an expression for $\mathrm{g}^{-1}(x)$. [3]

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