Cambridge Assessment International Education

Cambridge International AS & A Level

CANDIDATE NAME

CENTRE NUMBER | | | | | |

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5105685851

MATHEMATICS **9709/63**

Paper 6 Probability & Statistics 2 **October/November 2022**

1 hour 15 minutes

You must answer on the question paper.

You will need: List of formulae (MF19)

INSTRUCTIONS

- Answer **all** questions.
- Use a black or dark blue pen. You may use an HB pencil for any diagrams or graphs.
- Write your name, centre number and candidate number in the boxes at the top of the page.
- Write your answer to each question in the space provided.
- Do **not** use an erasable pen or correction fluid.
- Do **not** write on any bar codes.
- If additional space is needed, you should use the lined page at the end of this booklet; the question number or numbers must be clearly shown.
- You should use a calculator where appropriate.
- You must show all necessary working clearly; no marks will be given for unsupported answers from a calculator.
- Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place for angles in degrees, unless a different level of accuracy is specified in the question.

INFORMATION

- The total mark for this paper is 50.
- The number of marks for each question or part question is shown in brackets [ ].

This document has **12** pages. Any blank pages are indicated.

[Turn over

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1 The heights, in metres, of a random sample of 10 mature trees of a certain variety are given below.

5.9 6.5 6.7 5.9 6.9 6.0 6.4 6.2 5.8 5.8

Find unbiased estimates of the population mean and variance of the heights of all mature trees of this variety. [3]

2 A spinner has five sectors, each printed with a different colour. Susma and Sanjay both wish to test whether the spinner is biased so that it lands on red on fewer spins than it would if it were fair. Susma spins the spinner 40 times. She finds that it lands on red exactly 4 times.

(a) Use a binomial distribution to carry out the test at the 5% significance level. [5]

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Sanjay also spins the spinner 40 times. He finds that it lands on red r times.

(b) Use a binomial distribution to find the largest value of r that lies in the rejection region for the test at the 5% significance level. [3]

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3 Drops of water fall randomly from a leaking tap at a constant average rate of 5.2 per minute.

(a) Find the probability that at least 3 drops fall during a randomly chosen 30-second period. [3]

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(b) Use a suitable approximating distribution to find the probability that at least 650 drops fall during a randomly chosen 2-hour period. [4]

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4 Each month a company sells X kg of brown sugar and Y kg of white sugar, where X and Y have the independent distributions $N(2500, 120^2)$ and $N(3700, 130^2)$ respectively.

(a) Find the mean and standard deviation of the total amount of sugar that the company sells in 3 randomly chosen months. [3]

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The company makes a profit of \$1.50 per kilogram of brown sugar sold and makes a loss of \$0.20 per kilogram of white sugar sold.

(b) Find the probability that, in a randomly chosen month, the total profit is less than \$3000. [5]

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5 A builders' merchant sells stones of different sizes.

(a) The masses of size A stones have standard deviation 6 grams. The mean mass of a random sample of 200 size A stones is 45 grams.

Find a 95% confidence interval for the population mean mass of size A stones. [3]

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(b) The masses of size B stones have standard deviation 11 grams. Using a random sample of size 200, an α% confidence interval for the population mean mass is found to have width 4 grams.

Find α. [4]

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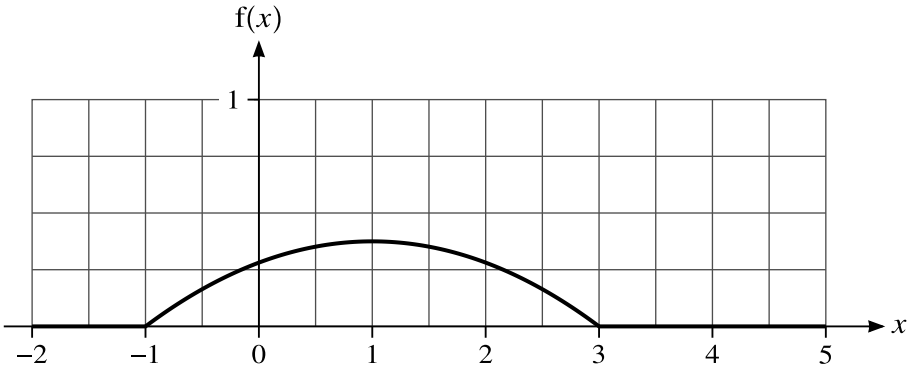
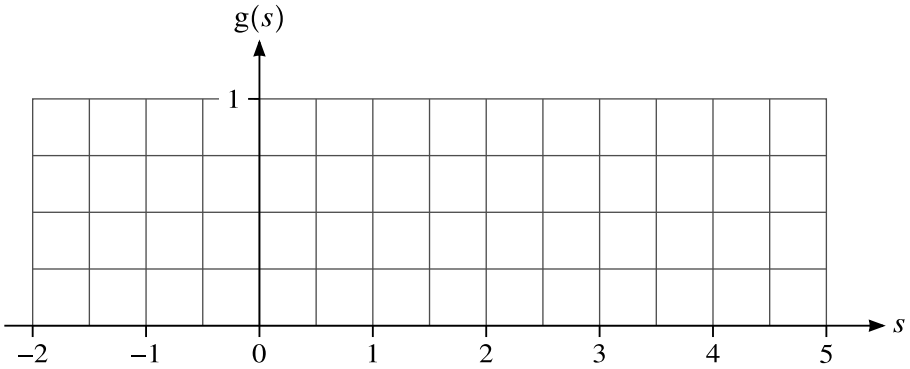


The diagram shows the graph of the probability density function of a random variable X that takes values between -1 and 3 only. It is given that the graph is symmetrical about the line $x = 1$. Between $x = -1$ and $x = 3$ the graph is a quadratic curve.

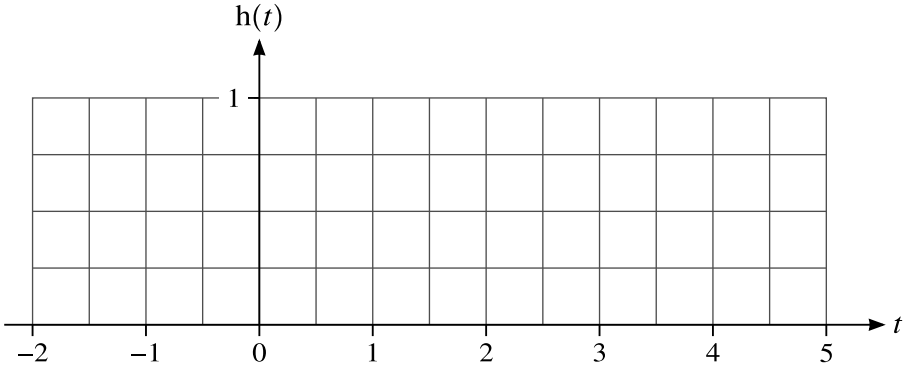
The random variable S is such that $\mathrm{E}(S) = 2 \times \mathrm{E}(X)$ and $\mathrm{Var}(S) = \mathrm{Var}(X)$.

(a) On the grid below, sketch a quadratic graph for the probability density function of S. [1]



The random variable T is such that $\mathrm{E}(T) = \mathrm{E}(X)$ and $\mathrm{Var}(T) = \frac{1}{4}\mathrm{Var}(X)$.

(b) On the grid below, sketch a quadratic graph for the probability density function of T. [2]



It is now given that

$$f(x) = \begin{cases} \frac{3}{32}(3 + 2x - x^2) & -1 \leqslant x \leqslant 3, \\ 0 & \text{otherwise.} \end{cases}$$

(c) Given that $P(1 - a < X < 1 + a) = 0.5$, show that $a^3 - 12a + 8 = 0$. [3]

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(d) Hence verify that $0.69 < a < 0.70$. [1]

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7 In the past Laxmi's time, in minutes, for her journey to college had mean 32.5 and standard deviation 3.1. After a change in her route, Laxmi wishes to test whether the mean time has decreased. She notes her journey times for a random sample of 50 journeys and she finds that the sample mean is 31.8 minutes. You should assume that the standard deviation is unchanged.

(a) Carry out a hypothesis test, at the 8% significance level, of whether Laxmi's mean journey time has decreased. [5]

Later Laxmi carries out a similar test with the same hypotheses, at the 8% significance level, using another random sample of size 50.

(b) Given that the population mean is now 31.5, find the probability of a Type II error. [5]

Additional Page

If you use the following lined page to complete the answer(s) to any question(s), the question number(s) must be clearly shown.

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