



## Cambridge International AS & A Level

CANDIDATE  
NAME

--

CENTRE  
NUMBER

--	--	--	--	--

CANDIDATE  
NUMBER

--	--	--	--



**MATHEMATICS**

**9709/32**

Paper 3 Pure Mathematics 3

**February/March 2021**

**1 hour 50 minutes**

You must answer on the question paper.

You will need: List of formulae (MF19)

### INSTRUCTIONS

- Answer **all** questions.
- Use a black or dark blue pen. You may use an HB pencil for any diagrams or graphs.
- Write your name, centre number and candidate number in the boxes at the top of the page.
- Write your answer to each question in the space provided.
- Do **not** use an erasable pen or correction fluid.
- Do **not** write on any bar codes.
- If additional space is needed, you should use the lined page at the end of this booklet; the question number or numbers must be clearly shown.
- You should use a calculator where appropriate.
- You must show all necessary working clearly; no marks will be given for unsupported answers from a calculator.
- Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place for angles in degrees, unless a different level of accuracy is specified in the question.

### INFORMATION

- The total mark for this paper is 75.
- The number of marks for each question or part question is shown in brackets [ ].

This document has **20** pages. Any blank pages are indicated.

**BLANK PAGE**



- 2 The polynomial  $ax^3 + 5x^2 - 4x + b$ , where  $a$  and  $b$  are constants, is denoted by  $p(x)$ . It is given that  $(x + 2)$  is a factor of  $p(x)$  and that when  $p(x)$  is divided by  $(x + 1)$  the remainder is 2.

Find the values of  $a$  and  $b$ .

[5]

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....



4 The variables  $x$  and  $y$  satisfy the differential equation

$$(1 - \cos x) \frac{dy}{dx} = y \sin x.$$

It is given that  $y = 4$  when  $x = \pi$ .

(a) Solve the differential equation, obtaining an expression for  $y$  in terms of  $x$ . [6]

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

(b) Sketch the graph of  $y$  against  $x$  for  $0 < x < 2\pi$ .

[1]









(b) Hence show that  $\int_a^{2a} f(x) dx = \ln 6.$  [4]

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

## 12

7 Two lines have equations  $\mathbf{r} = \begin{pmatrix} 1 \\ 3 \\ 2 \end{pmatrix} + s \begin{pmatrix} 2 \\ -1 \\ 3 \end{pmatrix}$  and  $\mathbf{r} = \begin{pmatrix} 2 \\ 1 \\ 4 \end{pmatrix} + t \begin{pmatrix} 1 \\ -1 \\ 4 \end{pmatrix}$ .

(a) Show that the lines are skew.

[5]

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

13

- (b) Find the acute angle between the directions of the two lines. [3]

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

8 The complex numbers  $u$  and  $v$  are defined by  $u = -4 + 2i$  and  $v = 3 + i$ .

(a) Find  $\frac{u}{v}$  in the form  $x + iy$ , where  $x$  and  $y$  are real. [3]

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

(b) Hence express  $\frac{u}{v}$  in the form  $re^{i\theta}$ , where  $r$  and  $\theta$  are exact. [2]

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

In an Argand diagram, with origin  $O$ , the points  $A$ ,  $B$  and  $C$  represent the complex numbers  $u$ ,  $v$  and  $2u + v$  respectively.

- (c) State fully the geometrical relationship between  $OA$  and  $BC$ . [2]

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

- (d) Prove that angle  $AOB = \frac{3}{4}\pi$ . [2]

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

9 Let  $f(x) = \frac{e^{2x} + 1}{e^{2x} - 1}$ , for  $x > 0$ .

- (a) The equation  $x = f(x)$  has one root, denoted by  $a$ .

Verify by calculation that  $a$  lies between 1 and 1.5.

[2]

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

- (b) Use an iterative formula based on the equation in part (a) to determine  $a$  correct to 2 decimal places. Give the result of each iteration to 4 decimal places.

[3]

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....



- (c) Find  $f'(x)$ . Hence find the exact value of  $x$  for which  $f'(x) = -8$ . [6]

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

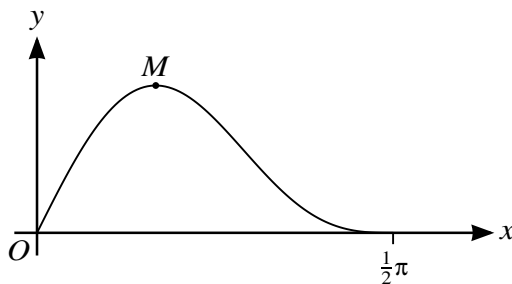
.....

.....

.....

.....

10



The diagram shows the curve  $y = \sin 2x \cos^2 x$  for  $0 \leq x \leq \frac{1}{2}\pi$ , and its maximum point  $M$ .

- (a) Using the substitution  $u = \sin x$ , find the exact area of the region bounded by the curve and the  $x$ -axis. [5]

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....



Additional Page

If you use the following lined page to complete the answer(s) to any question(s), the question number(s) must be clearly shown.

.....  
.....  
.....  
.....  
.....  
.....  
.....  
.....  
.....  
.....  
.....  
.....  
.....  
.....  
.....  
.....  
.....  
.....  
.....  
.....  
.....  
.....  
.....  
.....  
.....  
.....  
.....  
.....  
.....  
.....  
.....  
.....  
.....

Permission to reproduce items where third-party owned material protected by copyright is included has been sought and cleared where possible. Every reasonable effort has been made by the publisher (UCLES) to trace copyright holders, but if any items requiring clearance have unwittingly been included, the publisher will be pleased to make amends at the earliest possible opportunity.

To avoid the issue of disclosure of answer-related information to candidates, all copyright acknowledgements are reproduced online in the Cambridge Assessment International Education Copyright Acknowledgements Booklet. This is produced for each series of examinations and is freely available to download at [www.cambridgeinternational.org](http://www.cambridgeinternational.org) after the live examination series.

Cambridge Assessment International Education is part of the Cambridge Assessment Group. Cambridge Assessment is the brand name of the University of Cambridge Local Examinations Syndicate (UCLES), which itself is a department of the University of Cambridge.