Cambridge Assessment International Education

# Cambridge International AS & A Level

CANDIDATE NAME

CENTRE NUMBER

CANDIDATE NUMBER



**MATHEMATICS** **9709/62**

Paper 6 Probability & Statistics 2 **May/June 2023**

**1 hour 15 minutes**

You must answer on the question paper.

You will need: List of formulae (MF19)

**INSTRUCTIONS**

- Answer **all** questions.
- Use a black or dark blue pen. You may use an HB pencil for any diagrams or graphs.
- Write your name, centre number and candidate number in the boxes at the top of the page.
- Write your answer to each question in the space provided.
- Do **not** use an erasable pen or correction fluid.
- Do **not** write on any bar codes.
- If additional space is needed, you should use the lined page at the end of this booklet; the question number or numbers must be clearly shown.
- You should use a calculator where appropriate.
- You must show all necessary working clearly; no marks will be given for unsupported answers from a calculator.
- Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place for angles in degrees, unless a different level of accuracy is specified in the question.

**INFORMATION**

- The total mark for this paper is 50.
- The number of marks for each question or part question is shown in brackets [ ].

This document has **16** pages. Any blank pages are indicated.




**[Turn over**

**1** In a survey of 200 randomly chosen students from a certain college, 23% of the students said that they owned a car.

Calculate an approximate 93% confidence interval for the proportion of students from the college who own a car. [3]

**2** **(a)** The random variable $W$ has a Poisson distribution.

State the relationship between $\mathrm{E}(W)$ and $\mathrm{Var}(W)$. [1]

.....

.....

**(b)** The random variable $X$ has the distribution $\mathrm{B}(n, p)$. Jyothi wishes to use a Poisson distribution as an approximate distribution for $X$.

Use the formulae for $\mathrm{E}(X)$ and $\mathrm{Var}(X)$ to explain why it is necessary for $p$ to be close to 0 for this to be a reasonable approximation. [1]

.....

.....

.....

.....

.....

**(c)** Given that $Y$ has the distribution $\mathrm{B}(20\,000, 0.000\,07)$, use a Poisson distribution to calculate an estimate of $\mathrm{P}(Y > 2)$. [3]

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

**3** The masses, in kilograms, of newborn babies in country $A$ are represented by the random variable $X$, with mean $\mu$ and variance $\sigma^2$. The masses of a random sample of 500 newborn babies in this country were found and the results are summarised below.

$$n = 500 \qquad \Sigma x = 1625 \qquad \Sigma x^2 = 5663.5$$

**(a)** Calculate unbiased estimates of $\mu$ and $\sigma^2$. [3]

A researcher wishes to test whether the mean mass of newborn babies in a neighbouring country, $B$, is different from that in country $A$. He chooses a random sample of 60 newborn babies in country $B$ and finds that their sample mean mass is 2.95 kg.

Assume that your unbiased estimates in part **(a)** are the correct values for $\mu$ and $\sigma^2$. Assume also that the variance of the masses of newborn babies in country $B$ is the same as in country $A$.

**(b)** Carry out the test at the 1% significance level. [5]

**4** The number, $X$, of books received at a charity shop has a constant mean of 5.1 per day.

**(a)** State, in context, one condition for $X$ to be modelled by a Poisson distribution. [1]

.....

.....

.....

Assume now that $X$ can be modelled by a Poisson distribution.

**(b)** Find the probability that exactly 10 books are received in a 3-day period. [2]

.....

.....

.....

.....

**(c)** Use a suitable approximating distribution to find the probability that more than 180 books are received in a 30-day period. [4]

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

The number of DVDs received at the same shop is modelled by an independent Poisson distribution with mean 2.5 per day.

**(d)** Find the probability that the total number of books and DVDs that are received at the shop in 1 day is more than 3. [3]

**5** **(a)** Two random variables $X$ and $Y$ have the independent distributions N(7, 3) and N(6, 2) respectively. A random value of each variable is taken.

Find the probability that the two values differ by more than 2. [5]

**(b)** Each candidate's overall score in a science test is calculated as follows. The mark for theory is denoted by $T$, the mark for practical is denoted by $P$, and the overall score is given by $T + 1.5P$. The variables $T$ and $P$ are assumed to be independent with distributions N(62, 158) and N(42, 108) respectively. You should assume that no continuity corrections are needed when using these distributions.

**(i)** A pass is awarded to candidates whose overall score is at least 90.

Find the proportion of candidates who pass. [5]

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

**(ii)** Comment on the assumption that the variables $T$ and $P$ are independent. [1]

.....

.....

.....

.....

.....

**BLANK PAGE**

**6** When a child completes an online exercise called a Mathlit, they might be awarded a medal. The publishers claim that the probability that a randomly chosen child who completes a Mathlit will be awarded a medal is $\frac{1}{3}$. Asha wishes to test this claim. She decides that if she is awarded no medals while completing 10 Mathlits, she will conclude that the true probability is less than $\frac{1}{3}$.

**(a)** Use a binomial distribution to find the probability of a Type I error. [2]

.....

.....

.....

.....

.....

.....

.....

.....

The true probability of being awarded a medal is denoted by $p$.

**(b)** Given that the probability of a Type II error is 0.8926, find the value of $p$. [3]

.....

.....

.....

.....

.....

.....

.....

.....

.....

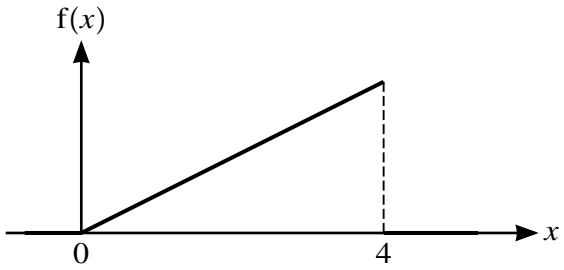
.....

.....

.....

.....

**7 (a)**




The diagram shows the graph of the probability density function, f, of a random variable $X$ which takes values between 0 and 4 only. Between these two values the graph is a straight line.

**(i)** Show that $f(x) = kx$ for $0 \leqslant x \leqslant 4$, where $k$ is a constant to be determined. [2]

.....

.....

.....

.....

.....

.....

.....

.....

.....

**(ii)** Hence, or otherwise, find $E(X)$. [3]

.....

.....

.....

.....

.....

.....

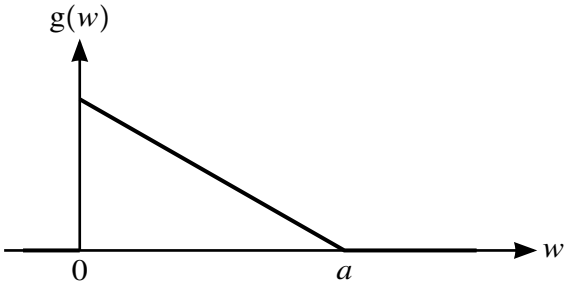
.....

.....

.....

.....

**(b)**




The diagram shows the graph of the probability density function, g, of a random variable $W$ which takes values between 0 and $a$ only, where $a > 0$. Between these two values the graph is a straight line.

Given that the median of $W$ is 1, find the value of $a$. [3]

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

## Additional Page

If you use the following lined page to complete the answer(s) to any question(s), the question number(s) must be clearly shown.

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

**BLANK PAGE**

**BLANK PAGE**