Cambridge International AS & A Level

Cambridge International Examinations
Cambridge International Advanced Subsidiary and Advanced Level

CANDIDATE NAME

CENTRE NUMBER | | | | | |

CANDIDATE NUMBER | | | | |

6898538094

MATHEMATICS **9709/11**

Paper 1 Pure Mathematics 1 **(P1)** **May/June 2017**

1 hour 45 minutes

Candidates answer on the Question Paper.

Additional Materials: List of Formulae (MF9)

READ THESE INSTRUCTIONS FIRST

Write your Centre number, candidate number and name in the spaces at the top of this page.
Write in dark blue or black pen.
You may use an HB pencil for any diagrams or graphs.
Do not use staples, paper clips, glue or correction fluid.
DO **NOT** WRITE IN ANY BARCODES.

Answer **all** the questions.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
The use of an electronic calculator is expected, where appropriate.
You are reminded of the need for clear presentation in your answers.

At the end of the examination, fasten all your work securely together.
The number of marks is given in brackets [ ] at the end of each question or part question.
The total number of marks for this paper is 75.

This document consists of **19** printed pages and **1** blank page.

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1 The coefficients of x^2 and x^3 in the expansion of $(3 - 2x)^6$ are a and b respectively. Find the value of $\frac{a}{b}$. [4]

2 Relative to an origin O, the position vectors of points A and B are given by

$$\overrightarrow{OA} = \begin{pmatrix} 3 \\ -6 \\ p \end{pmatrix} \quad \text{and} \quad \overrightarrow{OB} = \begin{pmatrix} 2 \\ -6 \\ -7 \end{pmatrix},$$

and angle $AOB = 90°$.

(i) Find the value of p. [2]

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The point C is such that $\overrightarrow{OC} = \frac{2}{3}\overrightarrow{OA}$.

(ii) Find the unit vector in the direction of $\overrightarrow{BC}$. [4]

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3 **(i)** Prove the identity $\dfrac{1+\cos\theta}{\sin\theta}+\dfrac{\sin\theta}{1+\cos\theta}\equiv\dfrac{2}{\sin\theta}$. [3]

(ii) Hence solve the equation $\frac{1+\cos\theta}{\sin\theta} + \frac{\sin\theta}{1+\cos\theta} = \frac{3}{\cos\theta}$ for $0° \leqslant \theta \leqslant 360°$. [3]

4 **(a)** An arithmetic progression has a first term of 32, a 5th term of 22 and a last term of −28. Find the sum of all the terms in the progression. [4]

(b) Each year a school allocates a sum of money for the library. The amount allocated each year increases by 2.5% of the amount allocated the previous year. In 2005 the school allocated \$2000. Find the total amount allocated in the years 2005 to 2014 inclusive. [3]

5 The equation of a curve is $y = 2\cos x$.

(i) Sketch the graph of $y = 2\cos x$ for $-\pi \leqslant x \leqslant \pi$, stating the coordinates of the point of intersection with the y-axis. [2]

Points P and Q lie on the curve and have x-coordinates of $\frac{1}{3}\pi$ and π respectively.

(ii) Find the length of PQ correct to 1 decimal place. [2]

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The line through P and Q meets the x-axis at $H\,(h,\,0)$ and the y-axis at $K\,(0,\,k)$.

(iii) Show that $h = \frac{5}{9}\pi$ and find the value of k. [3]

6 The horizontal base of a solid prism is an equilateral triangle of side x cm. The sides of the prism are vertical. The height of the prism is h cm and the volume of the prism is $2000\,\text{cm}^3$.

(i) Express h in terms of x and show that the total surface area of the prism, $A\,\text{cm}^2$, is given by

$$A = \frac{\sqrt{3}}{2}x^2 + \frac{24\,000}{\sqrt{3}}x^{-1}.$$ [3]

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(ii) Given that x can vary, find the value of x for which A has a stationary value. [3]

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(iii) Determine, showing all necessary working, the nature of this stationary value. [2]

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7 A curve for which $\frac{dy}{dx} = 7 - x^2 - 6x$ passes through the point $(3, -10)$.

(i) Find the equation of the curve. [3]

(ii) Express $7 - x^2 - 6x$ in the form $a - (x + b)^2$, where a and b are constants. [2]

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(iii) Find the set of values of x for which the gradient of the curve is positive. [3]

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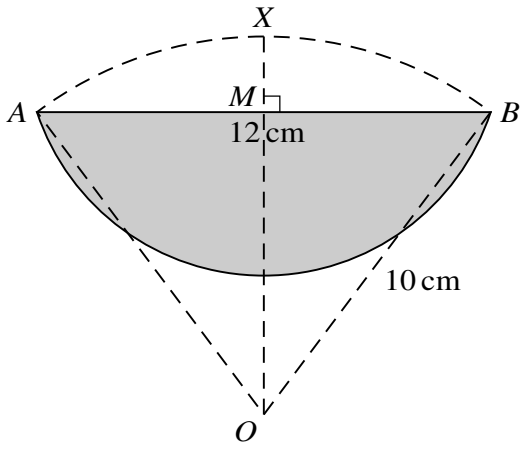

In the diagram, $OAXB$ is a sector of a circle with centre O and radius 10 cm. The length of the chord AB is 12 cm. The line OX passes through M, the mid-point of AB, and OX is perpendicular to AB. The shaded region is bounded by the chord AB and by the arc of a circle with centre X and radius XA.

(i) Show that angle AXB is 2.498 radians, correct to 3 decimal places. [3]

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(ii) Find the perimeter of the shaded region. [3]

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(iii) Find the area of the shaded region. [3]

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9 The function f is defined by $\mathrm{f} : x \mapsto \dfrac{2}{3-2x}$ for $x \in \mathbb{R},\ x \neq \frac{3}{2}$.

(i) Find an expression for $\mathrm{f}^{-1}(x)$. [3]

The function g is defined by $g : x \mapsto 4x + a$ for $x \in \mathbb{R}$, where a is a constant.

(ii) Find the value of a for which $gf(-1) = 3$. [3]

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(iii) Find the possible values of a given that the equation $f^{-1}(x) = g^{-1}(x)$ has two equal roots. [4]

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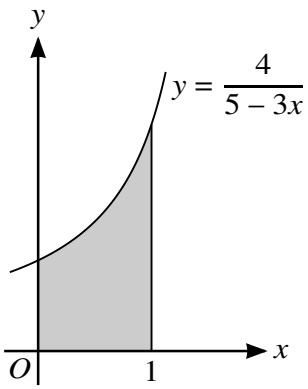

The diagram shows part of the curve $y = \frac{4}{5 - 3x}$.

(i) Find the equation of the normal to the curve at the point where $x = 1$ in the form $y = mx + c$, where m and c are constants. [5]

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The shaded region is bounded by the curve, the coordinate axes and the line $x = 1$.

(ii) Find, showing all necessary working, the volume obtained when this shaded region is rotated through 360° about the x-axis. [5]

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