Cambridge International Examinations
Cambridge International Advanced Subsidiary and Advanced Level

Cambridge International AS & A Level

MATHEMATICS **9709/12**

Paper 1 Pure Mathematics 1 **(P1)** **February/March 2016**

1 hour 45 minutes

Additional Materials: Answer Booklet/Paper
Graph Paper
List of Formulae (MF9)

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READ THESE INSTRUCTIONS FIRST

If you have been given an Answer Booklet, follow the instructions on the front cover of the Booklet.
Write your Centre number, candidate number and name on all the work you hand in.
Write in dark blue or black pen.
You may use an HB pencil for any diagrams or graphs.
Do not use staples, paper clips, glue or correction fluid.
DO **NOT** WRITE IN ANY BARCODES.

Answer **all** the questions.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
The use of an electronic calculator is expected, where appropriate.
You are reminded of the need for clear presentation in your answers.

At the end of the examination, fasten all your work securely together.
The number of marks is given in brackets [ ] at the end of each question or part question.
The total number of marks for this paper is 75.
Questions carrying smaller numbers of marks are printed earlier in the paper, and questions carrying larger numbers of marks later in the paper.

This document consists of **5** printed pages and **3** blank pages.

CAMBRIDGE International Examinations

1 **(i)** Find the coefficients of x^4 and x^5 in the expansion of $(1 - 2x)^5$. [2]

(ii) It is given that, when $(1 + px)(1 - 2x)^5$ is expanded, there is no term in x^5. Find the value of the constant p. [2]

2 A curve for which $\dfrac{dy}{dx} = 3x^2 - \dfrac{2}{x^3}$ passes through $(-1, 3)$. Find the equation of the curve. [4]

3 The 12th term of an arithmetic progression is 17 and the sum of the first 31 terms is 1023. Find the 31st term. [5]

4 **(a)** Solve the equation $\sin^{-1}(3x) = -\frac{1}{3}\pi$, giving the solution in an exact form. [2]

(b) Solve, by factorising, the equation $2\cos\theta\sin\theta - 2\cos\theta - \sin\theta + 1 = 0$ for $0 \leqslant \theta \leqslant \pi$. [4]

5 Two points have coordinates $A\,(5, 7)$ and $B\,(9, -1)$.

(i) Find the equation of the perpendicular bisector of AB. [3]

The line through $C\,(1, 2)$ parallel to AB meets the perpendicular bisector of AB at the point X.

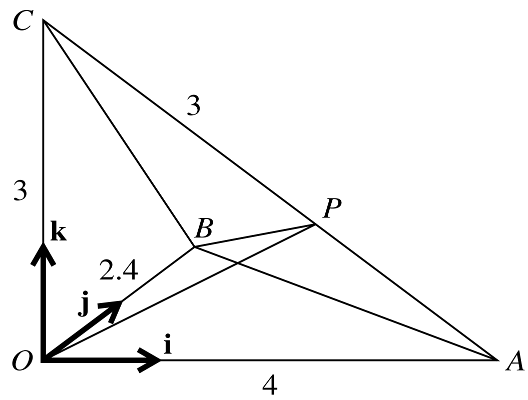
(ii) Find, by calculation, the distance BX. [5]

6 A vacuum flask (for keeping drinks hot) is modelled as a closed cylinder in which the internal radius is r cm and the internal height is h cm. The volume of the flask is $1000\,\text{cm}^3$. A flask is most efficient when the total internal surface area, $A\,\text{cm}^2$, is a minimum.

(i) Show that $A = 2\pi r^2 + \dfrac{2000}{r}$. [3]

(ii) Given that r can vary, find the value of r, correct to 1 decimal place, for which A has a stationary value and verify that the flask is most efficient when r takes this value. [5]

7



The diagram shows a pyramid $OABC$ with a horizontal triangular base OAB and vertical height OC. Angles AOB, BOC and AOC are each right angles. Unit vectors $\mathbf{i}$, $\mathbf{j}$ and $\mathbf{k}$ are parallel to OA, OB and OC respectively, with $OA = 4$ units, $OB = 2.4$ units and $OC = 3$ units. The point P on CA is such that $CP = 3$ units.

(i) Show that $\overrightarrow{CP} = 2.4\mathbf{i} - 1.8\mathbf{k}$. [2]

(ii) Express $\overrightarrow{OP}$ and $\overrightarrow{BP}$ in terms of $\mathbf{i}$, $\mathbf{j}$ and $\mathbf{k}$. [2]

(iii) Use a scalar product to find angle BPC. [4]

8 The function f is such that $\mathrm{f}(x) = a^2x^2 - ax + 3b$ for $x \leqslant \dfrac{1}{2a}$, where a and b are constants.

(i) For the case where $\mathrm{f}(-2) = 4a^2 - b + 8$ and $\mathrm{f}(-3) = 7a^2 - b + 14$, find the possible values of a and b. [5]

(ii) For the case where $a = 1$ and $b = -1$, find an expression for $\mathrm{f}^{-1}(x)$ and give the domain of f^{-1}. [5]

9 **(a)**

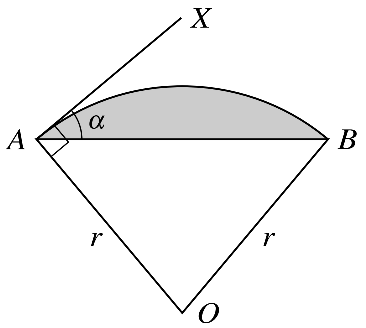


Fig. 1

In Fig. 1, OAB is a sector of a circle with centre O and radius r. AX is the tangent at A to the arc AB and angle $BAX = \alpha$.

(i) Show that angle $AOB = 2\alpha$. [2]

(ii) Find the area of the shaded segment in terms of r and α. [2]

(b)

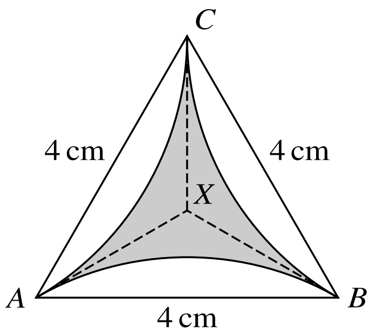


Fig. 2

In Fig. 2, ABC is an equilateral triangle of side 4 cm. The lines AX, BX and CX are tangents to the equal circular arcs AB, BC and CA. Use the results in part **(a)** to find the area of the shaded region, giving your answer in terms of π and $\sqrt{3}$. [6]

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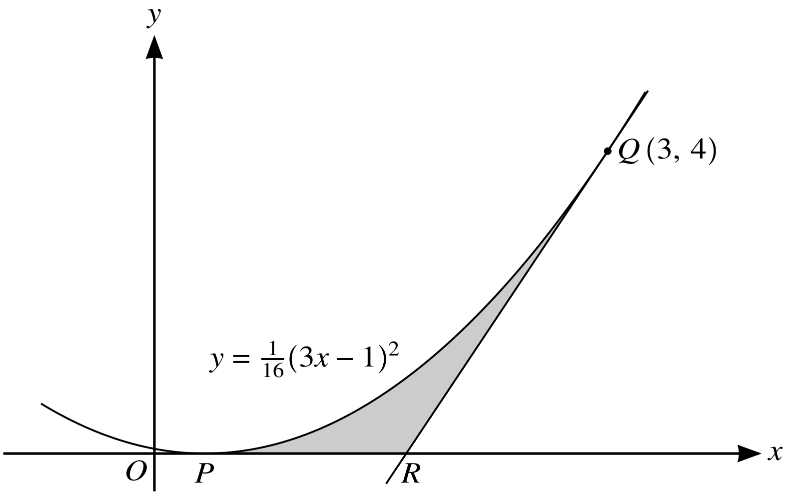

The diagram shows part of the curve $y = \frac{1}{16}(3x - 1)^2$, which touches the x-axis at the point P. The point $Q\,(3, 4)$ lies on the curve and the tangent to the curve at Q crosses the x-axis at R.

(i) State the x-coordinate of P. [1]

Showing all necessary working, find by calculation

(ii) the x-coordinate of R, [5]

(iii) the area of the shaded region PQR. [6]

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