Cambridge Assessment International Education

Cambridge International AS & A Level

MATHEMATICS **9709/51**

Paper 5 Probability & Statistics 1 **October/November 2022**

MARK SCHEME

Maximum Mark: 50

Published

This mark scheme is published as an aid to teachers and candidates, to indicate the requirements of the examination. It shows the basis on which Examiners were instructed to award marks. It does not indicate the details of the discussions that took place at an Examiners' meeting before marking began, which would have considered the acceptability of alternative answers.

Mark schemes should be read in conjunction with the question paper and the Principal Examiner Report for Teachers.

Cambridge International will not enter into discussions about these mark schemes.

Cambridge International is publishing the mark schemes for the October/November 2022 series for most Cambridge IGCSE™, Cambridge International A and AS Level components and some Cambridge O Level components.

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Generic Marking Principles

These general marking principles must be applied by all examiners when marking candidate answers. They should be applied alongside the specific content of the mark scheme or generic level descriptors for a question. Each question paper and mark scheme will also comply with these marking principles.

GENERIC MARKING PRINCIPLE 1:
Marks must be awarded in line with:

- the specific content of the mark scheme or the generic level descriptors for the question
- the specific skills defined in the mark scheme or in the generic level descriptors for the question
- the standard of response required by a candidate as exemplified by the standardisation scripts.

GENERIC MARKING PRINCIPLE 2:
Marks awarded are always **whole marks** (not half marks, or other fractions).

GENERIC MARKING PRINCIPLE 3:
Marks must be awarded **positively**:

- marks are awarded for correct/valid answers, as defined in the mark scheme. However, credit is given for valid answers which go beyond the scope of the syllabus and mark scheme, referring to your Team Leader as appropriate
- marks are awarded when candidates clearly demonstrate what they know and can do
- marks are not deducted for errors
- marks are not deducted for omissions
- answers should only be judged on the quality of spelling, punctuation and grammar when these features are specifically assessed by the question as indicated by the mark scheme. The meaning, however, should be unambiguous.

GENERIC MARKING PRINCIPLE 4:
Rules must be applied consistently, e.g. in situations where candidates have not followed instructions or in the application of generic level descriptors.

GENERIC MARKING PRINCIPLE 5:
Marks should be awarded using the full range of marks defined in the mark scheme for the question (however; the use of the full mark range may be limited according to the quality of the candidate responses seen).

GENERIC MARKING PRINCIPLE 6:
Marks awarded are based solely on the requirements as defined in the mark scheme. Marks should not be awarded with grade thresholds or grade descriptors in mind.

Mathematics Specific Marking Principles	
1	Unless a particular method has been specified in the question, full marks may be awarded for any correct method. However, if a calculation is required then no marks will be awarded for a scale drawing.
2	Unless specified in the question, answers may be given as fractions, decimals or in standard form. Ignore superfluous zeros, provided that the degree of accuracy is not affected.
3	Allow alternative conventions for notation if used consistently throughout the paper, e.g. commas being used as decimal points.
4	Unless otherwise indicated, marks once gained cannot subsequently be lost, e.g. wrong working following a correct form of answer is ignored (isw).
5	Where a candidate has misread a number in the question and used that value consistently throughout, provided that number does not alter the difficulty or the method required, award all marks earned and deduct just 1 mark for the misread.
6	Recovery within working is allowed, e.g. a notation error in the working where the following line of working makes the candidate's intent clear.

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Mark Scheme Notes

The following notes are intended to aid interpretation of mark schemes in general, but individual mark schemes may include marks awarded for specific reasons outside the scope of these notes.

Types of mark

M Method mark, awarded for a valid method applied to the problem. Method marks are not lost for numerical errors, algebraic slips or errors in units. However, it is not usually sufficient for a candidate just to indicate an intention of using some method or just to quote a formula; the formula or idea must be applied to the specific problem in hand, e.g. by substituting the relevant quantities into the formula. Correct application of a formula without the formula being quoted obviously earns the M mark and in some cases an M mark can be implied from a correct answer.

A Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. Accuracy marks cannot be given unless the associated method mark is earned (or implied).

B Mark for a correct result or statement independent of method marks.

DM or **DB** When a part of a question has two or more 'method' steps, the M marks are generally independent unless the scheme specifically says otherwise; and similarly, when there are several B marks allocated. The notation DM or DB is used to indicate that a particular M or B mark is dependent on an earlier M or B (asterisked) mark in the scheme. When two or more steps are run together by the candidate, the earlier marks are implied and full credit is given.

FT Implies that the A or B mark indicated is allowed for work correctly following on from previously incorrect results. Otherwise, A or B marks are given for correct work only.

- A or B marks are given for correct work only (not for results obtained from incorrect working) unless follow through is allowed (see abbreviation FT above).
- For a numerical answer, allow the A or B mark if the answer is correct to 3 significant figures or would be correct to 3 significant figures if rounded (1 decimal place for angles in degrees).
- The total number of marks available for each question is shown at the bottom of the Marks column.
- Wrong or missing units in an answer should not result in loss of marks unless the guidance indicates otherwise.
- Square brackets [ ] around text or numbers show extra information not needed for the mark to be awarded.

Abbreviations

AEF/OE	Any Equivalent Form (of answer is equally acceptable) / Or Equivalent
AG	Answer Given on the question paper (so extra checking is needed to ensure that the detailed working leading to the result is valid)
CAO	Correct Answer Only (emphasising that no 'follow through' from a previous error is allowed)
CWO	Correct Working Only
ISW	Ignore Subsequent Working
SOI	Seen Or Implied
SC	Special Case (detailing the mark to be given for a specific wrong solution, or a case where some standard marking practice is to be varied in the light of a particular circumstance)
WWW	Without Wrong Working
AWRT	Answer Which Rounds To

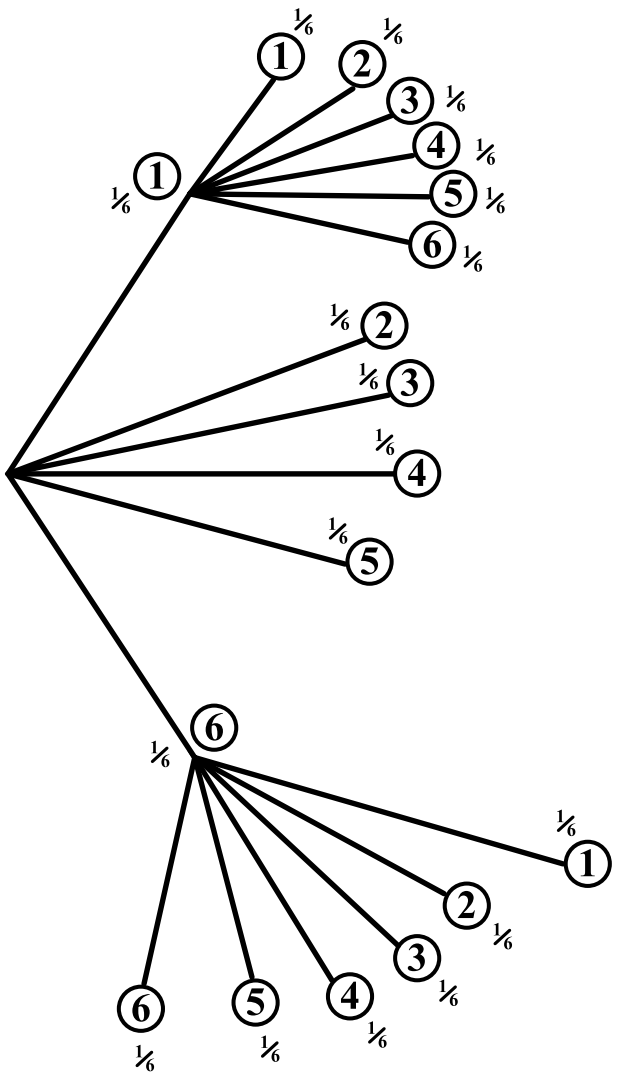
Question	Answer	Marks	Guidance
1	$0.12+p+q+0.16+0.3=1$	**B1**	Sum of probabilities = 1 $p+q = 0.42$ OE.
	$-0.24-p+0.5q+0.16+0.6=0.28$	**B1**	Form equation using E(X) = 0.28 $-p + 0.5q = -0.24$ OE. Accept unsimplified.
	Attempt to solve *their* two equations in p and q	**M1**	**Either** Substitution method to form a single equation in either p or q and finding values for both unknowns. **Or** Elimination method by writing both equations in the same form (usually $ap + bq = c$) and + or – to find an equation in one unknown and finding values for both unknowns.
	$q=0.12,\ p=0.3$	**A1**	CAO, both WWW. If M0 awarded **SC B1** for both correct WWW.
		4	

Question	Answer	Marks	Guidance
2(a)	[P(3, 4, …7) = 1 – P(0, 1, 2, 8)] $= 1 - ({}^{8}C_0\ 0.48^0\ 0.52^8 + {}^{8}C_1\ 0.48^1\ 0.52^7$ $+ {}^{8}C_2\ 0.48^2\ 0.52^6 + {}^{8}C_8\ 0.48^8\ 0.52^0)$	**M1**	One term ${}^{8}C_x\ p^x (1-p)^{8-x}$, for $0 < x < 8$, $0 < p < 1$
	$= 1 - (0.00534597 + 0.039478 + 0.127544 + 0.0028179)$	**A1**	Correct expression, accept unsimplified, no terms omitted, leading to final answer.
	0.825	**B1**	Mark the final answer at the most accurate value. $0.8248 < p \leqslant 0.825$ WWW.
	Alternative method for Question 2(a)		
	[P(3, 4, 5, 6, 7) =] ${}^{8}C_3\ 0.48^3\ 0.52^5 + {}^{8}C_4\ 0.48^4\ 0.52^4 + {}^{8}C_5\ 0.48^5\ 0.52^3 + {}^{8}C_6$ $0.48^6\ 0.52^2 + {}^{8}C_7\ 0.48^7\ 0.52^1$	**M1**	One term ${}^{8}C_x\ p^x (1-p)^{8-x}$, for $0<x<8$, $0<p<1$
		A1	Correct expression, accept unsimplified, no terms omitted, leading to final answer.
	0.825	**B1**	Final answer $0.8248 < p \leqslant 0.825$ WWW.
		3	

Question	Answer	Marks	Guidance
2(b)	[Mean = $0.52\times125=$]65, [var $=0.52\times0.48\times125=$]31.2	**B1**	65 and 31.2 seen, allow unsimplified. May be seen in standardisation formula. ($5.585 < \sigma \leqslant 5.586$ imply correct variance).
	[P($X > 72$) =]P($Z > \frac{72.5-65}{\sqrt{31.2}}$) [= P($Z > 1.343$)]	**M1**	Substituting *their* 65 and $\sqrt{their\,31.2}$ into ±standardisation formula (any number for 72·5), not *their* 31.2, √ *their* 5.586 .
		M1	Using continuity correction 72·5 or 71·5 in *their* standardisation formula . Note $\frac{\pm7.5}{\sqrt{31.2}}$ or $\frac{\pm7.5}{5.586}$ seen gains **M2** BOD
	= 1 – 0.9104	**M1**	Appropriate area Φ, from final process, must be probability.
	0.0896	**A1**	$0.0896 \leqslant p \leqslant 0.0897$ WWW.
		5	

Question	Answer	Marks	Guidance
3(a)	Lions | Tigers 9 | 16 9 8 | 17 | 9 9 7 6 1 0 | 18 | 0 3 4 7 6 0 0 | 19 | 0 1 4 5 7 | 20 | 1	**B1**	Correct stem can be upside down, ignore extra values (not in reverse).
		B1	Correct Lions labelled on left, leaves in order from right to left and lined up vertically, no commas or other punctuation.
		B1	Correct Tigers labelled on same diagram, leaves in order and lined up vertically, no commas or other punctuation. If the correct data for Lions and Tigers is transposed, treat as a single error in Lions and condone in Tigers.
	Key 1|18|3 means 181 cm for Lions and 183 cm for Tigers	**B1**	Correct single key for their diagram, need both teams identified and 'cm' stated at least once here or in leaf headings or title. **SC** If 2 separate diagrams drawn, **SC B1** if both keys meet these criteria (Max B1, B0, B0, B1).
		4	
3(b)	Median = 186 cm	**B1**	
	[UQ = 190 cm, LQ = 179 cm] IQR = 190 – 179	**M1**	$189 \leqslant$ UQ $\leqslant 190 - 178 \leqslant$ LQ $\leqslant 180$
	11[cm]	**A1**	WWW
		3	
3(c)	Tigers are (generally) taller	**B1**	Comparison about central tendency in context.
	Heights of Tigers are slightly less consistent than heights of Lions	**B1**	Comparison about spread in context. (Condone 'similar spread' in context.)
		2	

Question	Answer	Marks	Guidance
4(a)	$P(X<132) = P(Z < \frac{132-125.4}{18.6}) = P(Z < 0.3548)$	**M1**	Use of ±standardisation formula with 132 and 125.4 substituted, condone continuity correction 132±0.5 and use of 18.6^2, $\sqrt{18.6}$
	0.639	**A1**	$0.6385 < p \leqslant 0.639$ If M0 scored, **SC B1** for $0.6385 < p \leqslant 0.639$
		2	
4(b)	$\frac{108-117}{\sigma} = -1.175$	**B1**	$1.1749 < z \leqslant 1.175$ or $-1.175 \leqslant z < -1.1749$
		M1	108 and 117 substituted in ±standardisation formula, no continuity correction, not σ^2, $\sqrt{\sigma}$, equated to a z-value.
	$\sigma = 7.66$	**A1**	$7.659 \leqslant \sigma \leqslant 7.66$ If M0 scored, **SC B1** for $7.659 \leqslant \sigma \leqslant 7.66$
		3	
4(c)	$P(-1.5 < Z < 1.5)$ $[\Phi(1.5) - \Phi(-1.5)]$ $[= 2\Phi(1.5) - 1]$ $= 2 \times \textit{their}\ 0.9332 - 1$ **or** $\textit{their}\ 0.9332 - (1 - \textit{their}\ 0.9332)$ **or** $2 \times (\textit{their}\ 0.9332 - 0.5)$	**M1**	{Both 1.5 and –1.5 seen as z-values **or** appropriate use of 1.5 or −1.5} **and** {no other z-values in part}.
		M1	Calculating the appropriate area from stated phis of z-values which must be ± the same number. Condone *their* 0.0668 as (1 – *their* 0.9332).
	0.8664	**A1**	Accept answers wrt 0.866 If A0 scored **SC B1** for answers wrt 0.866
	$0.8664^3 = 0.650[36\ldots]$	**B1 FT**	FT *their* 4SF (or better) probability, accept final answers to 3SF.
		4	

Question	Answer	Marks	Guidance
5(a)		**B1**	1st throw fully correct with probabilities and outcomes identified. (Probabilities $\left(\text{all}\frac{1}{6}\right)$ and outcomes (1,2,3,4,5,6) on branches).
		B1	2nd throw fully correct with probabilities and outcomes identified. (Probabilities $\left(\text{all}\frac{1}{6}\right)$ and outcomes (1,2,3,4,5,6) on branches).
		2	

Question	Answer	Marks	Guidance
5(b)	5 comes from 1+4 or 5: $P(5) = \frac{1}{6}\times\frac{1}{6}+\frac{1}{6} = \frac{7}{36}$ 6 comes from 1+5 $P(6) = \frac{1}{6}\times\frac{1}{6} = \frac{1}{36}$ 7 comes from 1+6 or 6+1 $P(7) = \frac{1}{6}\times\frac{1}{6}+\frac{1}{6}\times\frac{1}{6} = \frac{2}{36}$ 8 comes from 6+2 $P(8) = \frac{1}{6}\times\frac{1}{6} = \frac{1}{36}$ 9 comes from 6+3 $P(9) = \frac{1}{6}\times\frac{1}{6} = \frac{1}{36}$	**B1**	P(5) or P(7) identified and correct unsimplified, accept if supported by correct scenarios shown or from tree diagram .
	$P(A) = \frac{7}{36}+\frac{1}{36}+\frac{2}{36}+\frac{1}{36}+\frac{1}{36}$	**M1**	Adding only the values from 5 correct scenarios.
	$=\frac{12}{36}=\frac{1}{3}$	**A1**	Scenarios identified (may be on tree diagram in **5(a)**), all probabilities seen, WWW AG.
		3	
5(c)	$P(B) = \frac{1}{3}$, $P(A\cap B) = \frac{6}{36}$	**M1**	Both identified and evaluated, consistent with *their* tree diagram or correct.
	$P(A)P(B) = \frac{1}{3}\times\frac{1}{3} = \frac{1}{9}$ $\frac{6}{36} \neq \frac{1}{9}$, so not independent	**A1**	P(A) × P(B) seen and evaluated, all notation present and correct. Correct conclusion WWW.
		2	

Question	Answer	Marks	Guidance
5(d)	$\mathrm{P}(B \mid A') = \dfrac{P(B \cap A')}{P(A')} = \dfrac{their\ \frac{6}{36}}{\frac{2}{3}}$	**B1**	$\frac{6}{36}$ oe as numerator of a fraction.
		M1	$\dfrac{their\ \frac{6}{36}\ \textbf{or}\ correct}{their\ 1-\frac{1}{3}\ \textbf{or}\ correct}$ seen, consistent with *their* tree diagram.
	$\frac{1}{4}$, 0.25	**A1**	
		3	

Question	Answer	Marks	Guidance
6(a)	5M0W $^{8}C_{5}$ [$\times$ $^{7}C_{0}$] = 56 4M1W $^{8}C_{4} \times {}^{7}C_{1} = 490$ 3M2W $^{8}C_{3} \times {}^{7}C_{2} = 1176$	**M1**	$^{8}C_{x} \times {}^{7}C_{5-x}$ for x = 1, 2, 3, 4,or 5
		B1	Outcome for 4M1W **or** 3M2W correct and identified, accept unsimplified.
		M1	Add 3 values of appropriate scenarios, no incorrect scenarios, no repeated scenarios, accept unsimplified. Addition may be implied by final answer.
	[Total =] 1722	**A1**	Value stated WWW.
	Alternative method for Question 6(a)		
	2M3W $^{8}C_{2} \times {}^{7}C_{3} = 980$ 1M4W $^{8}C_{1} \times {}^{7}C_{4} = 280$ 0M5W $^{8}C_{0} \times {}^{7}C_{5} = 21$	**M1**	$^{8}C_{x} \times {}^{7}C_{5-x}$ for x = 1, 2, 3, 4,or 5
		B1	Outcome for 2M3W **or** 1M4W correct and identified, accept unsimplified.
	[Total = $^{15}C_{5}$ – (980 + 280 + 21)] 3003 – (980 + 280 + 21)	**M1**	Subtract 3 values of appropriate scenarios from *their* identified total or correct, no incorrect scenarios, no repeated scenarios, accept unsimplified.
	[Total =] 1722	**A1**	Value stated WWW.
		4	
6(b)	$^{15}C_{3} \times {}^{12}C_{5}$ [$\times$ $^{7}C_{7}$] [= 455 × 792]	**M1**	$^{15}C_{r} \times q$, r = 3, 5, 7; q a positive integer >1
		M1	$^{15}C_{s} \times {}^{15-s}C_{t}$ [$\times$ $^{15-s-t}C_{u}$] s = 3,5,7; t = 3,5,7 $\neq s$; u = 3,5,7 $\neq s,t$
	360360	**A1**	Final answer. If A0 awarded **SC B1** for final answer 360360.
		3	

Question	Answer	Marks	Guidance
6(c)	**Method 1: Total number of arrangements with AB together – Arrangements with AB and FG together**		
	$6! \times 2 - 5! \times 2 \times 2$ $[= 1440 - 480]$	**M1**	$a! \times 2! \times b$, $a = 5, 6$; $b = 1,2$ seen.
		M1	Either $6! \times 2 - c$, $1 < c < 1440$ or $d - 5! \times 2 \times 2$, $1440 < d$
	960	**A1**	
	Method 2: arrangements with AB together with F and G not together.		
	$2 \times 4! \times 5 \times 4$	**M1**	$2 \times 4! \times e$, e positive integer >1
		M1	$f \times 5 \times 4$, f positive integer >1 condone $f \times 20$, $f \times {}^5C_2$, f positive integer >1
	960	**A1**	
		3	