



Cambridge International AS & A Level

CANDIDATE
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MATHEMATICS

9709/33

Paper 3 Pure Mathematics 3

May/June 2021

1 hour 50 minutes

You must answer on the question paper.

You will need: List of formulae (MF19)

INSTRUCTIONS

- Answer **all** questions.
- Use a black or dark blue pen. You may use an HB pencil for any diagrams or graphs.
- Write your name, centre number and candidate number in the boxes at the top of the page.
- Write your answer to each question in the space provided.
- Do **not** use an erasable pen or correction fluid.
- Do **not** write on any bar codes.
- If additional space is needed, you should use the lined page at the end of this booklet; the question number or numbers must be clearly shown.
- You should use a calculator where appropriate.
- You must show all necessary working clearly; no marks will be given for unsupported answers from a calculator.
- Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place for angles in degrees, unless a different level of accuracy is specified in the question.

INFORMATION

- The total mark for this paper is 75.
- The number of marks for each question or part question is shown in brackets [].

This document has **20** pages. Any blank pages are indicated.

3 The parametric equations of a curve are

$$x = t + \ln(t + 2), \quad y = (t - 1)e^{-2t},$$

where $t > -2$.

(a) Express $\frac{dy}{dx}$ in terms of t , simplifying your answer. [5]

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(b) Find the exact y-coordinate of the stationary point of the curve. [2]

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4 Let $f(x) = \frac{15 - 6x}{(1 + 2x)(4 - x)}$.

(a) Express $f(x)$ in partial fractions. [3]

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(b) Hence find $\int_1^2 f(x) dx$, giving your answer in the form $\ln\left(\frac{a}{b}\right)$, where a and b are integers. [4]

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- 5 (a) By first expanding $\tan(2\theta + 2\theta)$, show that the equation $\tan 4\theta = \frac{1}{2} \tan \theta$ may be expressed as $\tan^4 \theta + 2 \tan^2 \theta - 7 = 0$. [4]

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- 6 (a) By sketching a suitable pair of graphs, show that the equation $\cot \frac{1}{2}x = 1 + e^{-x}$ has exactly one root in the interval $0 < x \leq \pi$. [2]

- (b) Verify by calculation that this root lies between 1 and 1.5. [2]

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- (c) Use the iterative formula $x_{n+1} = 2 \tan^{-1} \left(\frac{1}{1 + e^{-x_n}} \right)$ to determine the root correct to 2 decimal places. Give the result of each iteration to 4 decimal places. [3]

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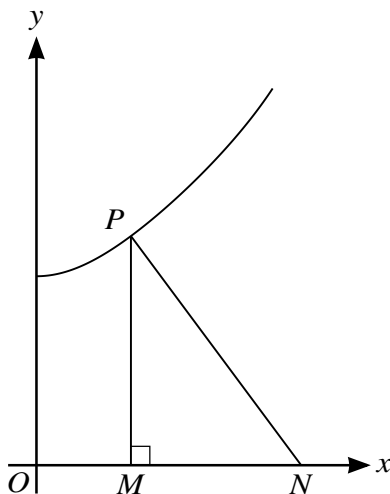
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For the curve shown in the diagram, the normal to the curve at the point P with coordinates (x, y) meets the x -axis at N . The point M is the foot of the perpendicular from P to the x -axis.

The curve is such that for all values of x in the interval $0 \leq x < \frac{1}{2}\pi$, the area of triangle PMN is equal to $\tan x$.

(a) (i) Show that $\frac{MN}{y} = \frac{dy}{dx}$. [1]

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(ii) Hence show that x and y satisfy the differential equation $\frac{1}{2}y^2 \frac{dy}{dx} = \tan x$. [2]

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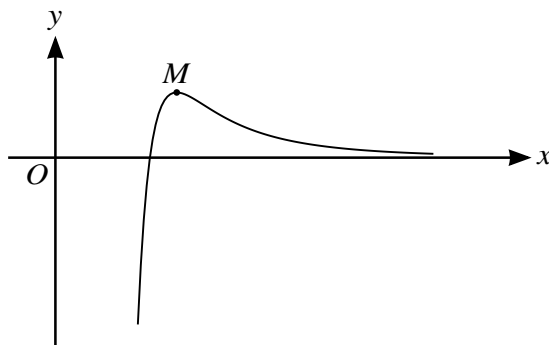
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The diagram shows the curve $y = \frac{\ln x}{x^4}$ and its maximum point M .

(a) Find the exact coordinates of M . [4]

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9 The quadrilateral $ABCD$ is a trapezium in which AB and DC are parallel. With respect to the origin O , the position vectors of A , B and C are given by $\vec{OA} = -\mathbf{i} + 2\mathbf{j} + 3\mathbf{k}$, $\vec{OB} = \mathbf{i} + 3\mathbf{j} + \mathbf{k}$ and $\vec{OC} = 2\mathbf{i} + 2\mathbf{j} - 3\mathbf{k}$.

(a) Given that $\vec{DC} = 3\vec{AB}$, find the position vector of D . [3]

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(b) State a vector equation for the line through A and B . [1]

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- 10 (a)** Verify that $-1 + \sqrt{2}i$ is a root of the equation $z^4 + 3z^2 + 2z + 12 = 0$. [3]

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- (b)** Find the other roots of this equation. [7]

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Additional Page

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