



Cambridge International AS & A Level

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MATHEMATICS

9709/33

Paper 3 Pure Mathematics 3

May/June 2020

1 hour 50 minutes

You must answer on the question paper.

You will need: List of formulae (MF19)

INSTRUCTIONS

- Answer **all** questions.
- Use a black or dark blue pen. You may use an HB pencil for any diagrams or graphs.
- Write your name, centre number and candidate number in the boxes at the top of the page.
- Write your answer to each question in the space provided.
- Do **not** use an erasable pen or correction fluid.
- Do **not** write on any bar codes.
- If additional space is needed, you should use the lined page at the end of this booklet; the question number or numbers must be clearly shown.
- You should use a calculator where appropriate.
- You must show all necessary working clearly; no marks will be given for unsupported answers from a calculator.
- Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place for angles in degrees, unless a different level of accuracy is specified in the question.

INFORMATION

- The total mark for this paper is 75.
- The number of marks for each question or part question is shown in brackets [].

This document has **20** pages. Blank pages are indicated.

- 3 (a) Show that the equation

$$\ln(1 + e^{-x}) + 2x = 0$$

can be expressed as a quadratic equation in e^x . [2]

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- (b) Hence solve the equation $\ln(1 + e^{-x}) + 2x = 0$, giving your answer correct to 3 decimal places. [4]

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4 The equation of a curve is $y = x \tan^{-1}\left(\frac{1}{2}x\right)$.

(a) Find $\frac{dy}{dx}$. [3]

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(b) The tangent to the curve at the point where $x = 2$ meets the y -axis at the point with coordinates $(0, p)$.

Find p . [3]

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- 6 (a) By sketching a suitable pair of graphs, show that the equation $x^5 = 2 + x$ has exactly one real root. [2]

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- (b) Show that if a sequence of values given by the iterative formula

$$x_{n+1} = \frac{4x_n^5 + 2}{5x_n^4 - 1}$$

converges, then it converges to the root of the equation in part (a). [2]

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(c) Use the iterative formula with initial value $x_1 = 1.5$ to calculate the root correct to 3 decimal places. Give the result of each iteration to 5 decimal places. [3]

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7 Let $f(x) = \frac{2}{(2x - 1)(2x + 1)}$.

(a) Express $f(x)$ in partial fractions. [2]

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(b) Using your answer to part (a), show that

$$(f(x))^2 = \frac{1}{(2x - 1)^2} - \frac{1}{2x - 1} + \frac{1}{2x + 1} + \frac{1}{(2x + 1)^2}. \quad [2]$$

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8 Relative to the origin O , the points A , B and D have position vectors given by

$$\vec{OA} = \mathbf{i} + 2\mathbf{j} + \mathbf{k}, \quad \vec{OB} = 2\mathbf{i} + 5\mathbf{j} + 3\mathbf{k} \quad \text{and} \quad \vec{OD} = 3\mathbf{i} + 2\mathbf{k}.$$

A fourth point C is such that $ABCD$ is a parallelogram.

- (a) Find the position vector of C and verify that the parallelogram is not a rhombus. [5]

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- (b) Find angle BAD , giving your answer in degrees. [3]

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- (c) Find the area of the parallelogram correct to 3 significant figures. [2]

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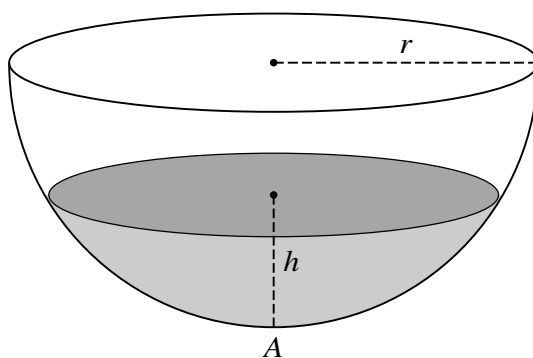
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- (b) On a sketch of an Argand diagram, shade the region whose points represent complex numbers z satisfying the inequalities

$$|z - 2 - 2i| \leq 2, \quad 0 \leq \arg z \leq \frac{1}{4}\pi \quad \text{and} \quad \operatorname{Re} z \leq 3. \quad [5]$$

10



A tank containing water is in the form of a hemisphere. The axis is vertical, the lowest point is A and the radius is r , as shown in the diagram. The depth of water at time t is h . At time $t = 0$ the tank is full and the depth of the water is r . At this instant a tap at A is opened and water begins to flow out at a rate proportional to \sqrt{h} . The tank becomes empty at time $t = 14$.

The volume of water in the tank is V when the depth is h . It is given that $V = \frac{1}{3}\pi(3rh^2 - h^3)$.

(a) Show that h and t satisfy a differential equation of the form

$$\frac{dh}{dt} = -\frac{B}{2rh^{\frac{1}{2}} - h^{\frac{3}{2}}},$$

where B is a positive constant.

[4]

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Additional Page

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