



Cambridge International AS & A Level

CANDIDATE
NAME

CENTRE
NUMBER

--	--	--	--	--

CANDIDATE
NUMBER

--	--	--	--

* 1 4 7 2 6 4 1 3 2 3 *

MATHEMATICS

9709/33

Paper 3 Pure Mathematics 3

October/November 2021

1 hour 50 minutes

You must answer on the question paper.

You will need: List of formulae (MF19)

INSTRUCTIONS

- Answer **all** questions.
- Use a black or dark blue pen. You may use an HB pencil for any diagrams or graphs.
- Write your name, centre number and candidate number in the boxes at the top of the page.
- Write your answer to each question in the space provided.
- Do **not** use an erasable pen or correction fluid.
- Do **not** write on any bar codes.
- If additional space is needed, you should use the lined page at the end of this booklet; the question number or numbers must be clearly shown.
- You should use a calculator where appropriate.
- You must show all necessary working clearly; no marks will be given for unsupported answers from a calculator.
- Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place for angles in degrees, unless a different level of accuracy is specified in the question.

INFORMATION

- The total mark for this paper is 75.
- The number of marks for each question or part question is shown in brackets [].

This document has **20** pages. Any blank pages are indicated.

2 (a) Sketch the graph of $y = |2x - 3|$.

[1]

(b) Solve the inequality $|2x - 3| < 3x + 2$.

[3]

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

7 The equation of a curve is $\ln(x + y) = x - 2y$.

(a) Show that $\frac{dy}{dx} = \frac{x + y - 1}{2(x + y) + 1}$. [4]

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

(b) Find the coordinates of the point on the curve where the tangent is parallel to the x -axis. [3]

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

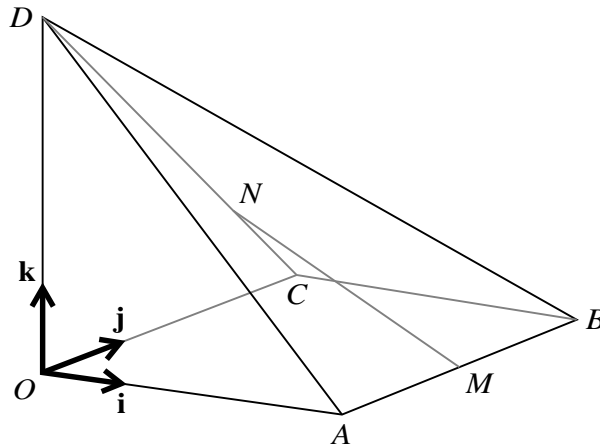
.....

.....

.....

.....

8



In the diagram, $OABCD$ is a pyramid with vertex D . The horizontal base $OABC$ is a square of side 4 units. The edge OD is vertical and $OD = 4$ units. The unit vectors \mathbf{i} , \mathbf{j} and \mathbf{k} are parallel to OA , OC and OD respectively.

The midpoint of AB is M and the point N on CD is such that $DN = 3NC$.

- (a) Find a vector equation for the line through M and N . [5]

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

(b) Show that the length of the perpendicular from O to MN is $\frac{1}{3}\sqrt{82}$.

[4]

.....
.....
.....
.....
.....
.....
.....
.....
.....
.....
.....
.....
.....
.....
.....
.....
.....
.....
.....
.....
.....
.....
.....
.....

- 10** A large plantation of area 20 km^2 is becoming infected with a plant disease. At time t years the area infected is $x \text{ km}^2$ and the rate of increase of x is proportional to the ratio of the area infected to the area not yet infected.

When $t = 0$, $x = 1$ and $\frac{dx}{dt} = 1$.

- (a)** Show that x and t satisfy the differential equation

$$\frac{dx}{dt} = \frac{19x}{20 - x} \quad [2]$$

.....

.....

.....

.....

.....

.....

.....

- (b)** Solve the differential equation and show that when $t = 1$ the value of x satisfies the equation $x = e^{0.9+0.05x}$. [5]

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

(c) Use an iterative formula based on the equation in part (b), with an initial value of 2, to determine x correct to 2 decimal places. Give the result of each iteration to 4 decimal places. [3]

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

(d) Calculate the value of t at which the entire plantation becomes infected. [1]

.....

.....

.....

.....

11 The complex number $-\sqrt{3} + i$ is denoted by u .

(a) Express u in the form $re^{i\theta}$, where $r > 0$ and $-\pi < \theta \leq \pi$, giving the exact values of r and θ . [2]

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

(b) Hence show that u^6 is real and state its value. [2]

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

- (c) (i) On a sketch of an Argand diagram, shade the region whose points represent complex numbers z satisfying the inequalities $0 \leq \arg(z - u) \leq \frac{1}{4}\pi$ and $\operatorname{Re} z \leq 2$. [4]

- (ii) Find the greatest value of $|z|$ for points in the shaded region. Give your answer correct to 3 significant figures. [2]

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

BLANK PAGE

Permission to reproduce items where third-party owned material protected by copyright is included has been sought and cleared where possible. Every reasonable effort has been made by the publisher (UCLES) to trace copyright holders, but if any items requiring clearance have unwittingly been included, the publisher will be pleased to make amends at the earliest possible opportunity.

To avoid the issue of disclosure of answer-related information to candidates, all copyright acknowledgements are reproduced online in the Cambridge Assessment International Education Copyright Acknowledgements Booklet. This is produced for each series of examinations and is freely available to download at www.cambridgeinternational.org after the live examination series.

Cambridge Assessment International Education is part of the Cambridge Assessment Group. Cambridge Assessment is the brand name of the University of Cambridge Local Examinations Syndicate (UCLES), which itself is a department of the University of Cambridge.